

THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2013
Campus: City

COMPUTER SCIENCE

Algorithms and Data Structures

(Time allowed: 40 minutes)

NOTE:

- Enter your name and student ID into the Teleform sheet FIRST.
- THEN: Attempt *all* questions!
- All questions have ONE correct answer.
- DO NOT tick two answers as correct for the same question.
- If you believe that there is an error in a question (multiple correct answers or no correct answer), select the answer you believe was intended as the correct one and contact the exam room supervisor after the exam.
- Keep your question book. Writing on the question book will not be marked.
- Use of calculators is NOT permitted.
- Good luck!

1. Which of the following functions is **not** $O(n^2)$?

- A. $f(n) = (n^2 + 2)(2n + 7) - n^3$
- B. $f(n) = e^3 n^2$
- C. $f(n) = n(2n + 7) - 1$
- D. $f(n) = n^2(n + 7) - n^3$
- E. $f(n) = e^{-n}$

2. Which of the following functions is $\Theta(n^2)$?

- A. $f(n) = n^2 \sin(15n)$
- B. $f(n) = n(2n + 7) - 1$
- C. $f(n) = n^3 + n$
- D. $f(n) = \frac{n(n+1)}{2n}$
- E. $f(n) = n^3 + n^2$

3. If $g(n)$ is $O(n)$ and $h(n)$ is $\Theta(n^2)$, which of the following statements about the function $f(n) = g(n) + h(n)$ is true?

- A. $f(n)$ is $\Theta(n^2)$
- B. $f(n)$ is $\Omega(n^3)$
- C. $f(n)$ is $O(n)$
- D. $f(n)$ is $\Theta(n^3)$
- E. $f(n)$ is $\Theta(n)$

4. How many comparisons does mergesort make when merging two lists of length n each?

- A. At most $2n - 1$
- B. At most $n - 1$
- C. $n \log n$
- D. At most n
- E. Exactly $2n - 1$

5. A certain sorting algorithm sorts its list as follows: Select an element on the existing list, then move all elements with a higher key to the right of the selected element, and all elements with a lower key to the left of the selected element. Then sort the lists to the left/right of the selected element each by recursively applying the algorithm again. This sorting algorithm is known as:

- A. quick
- B. heap
- C. selection
- D. merge
- E. insertion

6. Which of the following statements about selection sort is **not** true?

- A. Selection sort has an average time complexity of $\Omega(n^2)$
- B. Selection sort has a best-case time-complexity that is better than cubic
- C. Selection sort always takes the same number of comparisons for a given input size
- D. Selection sort sorts pre-sorted lists of the same length in the same time regardless of sorting direction
- E. Selection sort has a worst-case time complexity of $\Omega(n^2 \log n)$

7. An algorithm takes approximately 1 second for a input of size $n = 10$, approximately 200 seconds for an input of size $n = 100$, and approximately 20000 seconds for $n = 1000$. The algorithm's observed time complexity is:

- A. numeric
- B. linear
- C. cubic
- D. exponential
- E. quadratic

8. Before Facebook, a COMPSCI220 class had an idea. They wanted to produce a book as a souvenir, with one page for each student in the class. The book was to be printed and a copy given to each student. Assume that the cost of printing is proportional to the amount of paper used. If so, the cost of printing the book for a class of n students would have been:

- A. $O(n \log n)$
- B. $\Theta(n^2)$
- C. $\Theta(n^3)$
- D. $O(n)$
- E. $\Omega(n^3)$

9. The Table Abstract Data Type (ADT) may be implemented as a hash table, or as a balanced tree. For which of the following Table operations is a hash table slower than a balanced tree?

- A. Enumerate
- B. Delete
- C. Insert
- D. Update
- E. Search

10. What property of a maximum-heap allows a non-recursive implementation of heapsort?

- A. A heap can be stored simply in an array of nodes
- B. Inserting or deleting heap nodes is a $\Theta(\log n)$ operation
- C. The highest-value key is always at the root of a maximum-heap
- D. Every maximum-heap node has a key value \geq the key values of its child nodes
- E. A heap is a complete binary tree

11. Which recurrence relation has the exact solution $T(n) = 2^n + 1$ for $T(0) = 2$?

- A. $T(n) = 2(T(n-1) + 1)$
- B. $T(n) = 2T(n-1) - 1$
- C. $T(n) = T(n-1) + 1$
- D. $T(n) = T(n-1) + 2$
- E. $T(n) = 2T(n-1) + 1$

12. Which of the following statements about m -ary search trees is **not** true?

- A. They cannot be used when all their nodes are held in memory
- B. Their nodes must be larger so that they can hold more keys
- C. They reduce the amount of disk I/O when working with very large datasets
- D. Searches may have to make $m - 1$ comparisons at each node
- E. Their height is less than that of a binary tree with the same number of keys

13. You test an algorithm that is known to exhibit cubic time complexity. With an input size of $n = 100$, the algorithm runs for a day. As you double the input size ($n = 200$), the algorithm takes 8 days to complete. How long is input size 1000 likely to take?

- A. two weeks
- B. just over a year
- C. almost three years
- D. nine months
- E. three months

14. Which of the following is not an elementary operation?

- A. Iterating five times through a for-loop that contains a single elementary operation
- B. Checking whether an input string contains the letter 'x'
- C. Incrementing an integer in Java
- D. Comparing two Boolean variables for equality
- E. Printing the string 'HELLO WORLD'

15. For a hash table that uses an array with n elements, why do we use the remainder $h(k) = k \% n$ as a hash value?

- A. This hash function will be faster than taking parts of key k and adding them
- B. Division is the most common basic method used in hash functions
- C. We may want to use $(k/n) \bmod n$ as our double hashing decrement
- D. Doing so guarantees that $h(k)$ lies within the hash array
- E. Computing a remainder is faster than computing a quotient

16. Which of the following sort algorithms is *stable* and has the same complexity for worst, average and best cases of its input data?

A. mergesort
B. shellsort
C. insertionsort
D. quicksort
E. heapsort

17. Which of the following pairs of time complexities is most difficult to tell apart by observation?

A. $\Theta(n)$ and $\Theta(n \log n)$
B. $\Theta(2^n)$ and $\Theta(n^2)$
C. $\Theta(\log n)$ and $\Theta(n \log n)$
D. $\Theta(n^2)$ and $\Theta(n \log n)$
E. $\Theta(n)$ and $\Theta(n^2)$

18. An *inversion* in an array of integers is a pair of array elements $a[i]$ and $a[j]$ for which $i < j$ and $a[i] > a[j]$. Which of the following statements is **not** true?

A. A sorted array has no inversions
B. Exactly one inversion may be removed if we swap $a[i]$ and $a[i + 1]$
C. Swapping $a[i]$ and $a[i + gap]$ may remove more than one inversion
D. Shellsort can eliminate only *gap* inversions for each gap size it uses
E. Shellsort's performance depends on its sequence of gap sizes

Working page 1

Working page 2

