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Computer Games Technology

—Fuzzy Maps—

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Imprecision and Uncertainty in Maps

- Areas of a map are often imprecise:
  - Where does the urban area end and the rural area start?
  - Where exactly is the shore line of the ocean?
- Knowledge about areas of a map is often uncertain:
  - Which area has been flooded by the last rainstorm?
  - Which territory is held by enemy troops?

Fuzzification of Maps

- A grid cell in a crisp raster map either has a certain feature or does not have that feature:
  - If the feature is present, the cell has a value of 1.
  - If the feature is not present, it has a value of 0.
- A grid cell in a fuzzy raster map has the feature to a certain degree:
  - The value of the cell is from the interval $[0, 1]$.
  - The greater the value of the cell, the higher the degree to which the feature is present.
- The set of all grid cells define a fuzzy set.

Example: Downtown on a City Map

- The grid cells on a city map are usually labeled with character–digit combinations:

$$\mathsf{A1}, \mathsf{A2}, \mathsf{A3}, \ldots, \mathsf{B1}, \mathsf{B2} \ldots \mathsf{L1}, \ldots, \mathsf{M1}, \ldots$$

- Some location are more considered to be in downtown than others.

- This fact can be represented by a fuzzy map that results in a fuzzy set like the following:

$$\tilde{A} = \{\langle \mathsf{M5}, 1\rangle, \langle \mathsf{M4}, 0.8\rangle, \langle \mathsf{L5}, 0.8\rangle, \ldots, \langle \mathsf{L4}, 0.7\rangle, \ldots\}$$

Operations on Maps

1. A fuzzy map can be inverted by computing the complement of the underlying fuzzy set, i.e., by replacing the membership grade m of each grid cell with $1 - m$.

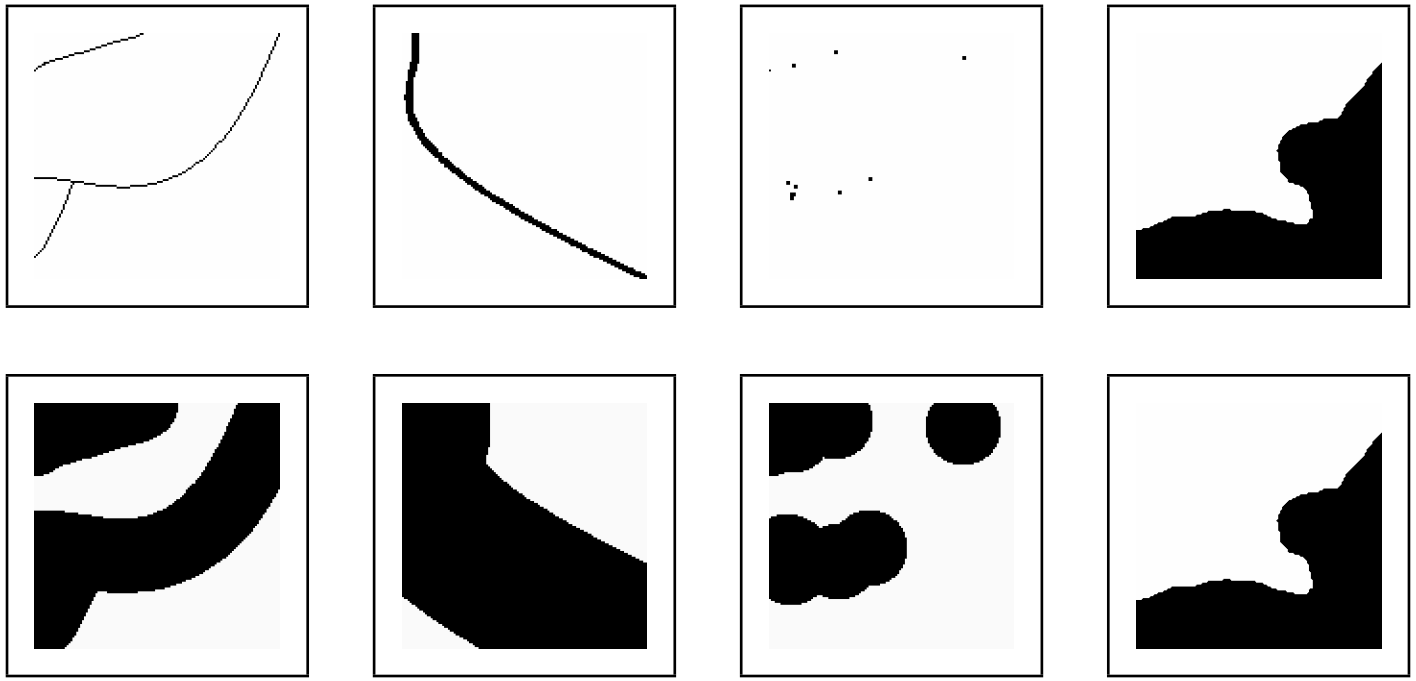
2. Two fuzzy maps can be combined by computing the intersection or union of the underlying fuzzy sets, i.e., by applying the t-norm or s-norm to each pair of corresponding grid cells.

3. A fuzzy map can be buffered by increasing the membership grade of grid cells.

Buffering Fuzzy Maps

- A crisp map is buffered by changing the value of a location x to 1, if there is at least one other location in the vicinity of x that has a value of 1.
- A fuzzy map can be buffered by applying the s-norm to the current membership grade of x and the membership grades of all locations in the vicinity of x.
- This type of buffer operation is still crisp!

Maps Buffered in a Crisp Way



Locations of roads, water, residences, and native forest.

Buffering Depending on Proximity

- Buffer operation should depend on the proximity of the locations under consideration.

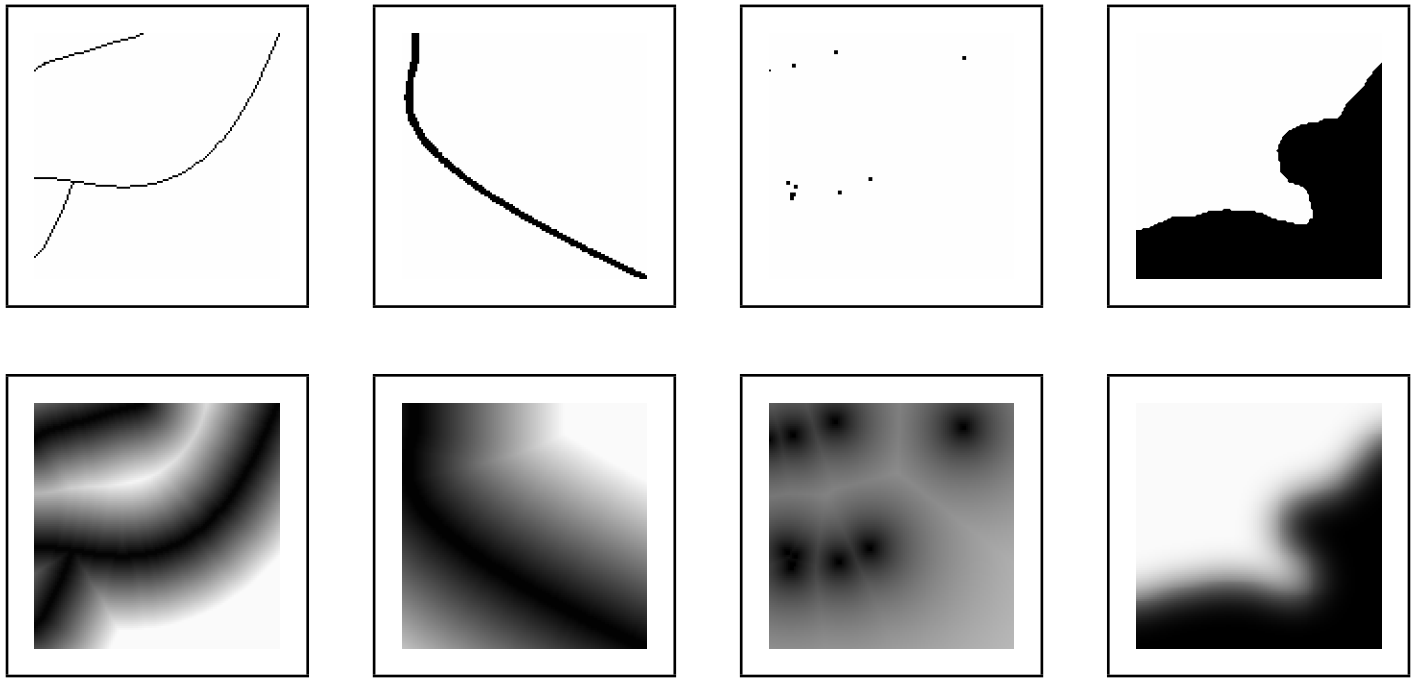
- For example, if there is an area on the map with very high membership grades, then the buffer operation should assign:
  - High membership grades to locations that are very close to that area
  - Medium high membership grades to locations close to the area
  - Low membership grades to locations further away

Neighborhood Relations and Distance Measures

A neighbor relation induces a distance measure in the following way (where x_0 and x_1 are two arbitrary locations):

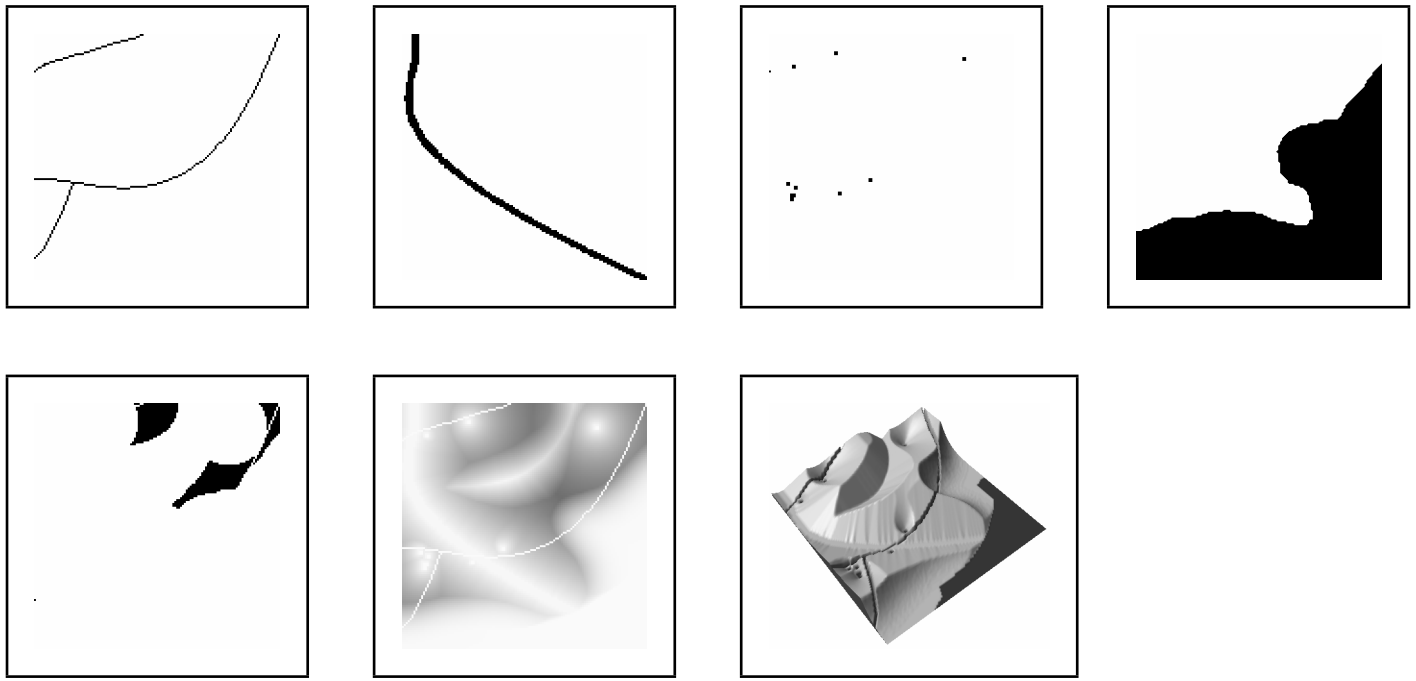
$$\delta(x_0, x_1) = \begin{cases} 0, \text{ if } x_0 = x_1 \\ \min\{\delta(x, x_1) \mid x \text{ is a neighbor of } x_0\} + 1, \text{ otherwise} \end{cases}$$

Maps Buffered in a Fuzzy Way



Locations of roads, water, residences, and native forest.

Map Overlay



Locations close to roads, not close to rivers or residences, and not on native forest.

Complexity of the Operations

- Performing a set operation (complement, union, and intersection) on fuzzy raster maps is straightforward, as they are defined cell-wise.
- Unlike the set operations, buffer operations cannot be defined cell-wise.
- They usually involve a number of cells that are in the same neighborhood or even all the cells of the raster map.

Brute-Force Buffering

- A brute-force algorithm for buffering a fuzzy raster map visits each cell of the map and updates its membership grade based on the membership grades of the neighboring cells.

- If any of the membership grades is changed, the algorithm repeats the updating process until all membership grades become stable.

- Since the algorithm revisits each cell when repeating the updating process, even the ones whose neighbors have not been changed in the previous iteration, it performs many unnecessary checks.

Local Propagation Strategy

- An improved algorithm keeps track of the changed cells and revisits a cell only if at least one of its neighbors has been changed.
- The membership grade of a cell is propagated to the neighbors of the cell, which are then put onto the list of cells to be visited in the future.
- The local propagation algorithm is guaranteed to terminate, since there is only a finite number of cells and therefore only a finite number of membership grades.

Ordered-Cells Strategy

- During local propagation, a large number of cells may be revisisted before the list of cells finally becomes empty.
- The reason is that it is always possible for a cell to receive a larger membership grade because of the buffer operation.
- To prevent this from happening, we can select a cell from the list with a maximum membership grade.
- The grade for such a cell cannot be increased by any buffer operation and therefore buffering the neighbors of a cell with maximum membership grade results in assigning a final membership grade to the neighbors of that cell.

Further Strategies

- Hardware-based algorithms that use the z-buffer of graphics cards to buffer fuzzy maps.

- Source-cell memory algorithms, which for each cell keep track of the currently highest membership grade and its origin.

Some Basics on Graphics Hardware

- The frame buffer stores color information for each pixel of an image.
- The z-buffer (or depth buffer) stores the depth of the closest object found so far that covers that pixel.
- Before a pixel is given the color of a new object, the depth of the object at that particular pixel is computed and compared with the depth stored in the z-buffer.
- If the new object is closer, its color will be stored in the frame buffer and its depth value in the z-buffer.

Membership Grades in the Z-Buffer

- Use the z-buffer to mimic the current fuzzy map.
- Take the depth of a given pixel (location) to uniquely represent its membership grade.
- A close depth indicates a high membership grade, whereas a far depth stands for a low membership grade.

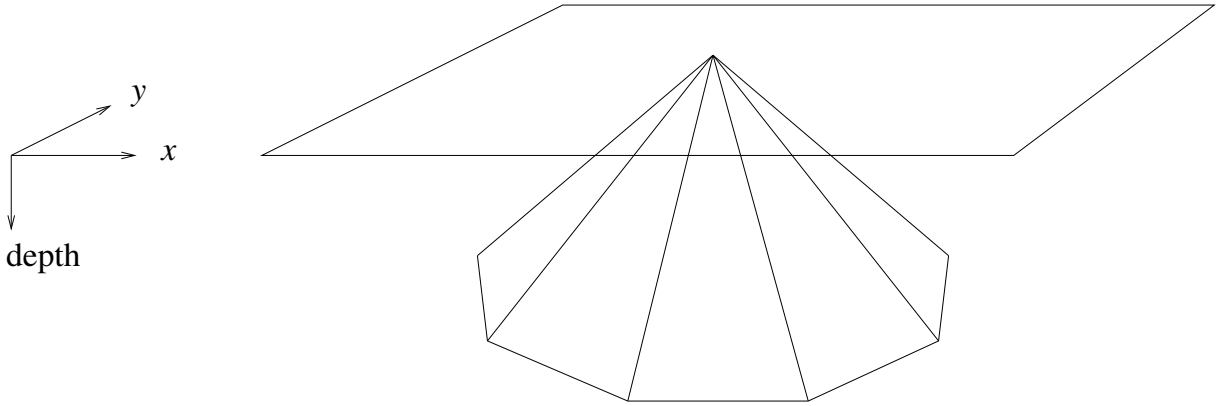
Buffering Membership Grades

- To buffer the membership grade represented by the depth of the pixel, render an object that corresponds to the buffer function applied that membership grade.

- The object is a right circular cone if the buffer function is of type $\max\{0, m - kd)$, where the factor k determines how fast the original membership m diminishes with the distance d.

- A membership grade of 1 is mapped to a depth of zero, whereas a membership grade of 0 is mapped to a depth of 1.

Approximating Buffering Cones



- If the distance between cells is defined based on the vertex neighbor relation, $\max\{0, m - kd)$ can be approximated perfectly by a pyramid that is aligned parallel to the grid structure.

- If distance is defined based on the edge neighbor relation, this is also possible, but the pyramid is rotated by 45 degrees.