

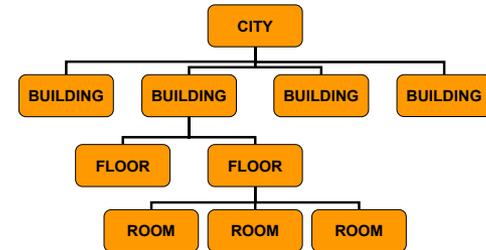
## Space Subdivision

- Divides space into smaller regions
- May be hierarchical
- Can be used in 2D or 3D
- Only load/process/render relevant regions
- Several categories:
  - Structural
  - Partitions
  - Bounding volumes

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## Structural subdivision

- World organised according to the natural structure and sub-structure of items in the scene



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## Structural subdivision (cont.)

```
class City
{ // A city is a bunch of buildings
  vector<Building> buildings;
};
class Building
{ // A building has some floors
  vector<Floor> floors;
  City* city;
  BBox boundingbox;
};
class Floor
{ // A floor has a number of rooms
  vector<Room> rooms;
  Building* building;
  int floor_number;
};
class Room
{ // A room has some stuff in it, and access to other rooms
  Floor* floor;
  vector<Room*> adjacent_rooms;
  ...
};
```

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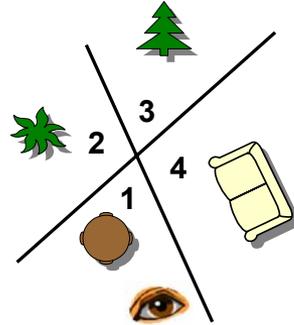
## Partition planes

- Space is subdivided by placing on or more partition planes
- A partition plane divides space into three:
  1. Region in front of the plane
  2. Region behind the plane
  3. Region on the plane
- Some subdivision schemes use only two regions:
  1. Region in front of the plane
  2. Region behind the plane
  - Region on the plane is considered to be either in front or behind (but be consistent!)

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## Partition planes

- Partition planes can define a depth order
  - Everything on the other side of a partition plane from the camera is further away than everything on the same side of the partition plane as the camera

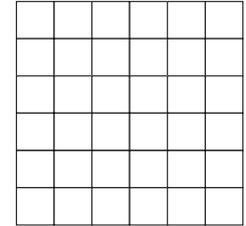


Anything in {1,2} appears in front of anything in {3,4}  
Anything in {1,4} appears in front of anything in {2,3}  
1 is always in front of {2,3,4}  
{2, 4} is always in front of 3  
Nothing in 2 occludes anything in 4, and vice versa  
Depth order: 1 (nearest), 2&4 (any order), 3 (furthest)

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## Regular grid

- Place axis-aligned partition planes at regular intervals
- Forms a regular grid over world
- Each grid square is a small block of the world
- Each block contains info about what is in the block
- Ideal for map-based worlds
- Easily implemented as a 2D array
- Usually the grid is square



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## Regular grid (cont.)

```
// A block contains whatever
class Block
{
    list<Monster> monsters;
    list<Model> objects;
    ...
};

// The world is a regular 2D array of blocks
Block world[WORLD_BLOCKS][WORLD_BLOCKS];
```

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## Regular grid (cont.)

```
class Player
{
    float world_x, world_y; // World position of player
    int block_x, block_y; // Block coordinates
};

Player::Move(float dx, float dy)
{
    world_x += dx;
    block_x = (int)floor(world_x);
    world_y += dy;
    block_y = (int)floor(world_y);
}
```

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## Regular grid (cont.)

```
const int subbits = 8;

class Player
{
    unsigned int world_x, world_y; // World coordinates

    unsigned int block_x, block_y; // Block coordinates
    unsigned int sub_x, sub_y; // Sub-block coordinates
};

Player::Move(int dx, int dy)
{
    world_x += dx;

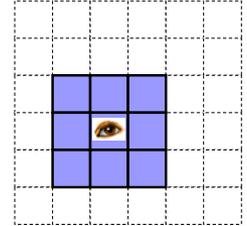
    block_x = world_x >> subbits; // Remove lower 8 bits
    sub_x = world_x & ((1 << subbits) - 1); // Mask out upper bits

    ... // Ditto for y
}
```

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## Regular grid (cont.)

- Only process blocks near the player:
  - Geometry upload
  - Monster AI
- Far-away blocks can be removed
- Incremental loading as player moves
- Limit memory usage



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## Regular grid (cont.)

```
Render(Player& player, int range)
{
    int bx = player.block_x;
    int by = player.block_y;
    for(int y = by-range; y <= by+range; y++)
    {
        for(int x = bx-range; x <= bx+range; x++)
        {
            if(IsInWorld(x, y))
            {
                Block& block = FetchBlock(x, y);
                block.Render();
            }
        }
    }
}
```

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## Regular grid (cont.)

```
Block& FetchBlock(int x, int y)
{
    if(!BlockIsInCache(x, y))
    {
        if(CacheIsFull())
            RemoveLeastUsedBlockFromCache();
        Block block = ReadBlockFromDisk(x, y);
        AddBlockToCache(block);
    }

    return BlockFromCache(x, y);
}
```

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## Quadtrees

- Regular grid in a hierarchy
- Divides world into four blocks using two axis-aligned partition planes
- Divides each block into four sub-blocks
- ... and so on
- Each block is linked to its four sub-blocks
  - Forms a tree with four children per node

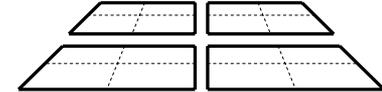
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## Quadtrees (cont.)

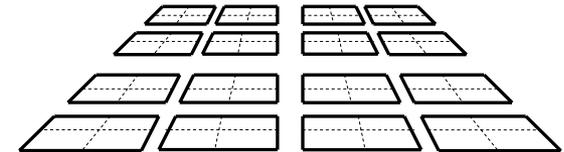
- Level 0



- Level 1



- Level 2



- ...

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## Quadtrees (cont.)

```
class Block
{
    Block* parent;
    Block* children[4];
    BBox boundingbox;
    ...
};

// The world is the top-level block
Block world;
```

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## Quadtrees (cont.)

- Number of blocks at a level:  
 $n(\text{level}) = 4^{\text{level}}$
- Total number of blocks for all levels:  
$$N(\text{level}) = 4^0 + 4^1 + 4^2 + \dots + 4^{\text{level}}$$
$$= (4^{(\text{level}+1)} - 1) / 3$$
- So overhead for using hierarchy instead of regular grid is:

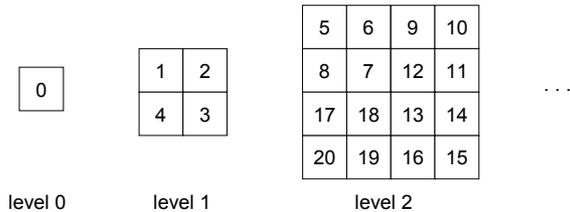
$$N(\text{level}) / n(\text{level}) < 4/3$$

No more than 33% extra blocks

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## Quadrees (cont.)

- Storing a quadtree without pointers
  - Can store as a single array
  - Enumerate the blocks as follows:



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## Quadrees (cont.)

- Block index of first block in level:
 
$$\text{start}(\text{level}) = N(\text{level}) - n(\text{level})$$

$$= (4^{\text{level}} - 1) / 3$$
- Block index of child block  $c \in \{0, 1, 2, 3\}$  of block  $b$ :
 
$$\text{child}(b, c) = \text{start}(\text{level}+1) + (b - \text{start}(\text{level})) * 4 + c$$

$$= (4^{(\text{level}+1)} - 1) / 3 + 4b - 4(4^{\text{level}} - 1) / 3 + c$$

$$= 4b + (4^{(\text{level}+1)} - 1) / 3 - (4^{\text{level}} - 1) / 3 + c$$

$$= 4b + 1 + c$$
- Block index of parent of block  $b$ :
 
$$\text{parent}(b) = (b - 1) / 4 \quad (\text{integer division})$$

Block  $b$  is child  $(b - 1) \% 4$  of parent

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## Quadrees (cont.)

- Level number of block  $b$ :
  - Not fast but simple way
 
$$\text{level}(b) = \text{floor}((3(b+1))^{1/4})$$
  - The “at least we’re avoiding floats” way:
    - Need to find largest power of  $4 \leq 3(b+1)$

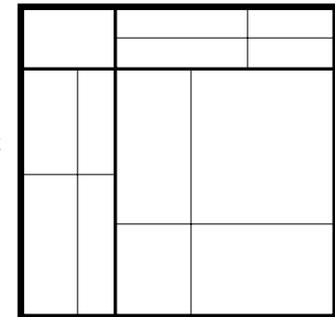
```

b' = 3 * (b + 1)
    if (b' & (3 << 30))   level(b) = 15
    else if (b' & (3 << 28)) level(b) = 14
    else if (b' & (3 << 26)) level(b) = 13
    . . .
    else if (b' & (3 << 4))   level(b) = 2
    else if (b' & (3 << 2))   level(b) = 1
    else                   level(b) = 0
// b' is never 0
    
```

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## Quadrees (cont.)

- Subdivision does not always need to be done in the middle
  - Split according to some criteria, e.g. same number of objects in each quadrant
- Subdivision does not always need to be done to the same level
  - E.g. stop when number of objects in quadrant less than threshold



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## Quadtrees (cont.)

```
class Block
{
    Pos bbox[2];                // Bounding box (x0,y0)-(x1,y1) of block
    Block* parent;
    Block* children[4];
};

Block* MakeBlock(float x0, float y0, float x1, float y1, Block *parent)
{
    Block* block = new Block();
    block->bbox[0].x = x0; block->bbox[0].y = y0;
    block->bbox[1].x = x1; block->bbox[1].y = y1;
    block->parent = parent;

    Pos split;
    if(NeedToSubdivide(block, split)) // Determine if block needs to be subdivided
    { // and if so, where to split
        block->children[0] = MakeBlock(x0, y0, split.x, split.y, block);
        block->children[1] = MakeBlock(split.x, y0, x1, split.y, block);
        block->children[2] = MakeBlock(split.x, split.y, x1, y1, block);
        block->children[3] = MakeBlock(x0, split.y, split.x, y1, block);
    }

    return block;
}
```

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## Quadtrees (cont.)

```
// Find the block that a given point is in
Block* FindBlock(Block* block, Pos p)
{
    for(int c = 0; c < 4; c++)
    {
        if(block->children[c])
            if(IsInBBox(p, block->children[c]->bbox))
                return FindBlock(block->children[c], p);
    }

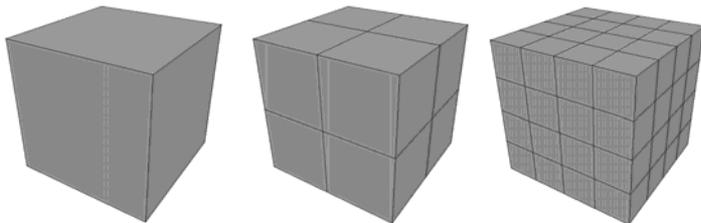
    return block;
}

bool IsInBBox(Pos p, Pos bbox[2])
{
    return (bbox[0].x <= p.x) && (p.x < bbox[1].x) &&
        (bbox[0].y <= p.y) && (p.y < bbox[1].y);
}
```

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## Octrees

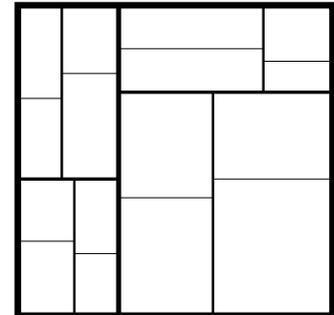
- Like quadtrees, extended to 3D
- Subdivides a volume
- Divides a cube into 8 sub-cubes using three axis-aligned partition planes



...

## kD-Trees

- A hierarchical binary 2D subdivision
- Splits world by alternating between axis-aligned partition planes
  - E.g. first split along Y axis, then X axis, then Y axis, etc.
- Obvious extension to 3D



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## kD-Trees (cont.)

```
class Block
{
    Pos bbox[2];           // Bounding box (x0,y0)-(x1,y1) of block
    Block* parent;
    Block* children[2];
};

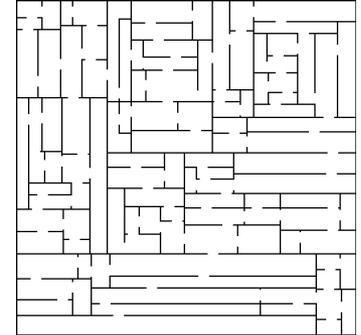
Block* MakeBlock(float x0, float y0, float x1, float y1, Block *parent, int direction)
{
    Block* block = new Block();
    block->bbox[0].x = x0; block->bbox[0].y = y0;
    block->bbox[1].x = x1; block->bbox[1].y = y1;
    block->parent = parent;

    Pos split;
    if(NeedToSubdivide(block, direction, split))
    {
        if(direction == X_AXIS)
        {
            block->children[0] = MakeBlock(x0, y0, split.y, block, Y_AXIS);
            block->children[1] = MakeBlock(split.y, x1, y1, block, Y_AXIS);
        }
        else if(direction == Y_AXIS)
        {
            block->children[0] = MakeBlock(x0, y0, split.x, y1, block, X_AXIS);
            block->children[1] = MakeBlock(split.x, x1, y1, block, X_AXIS);
        }
    }
    return block;
}
```

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## kD-Trees (cont.)

- Making a random maze with kD-trees
  - Place wall horizontally or vertically
  - Make door in wall
  - Repeat for sub-blocks
- Only place wall if it begins and ends at a wall
- Guarantees that there is exactly one path from any room to any other room



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## BSP trees

- Binary Space Partition
- Hierarchical, like quadtrees and kD-trees
- Splits a volume into two sub-volumes by plane
- Split each sub-volume into two
- ... and so on
- Generalisation of kD-trees, as the plane can be arbitrary, not just axis-aligned

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## BSP trees (cont.)

- BSP tree used primarily to organise polygons
  - Partition plane for a tree node chosen to be the plane of a polygon
  - Add polygons on the plane to the node
  - Add polygons in front of the plane to one child
  - Add polygons behind the plane to the other child
  - Split polygons which fall across the plane
- Partition plane does not *need* to be co-incident with a polygon plane

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## BSP trees (cont.)

```
struct BSPnode
{
    Plane plane;           // The partition plane
    list polygons;        // Polygons on the plane
    BSPnode *front, *back; // Links to the two children
};

void BuildTree(BSPnode *node, list polygons)
{
    Polygon *poly = polygons.GetPolygon(); // Take poly from list
    node->plane = plane->GetPlane();       // Get the plane of poly
    node->polygons.AddPolygon(poly);       // Add poly to node

    list backpolys, frontpolys;
```

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## BSP trees (cont.)

```
while(poly = polygons.GetPolygon()) // Go through remaining polys
{
    int type = ClassifyPoly(poly, plane);
    if(type == COINCIDENT) // Poly on plane
        node->polygons.AddPolygon(poly);
    else if(type == BEHIND) // Poly behind plane
        backpolys.AddPolygon(poly);
    else if(type == INFRONT) // Poly in front of plane
        frontpolys.AddPolygon(poly);
    else if(type == SPANNING) // Poly across plane
    {
        Polygon *frontpiece, *backpiece;
        SplitPolygon(poly, plane, frontpiece, backpiece);
        backpolys.AddPolygon(backpiece);
        frontpolys.AddPolygon(frontpiece);
    }
}
```

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## BSP trees (cont.)

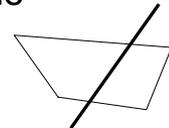
```
if(!frontpolys.Empty())
{
    node->front = new BSPnode();
    BuildTree(node->front, frontpolys);
}
if(!backpolys.Empty())
{
    node->back = new BSPnode();
    BuildTree(node->back, backpolys);
}
}
```

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## BSP trees (cont.)

### ■ Splitting a polygon by a plane

- Convex polygons: easy
  - Always creates 2 convex polys



- Concave polygons: hard
  - Can create any number of concave polys
  - Avoid using concave polygons



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## BSP trees (cont.)

```

void SplitPolygon(Polygon *poly, Plane plane, Polygon *&front, Polygon *&back)
{
    int numv = poly->NumVertices()
    Point p1, p2;
    float sidel, side2;

    front = new Polygon(); back = new Polygon();
    p1 = poly->Vertex(numv-1);
    sidel = ClassifyPoint(p1, plane);
    for(int v = 0; v < numv; v++)
    {
        p2 = poly->Vertex(v);
        side2 = ClassifyPoint(p2, plane);
        if(side2 > 0)
        {
            // Point p2 in front of plane
            if(sidel < 0)
            {
                // Edge crossed from back to front of plane
                Point intersect = EdgePlaneIntersect(p1, p2, plane);
                front->AddPoint(intersect);
                back->AddPoint(intersect);
            }
            front->AddPoint(p2);
        }
    }
}

```

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## BSP trees (cont.)

```

else if(side2 < 0)
{
    // Point p2 is behind plane
    if(sidel > 0)
    {
        // Edge crossed from front to back of plane
        Point intersect = EdgePlaneIntersect(p1, p2, plane);
        front->AddPoint(intersect);
        back->AddPoint(intersect);
    }
    back->AddPoint(p2);
}

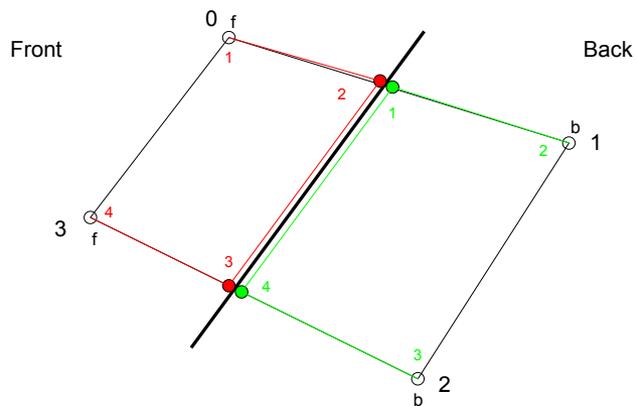
else
{
    // Point p2 is on plane
    front->AddPoint(p2);
    back->AddPoint(p2);
}

p1 = p2;
sidel = side2;
}
}

```

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## BSP trees (cont.)

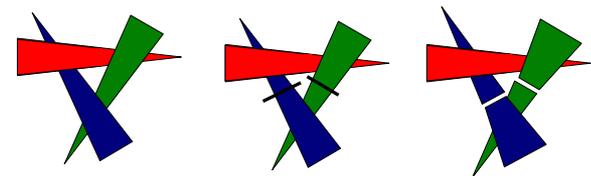


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## BSP trees (cont.)

### ■ The good thing about BSP trees:

- Depth ordering without ambiguity
  - Any polygon in the tree is either strictly in front of or behind any other polygon with respect to a partition plane (or coplanar)



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## BSP trees (cont.)

- The bad thing about BSP trees:
  - Creates a polygon soup, which makes efficient rendering difficult
  - Tree can be unbalanced
    - Find best polygon to place plane on, e.g. near centroid of all polygons, or with equal number of polygons in both children

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## Bounding volumes

- Instead of partitioning by a plane, divide space by a volume
- Use a mathematically simple shape to define a boundary
  - Bounding boxes
  - Bounding spheres
- Subdivide by dividing the bounded volume by multiple smaller bounding volumes
- Typically used to subdivide objects, but can also be used to subdivide a world
- Optimal bounding volumes typically determined by pre-processing

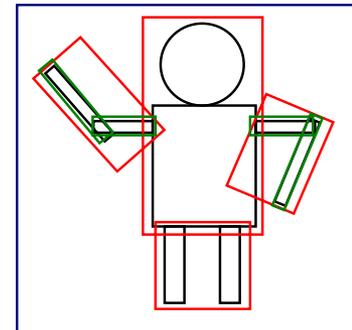
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## Bounding volumes (cont.)

- Bounding boxes
  - Have a big box around everything
  - Subdivide into smaller boxes
  - The union of children does not need to be the same volume as the parent
  - Boxes may be axis-aligned (AABB), or rotated to give a tighter fit

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## Bounding volumes (cont.)



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## Bounding volumes (cont.)

```
class AABB                // Axis-aligned bounding box
{
    bool isleaf;          // true if this is a leaf, false otherwise
    Pos box[2];           // Bounding box in world coordinates

    AABB* parent;
};

class AABBNode : public AABB // A non-leaf node in the AABB tree
{
    list<AABB*> children;
};

class AABBLeaf : public AABB // A leaf node in the AABB tree
{
    vector<Polygons> polygons;
};
```

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## Bounding volumes (cont.)

```
class BoundingBox
{
    bool isleaf;
    Pos centre;           // Centre of box, in parent's coordinates
    float size[3];       // Size of box
    Matrix rot;          // Rotation around centre, in parent coords

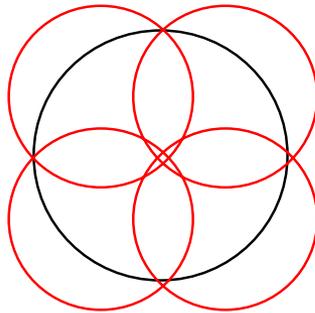
    BoundingBox* parent;
};
```

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## Bounding volumes (cont.)

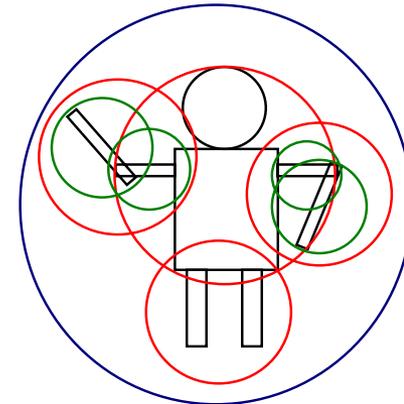
### ■ Spheres

- Easy to compute if point is in a sphere  
 $|p - \text{centre}| < \text{radius}$
- Often used for fast collision detection
  - Sphere defines boundary of some structure
  - Sub-spheres define boundaries of sub-structures



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## Bounding volumes (cont.)



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## Bounding volumes (cont.)

```
class BoundingSphere
{
    bool isleaf;           // true if this is a leaf, false otherwise
    Pos centre;
    float radius;

    BoundingSphere* parent;
};

class BoundingSphereNode : public BoundingSphere
{
    list<BoundingSphere*> children;
};

class BoundingSphereLeaf : public BoundingSphere
{
    vector<Polygons> polygons;
};
```

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## Space Subdivision (cont.)

- Various space subdivisions can be mixed together
  - E.g., quadtree down to some minimum area size, then a regular grid in the leaf nodes
    - Fast processing over large areas (hierarchical)
    - Fast rendering over small areas (display list, vertex array)

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