COMPSCI 777 S2 C 2004
Computer Games Technology

—Fuzzy Logic for Games—

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Fuzzy Logic vs. Traditional Logic

- Traditional logic works on the idea that something is either true or false.

- Fuzzy logic allows us to work with concepts that are not crisp, like for instance:
  - Size – “pretty big”, “awfully small”, “medium”, “gigantic”, . . .
  - Emotions – “irritated” vs, “incredibly angry”, “a bit nervous” vs. “terrified”, “happy” vs. “ecstatic”, . . .

Fuzzy Sets

- A fuzzy subset $\tilde{A}$ of a domain D is a set of ordered pairs, $\langle d, \mu_{\tilde{A}}(d) \rangle$, where $d \in D$ and $\mu_{\tilde{A}} : D \rightarrow [0, 1]$ is the membership function of $\tilde{A}$.
- The membership function replaces the characteristic function of a classical subset $A \subseteq D$.
- If the range of $\mu_{\tilde{A}}$ is $\{0, 1\}$, $\tilde{A}$ is nonfuzzy and $\mu_{\tilde{A}}(d)$ is identical with the characteristic function of a nonfuzzy set.

Examples of Fuzzy Sets (Formulas)

- Real numbers considerably larger than 10:

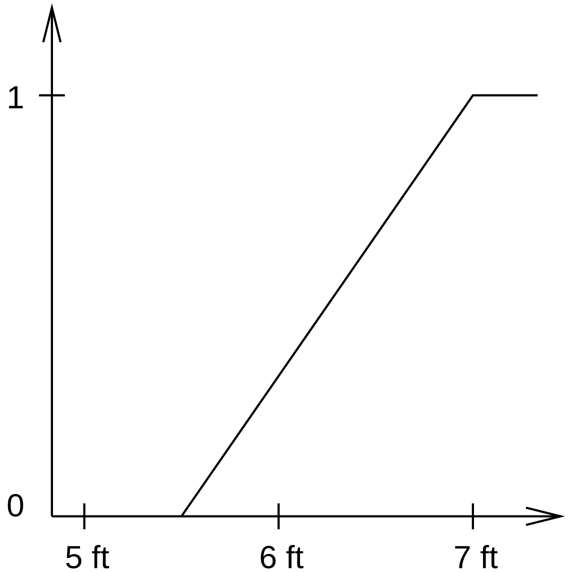
$$\tilde{A} = \{\langle d, \mu_{\tilde{A}}(d)\rangle \mid d \in \Re\} \text{ with } \mu_{\tilde{A}}(d) = \begin{cases} 0 & \text{for } d \leq 10 \\ \frac{1}{1+\frac{1}{(d-10)^2}} & \text{for } d > 10 \end{cases}$$

- Real numbers close to 10:

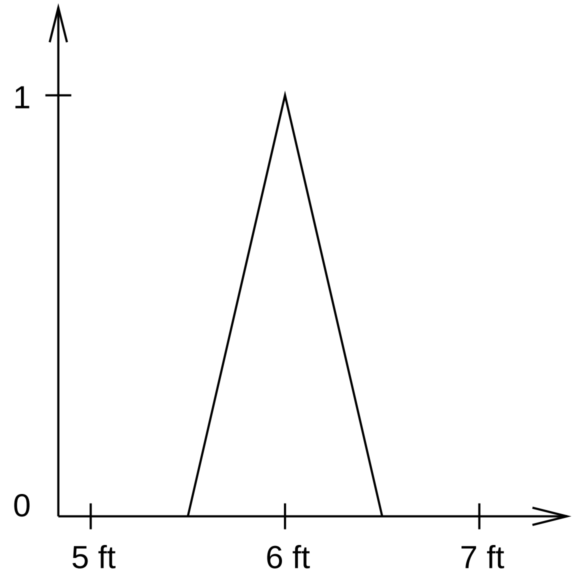
$$\tilde{A} = \{\langle d, \mu_{\tilde{A}}(d)\rangle \mid d \in \Re\} \text{ with } \mu_{\tilde{A}}(d) = \frac{1}{1+(d-10)^2}$$

Examples of Fuzzy Sets (Graphs)

- People who are tall:



- People who are about six feet:



(Strong) α-Level Sets

- Let $\tilde{A}$ be a fuzzy subset in D, then the (crisp) set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α-level set of $\tilde{A}$:

$$A_\alpha = \{d \in D \mid \mu_{\tilde{A}}(d) \geq \alpha\}$$

- If the degree of the elements is greater than α, the set is called the strong α-level set of $\tilde{A}$:

$$A_{\overline{\alpha}} = \{d \in D \mid \mu_{\tilde{A}}(d) > \alpha\}$$

Basic Operations on Fuzzy Sets

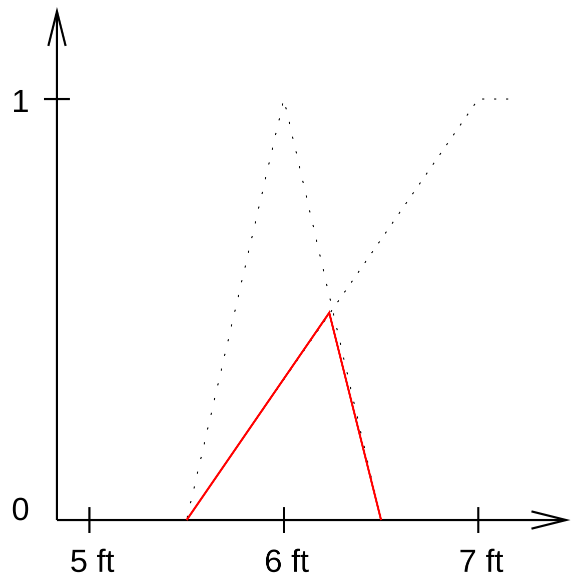
- The membership function $\mu_{\tilde{C}}(d)$ of the intersection $\tilde{C} = \tilde{A} \cap \tilde{B}$ is pointwise defined by $\mu_{\tilde{C}}(d) = \min\{\mu_{\tilde{A}}(d), \mu_{\tilde{B}}(d)\}$.

- The membership function $\mu_{\tilde{C}}(d)$ of the union $\tilde{C} = \tilde{A} \cup \tilde{B}$ is pointwise defined by $\mu_{\tilde{C}}(d) = \max\{\mu_{\tilde{A}}(d), \mu_{\tilde{B}}(d)\}$.

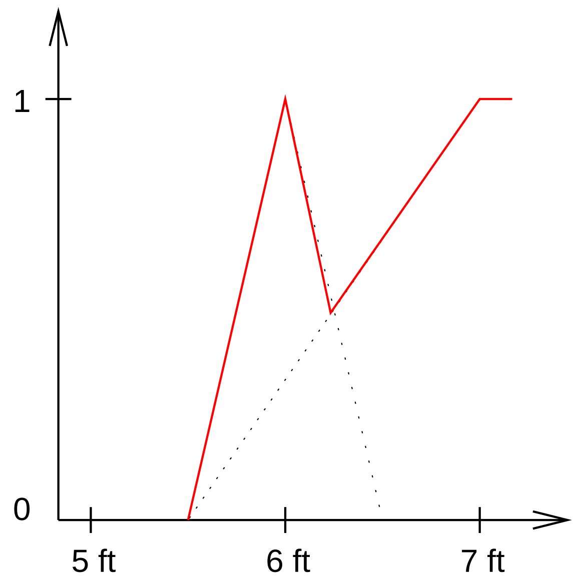
- The membership function $\mu_{\tilde{A}^c}(d)$ of the complement $\tilde{A}^c$ of a fuzzy set $\tilde{A}$ is pointwise defined by $\mu_{\tilde{A}^c}(d) = 1 - \mu_{\tilde{A}}(d)$.

Examples of Fuzzy Operations

- People who are tall and about six feet:



- People who are tall or about six feet:



Generalization of Intersection: t-Norms

t-norms are two-valued functions from $[0,1] \times [0,1]$ into $[0,1]$ that satisfy the following conditions:

- $t(0,0) = 0$
  $t(\mu_{\tilde{A}}(d), 1) = t(1, \mu_{\tilde{A}}(d)) = \mu_{\tilde{A}}(d), \quad d \in D$
- $t(\mu_{\tilde{A}}(d), \mu_{\tilde{B}}(d)) \leq t(\mu_{\tilde{U}}(d), \mu_{\tilde{V}}(d))$
  if $\mu_{\tilde{A}}(d) \leq \mu_{\tilde{U}}(d)$ and $\mu_{\tilde{B}}(d) \leq \mu_{\tilde{V}}(d)$ (monotonicity)
- $t(\mu_{\tilde{A}}(d), \mu_{\tilde{B}}(d)) = t(\mu_{\tilde{B}}(d), \mu_{\tilde{A}}(d))$ (commutativity)
- $t(\mu_{\tilde{A}}(d), t(\mu_{\tilde{B}}(d), \mu_{\tilde{C}}(d))) =$
  $t(t(\mu_{\tilde{A}}(d), \mu_{\tilde{B}}(d)), \mu_{\tilde{C}}(d))$ (associativity)

Generalization of Union: s-Norms

s-norms (or t-conorms) are two-valued functions from $[0,1] \times [0,1]$ into $[0,1]$ that satisfy the following conditions:

- $s(1,1) = 1$
  $s(\mu_{\tilde{A}}(d), 0) = s(0, \mu_{\tilde{A}}(d)) = \mu_{\tilde{A}}(d), \quad d \in D$
- $s(\mu_{\tilde{A}}(d), \mu_{\tilde{B}}(d)) \leq s(\mu_{\tilde{U}}(d), \mu_{\tilde{V}}(d))$
  if $\mu_{\tilde{A}}(d) \leq \mu_{\tilde{U}}(d)$ and $\mu_{\tilde{B}}(d) \leq \mu_{\tilde{V}}(d)$ (monotonicity)
- $s(\mu_{\tilde{A}}(d), \mu_{\tilde{B}}(d)) = s(\mu_{\tilde{B}}(d), \mu_{\tilde{A}}(d))$ (commutativity)
- $s(\mu_{\tilde{A}}(d), s(\mu_{\tilde{B}}(d), \mu_{\tilde{C}}(d))) =$
  $s(s(\mu_{\tilde{A}}(d), \mu_{\tilde{B}}(d)), \mu_{\tilde{C}}(d))$ (associativity)

Examples of t-Norms and s-Norms

- $\min\{\mu_{\tilde{A}}(d),\ \mu_{\tilde{B}}(d)\}$ *(minimum)*
  $\max\{\mu_{\tilde{A}}(d),\ \mu_{\tilde{B}}(d)\}$ *(maximum)*

- $\mu_{\tilde{A}}(d) \cdot \mu_{\tilde{B}}(d)$ *(algebraic product)*
  $\mu_{\tilde{A}}(d) + \mu_{\tilde{B}}(d) - \mu_{\tilde{A}}(d) \cdot \mu_{\tilde{B}}(d)$ *(algebraic sum)*

- $\max\{0,\ \mu_{\tilde{A}}(d) + \mu_{\tilde{B}}(d) - 1\}$ *(bounded difference)*
  $\min\{1,\ \mu_{\tilde{A}}(d) + \mu_{\tilde{B}}(d)\}$ *(bounded sum)*

Fuzzy Control

- Use fuzzy sets in rules to control actions.
- Parameter of the actions are represented as linguistic variables.
- A linguistic variable has a set of symbolic values as domain.

Example of Modeling Traffic

- Model traffic in a city simulation game.
- The cars in the simulation game are supposed to behave realistically.
- No cars can bump into the backs of other cars.
- Cars are supposed to maintain a reasonable distance to each other.

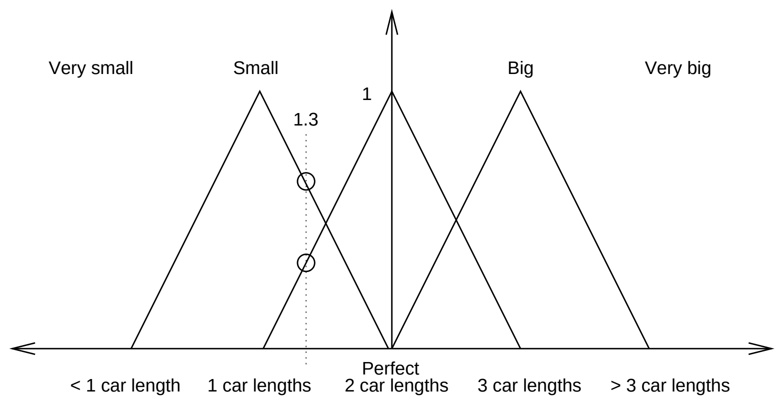
Input Linguistic Variables and their Fuzzy Sets

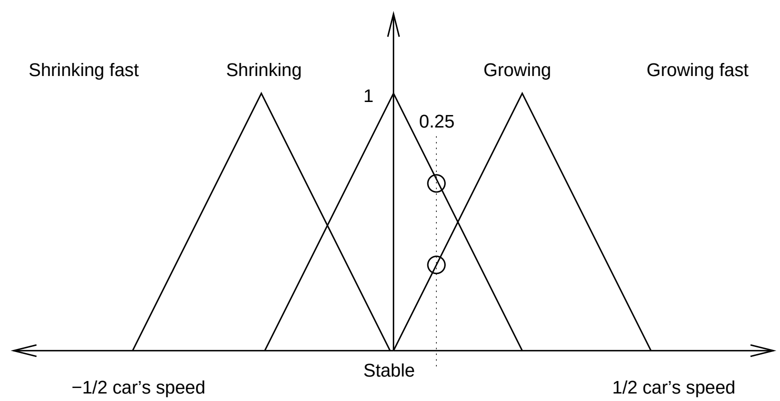
Distance

Linguistic value	Fuzzy Set
Very small	$<$ 1 car length
Small	$\sim$ 1 car length
Perfect	$\sim$ 2 car lengths
Big	$\sim$ 3 car lengths
Very big	$>$ 3 car lengths

Distance delta

Linguistic value	Fuzzy Set
Shrinking fast	$\sim -\frac{1}{2}$ car's speed
Shrinking	< 0
Stable	~ 0
Growing	> 0
Growing fast	$\sim \frac{1}{2}$ car's speed

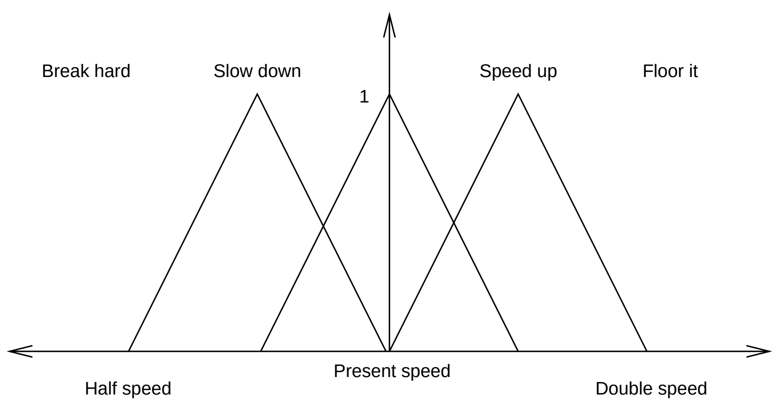




Output Linguistic Variable and its Fuzzy Set

Action

Linguistic value	Fuzzy Set
Brake hard	Speed $/=$ 2
Slow down	Speed $-=$ speed/2
Maintain speed	Do nothing
Speed up	Speed $+=$ speed/2
Floor it	Speed $*=$ 2



Fuzzy Rules

	Distance delta				
Distance	Shrinking fast	Shrinking	Stable	Growing	Growing fast
Very small	Brake hard	Brake hard	Slow down	Slow down	Maintain speed
Small	Brake hard	Slow down	Slow down	Maintain speed	Speed up
Perfect	Slow down	Slow down	Maintain speed	Speed up	Speed up
Big	Slow down	Maintain speed	Speed up	Speed up	Floor it
Very big	Maintain speed	Speed up	Speed up	Floor it	Floor it

Applying Fuzzy Rules

- If distance is small and distance delta is growing, maintain speed.
  True with a degree of 0.3.
- If distance is small and distance delta is stable, slow down.
  True with a degree of 0.6.
- If distance is perfect and distance delta is growing, speed up.
  True with a degree of 0.1.
- If distance is perfect and distance delta is stable, maintain speed.
  True with a degree of 0.1.

Defuzzification