Chapter 7 – Visualisation Techniques for Tensor Fields

7.1 Introduction

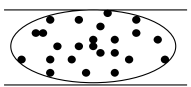
- Tensor fields are a common quantity in engineering and physical sciences
  - stress, strain, diffusion, velocity gradients,...
- Of particular interest are second-order tensors $\mathbf{T}=(T_{ij})$ which can be interpreted as linear transformations between vectors and are represented in 3D by 3x3 matrices.
- We consider only the visualisation of symmetric second-order tensors which are characterized by $T_{ij} = T_{ji}$ and are represented by symmetric matrices.
- Asymmetric second-order tensor fields can be visualised by decomposing the tensor into a symmetric and an antisymmetric part.

$$T_{ij} = \frac{T_{ij}+T_{ji}}{2} + \frac{T_{ij}-T_{ji}}{2}$$

Symmetric Part (red)
Antisymmetric Part (blue)

Introduction - Review

- Example: Diffusion Tensor
  - Water molecules move randomly due to Brownian motion (diffusion)
  - In inhomogeneous materials diffusion speed is different in each direction
  - Water molecules originating at fixed location form ellipsoidal shape
  - Shape described by a tensor

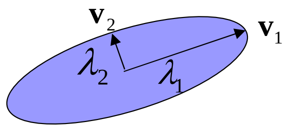


Introduction – Review (cont'd)

Any n-dimensional symmetric tensor T always has *n* eigenvalues λ_i and *n* mutually perpendicular eigenvectors $\mathbf{v}_i$ such that

$$\mathbf{T}\mathbf{v}_i = \lambda_i \mathbf{v}_i \qquad i = 1,\dots,n$$

The eigenvectors and eigenvalues of the diffusion tensor give the direction and length of the principal axes of the diffusion ellipsoid

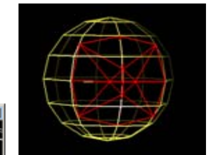


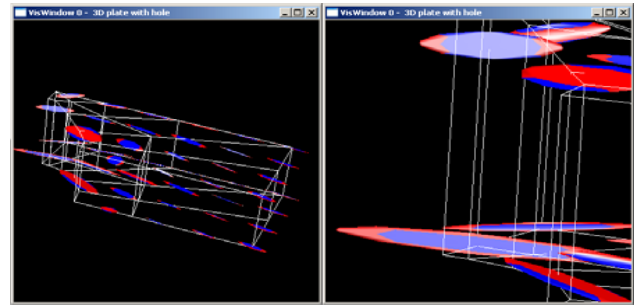
7.2 Tensor Glyphs

- A popular way to visualise tensors is by depicting their eigenvalues and eigenvectors.
- Can be achieved by drawing the eigenvectors as line segments whose length is proportional to the corresponding eigenvalues.
  - 3D perception of this representation is poor and using several of these glyphs simultaneously often leads to visual cluttering.
- An improved representation is achieved by using *tensor ellipsoids* which encode the eigenvalues and the eigenvectors of a tensor by the directions and lengths, respectively, of the principal axes of an ellipsoid.
- Solid tensor ellipsoids can occlude large areas of a visualisation.
  - “see-through” ellipsoids are defined by using bands along the equators (Post 1995).

Tensor Glyphs (cont’d)

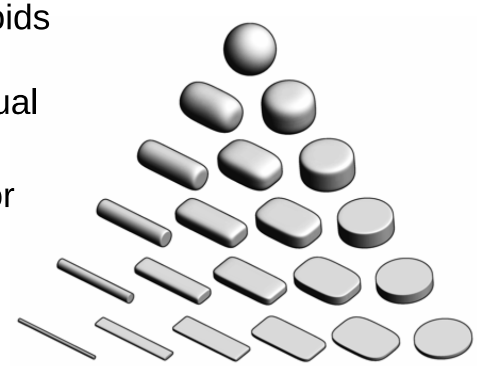
- Visualise the sign of an eigenvalue by dividing the ellipsoid into 6 segments using different colours for positive and negative values.





Tensor Glyphs (cont’d)

- Cuboids and ellipsoids have problems with asymmetry and visual ambiguities.
- Superquadric tensor glyphs represent a solution.



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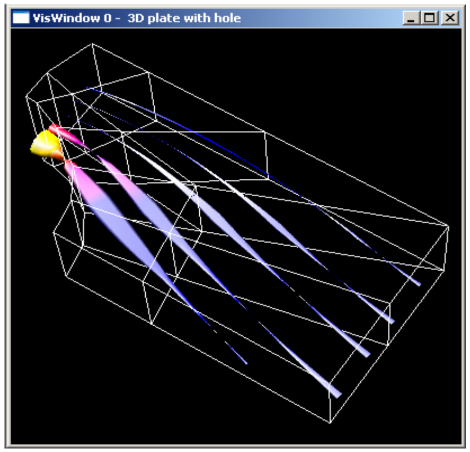
7.3 Hyperstreamlines

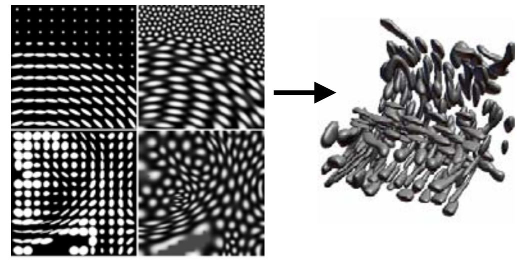
- Multiple tensor glyphs often cause visual cluttering.
- Perception is improved by replacing discretely spaced icons with a continuous representation such as *hyperstreamlines* (Delmarcelle & Hesselink 1993).
- The trajectory of a hyperstreamline is a streamline of an eigenvector field. The other two eigenvectors and corresponding eigenvalues define the directions and lengths, respectively, of the axes of the ellipsoidal cross section of the hyperstreamline.
- The hyperstreamline can be colour mapped with
  - the eigenvalue which corresponds to the eigenvector defining its trajectory.
  - other scalar quantities.

Hyperstreamlines (cont'd)



7.4 Direct Volume Rendering

- Can use 3D textures and colours to indicate properties of the tensor field



Ellipsoids Reaction-Diffusion Texture

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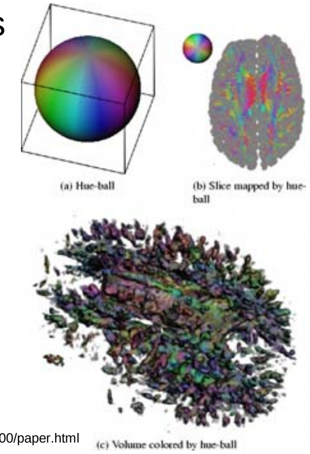
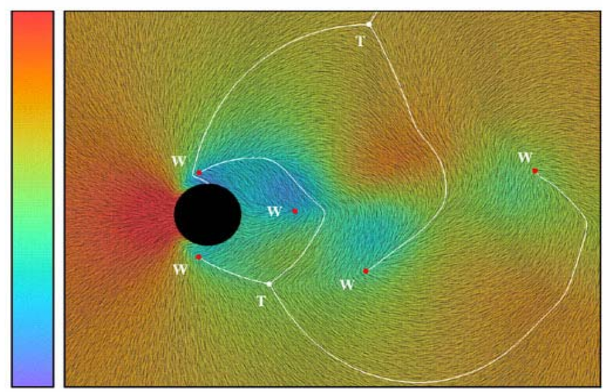


7.5 Tensor Field Topology

- As with vector fields the complex structure of a symmetric second-order tensor field can be represented by its topology.
- In two dimensions the topological skeleton consists of *degenerate points* which are connected by *separatrices*.
- Degenerate points are points for which both eigenvalues of the tensor are equal and hence every vector is an eigenvector.
  - Since the eigenvectors of a symmetric tensor are otherwise orthogonal degenerate points are the only points in the tensor field where the trajectories of an eigenvector field can cross!
- Separatrices are trajectories of the major eigenvector field connecting degenerate points and separating it into regions of similar behaviour.

Tensor Field Topology (cont'd)



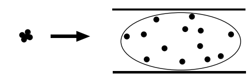
© 1994, Lambertus Hesselink and Thierry Delmarcelle, The Topology of Symmetric, Second-Order Tensor Fields, Proceedings of IEEE Visualization '94, October 1994.

7.6 Case Study - The Visualisation of Neuroanatomy from DTI Data

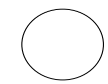
- Diffusion Tensor Imaging (DTI) detects water diffusion in the brain.
- The resulting diffusion tensor field provides *in vivo* information about the anatomy of the brain.
- In this case study we analyse the diffusion tensor field in a healthy brain using vector and tensor field visualization techniques.

Diffusion Tensor Imaging

The spatial distribution of water molecules diffusing outwards from a point location is described by an ellipsoid, the shape of which is determined. by the tissue microstructure. The ellipsoid is described mathematically by a diffusion tensor.



- Fluid filled compartments are characterized by a very high *isotropic diffusion.*



- Fiber tracts consisting of millions of parallel nerve fibers are characterized by a high *anisotropic diffusion*.

- Gray matter consists of cell bodies and intermingling supporting cells and has a low nearly isotropic diffusion.

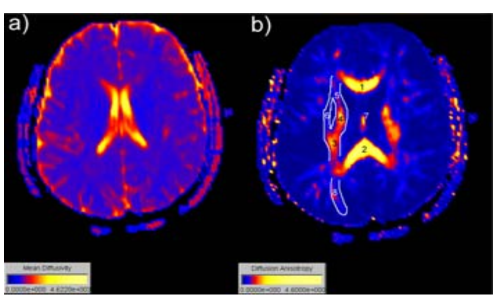
Visualising CSF and Gray Matter

Measures characterizing different tissue types are:

$$\lambda_{mean} = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3} = \frac{D_{11} + D_{22} + D_{33}}{3}$$

$$\lambda_{anisotropy} = trace\left(\left(\mathbf{D} - \lambda_{mean}\mathbf{I}\right)^T \left(\mathbf{D} - \lambda_{mean}\mathbf{I}\right)\right) / \lambda_{mean}^2$$

$$\text{where } trace(M) = m_{11} + m_{22} + m_{33}$$



(1,2) Genu and splenium of Corpus Callosum

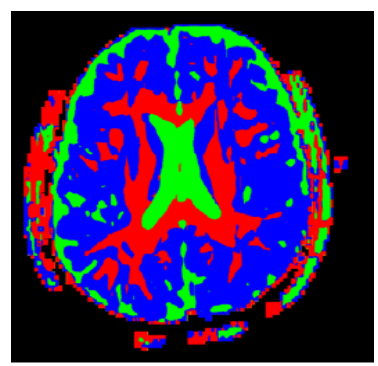
(3,4,5) Genu, anterioro limb and posterior limb of the internal capsule

(6) External capsule

(7) Fornix

(8) Optic radiation

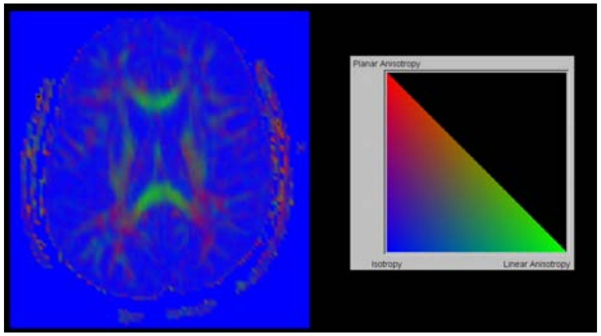
Visual Segmentation of Images



Regions of white matter, gray matter and cerebral spinal fluid can be displayed simultaneously using the segmentation function

$$\lambda_{segmented} = \begin{cases} 1 & \text{if } \lambda_{anisotropy} > 0.25 \text{ and } 5*10^{-6} < \lambda_{mean} < 10^{-3} \\ 2 & \text{if } \lambda_{mean} \geq 10^{-3} \\ 3 & \text{if } 5*10^{-6} < \lambda_{mean} < 10^{-3} \text{ and } \lambda_{anisotropy} < 0.25 \\ 0 & \text{otherwise} \end{cases}$$

Barycentric Color Map



Let $\lambda_1 \geq \lambda_2 \geq \lambda_3$ be the eigenvalues of the diffusion tensor. Then the planar anisotropy λ_p, linear anisotropy λ_l and isotropy λ_s are defined as:

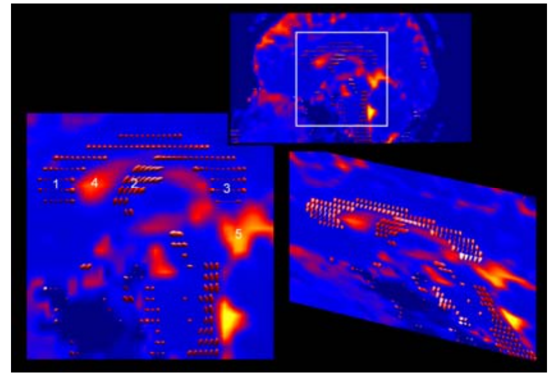
$$\lambda_l = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$\lambda_p = \frac{2(\lambda_2 - \lambda_3)}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$\lambda_s = \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$$

Barycentric color map visualising regions of isotropic, linear anisotropic and planar anisotropic diffusion.

Tensor Ellipsoids



(1,3) Genu and splenium of Corpus Callosum

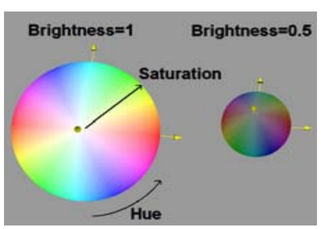
(2) Fornix

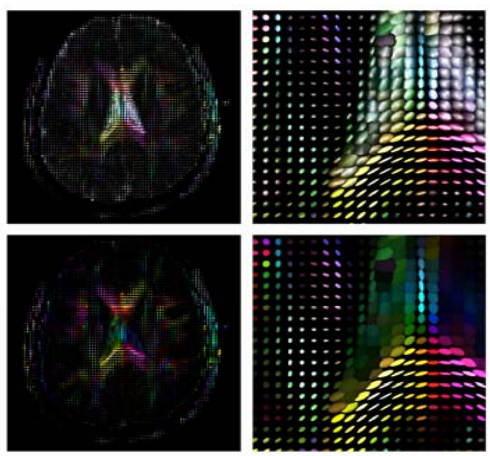
(4) Lateral ventricle

(5) Fourth ventricle

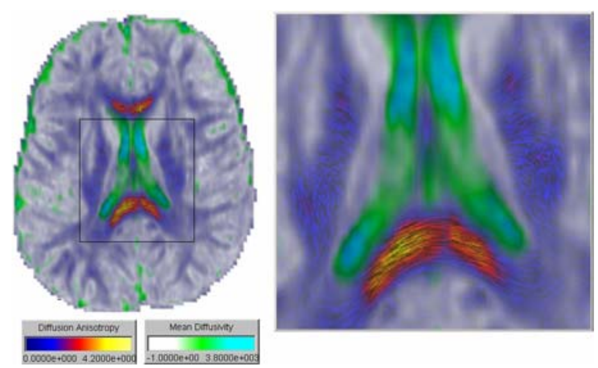
Saggital section color mapped with the mean diffusivity and overlayed with diffusion ellipsoids at MRI grid vertices.

Ellipsoids & Spherical Colour Maps



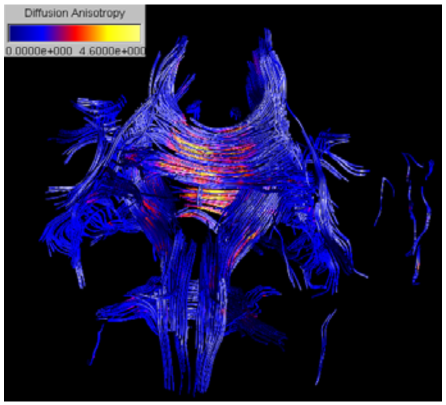


Anisotropic Modulated Line Integral Convolution (AMLIC)



The three dimensional direction of a nerve fiber is encoded by an AMLIC texture which is obtained by varying the length of a convolution kernel with the normal component of the principal diffusion direction. Tissue types are made evident by blending the colour mapped LIC texture with a color map of the mean diffusivity.

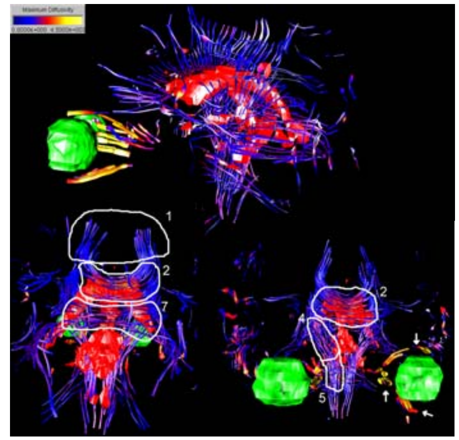
Streamtubes

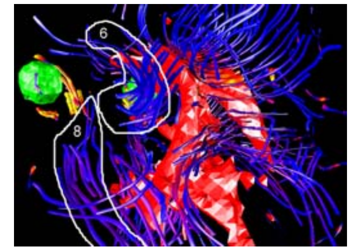


Nerve fiber tracts visualised using cylindrical stream-tubes color mapped with the diffusion anisotropy.

Streamtubes can be extended to hyperstreamlines (next slide) by scaling the cross sections with the transverse diffusivities.

Hyperstreamlines





(1) Corona radiata (2) Corpus callosum

(3) Optic radiation (4) Internal capsule

(5) Cerebral peduncles

(6) Superior longitudinal fasciculus

(7) Splenium of the corpus callosum

(8) Inferior occipitofrontal fasciculus

7.7 References

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