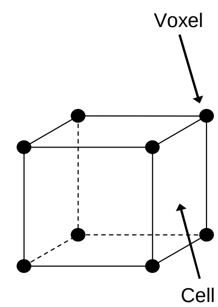
Chapter 5 – Visualization Techniques for Scalar Fields

5.1 Overview

Scalar data can be defined as

- *Continuous field f(x,y,z)* - defined for all (x,y,z)
  - Usually obtained as solution to a mathematically problem or by interpolating sampled data
- *Sampled volume data* f_{ijk} - defined only at particular points (x_i,y_j,z_k)
  - Most commonly on a cartesian grid
  - Sample values are called *voxels*
  - A cuboidal region with voxels at all 8 vertices is called a *cell*



Suitability of Visual Attributes for Displaying Quantitative Information

Highest accuracy of representation

Position on scale

Interval length

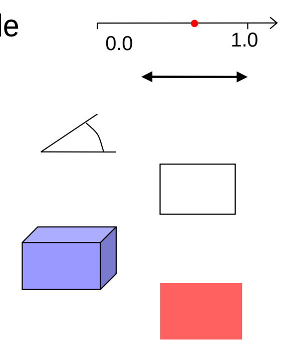
Slope/angle

Area

Volume

Colour

Lowest accuracy of representation

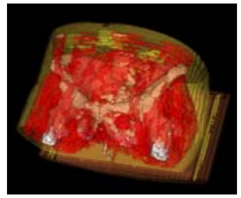


Visualisation Methods

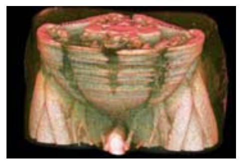
- Colour mapping
  - Associate scalar field values with colours
  - Visualizes field over a surface
  - Perception of qualitative information limited
- "Quick look" techniques
  - Easy to program & fast to compute
  - Weak visualization
- Surface-fitting methods
  - Define surface(s) of constant field values *f(x,y,z)=c*
  - Called *iso-level or iso-value surfaces*, often abbreviated to *isosurfaces*
  - Choose "interesting" values (isosurface levels)
    - e.g. between soft tissue and bone

Visualization Methods (cont'd)

- Surface-fitting methods (cont'd)
  - Usually use polygonal meshes
  - 3D equivalent of contour lines
  - Fast to display (e.g. OpenGL)
  - Only displays data at the selected isosurface level

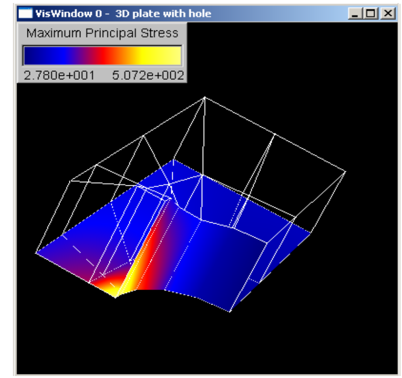


- Direct Volume Rendering
  - Use an optical model to define the colour and opacity/transparency of the continuous medium as a function of *f*
  - Display "whole volume" (e.g. by ray tracing)
  - Slow
  - Fuzzy
  - Contains more information (potentially)



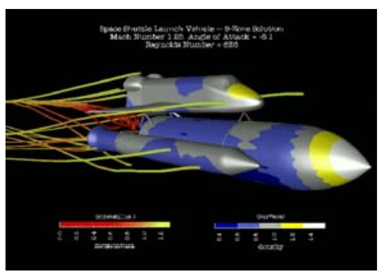
5.2 Colour Mapping

- Used to visualize scalar field over a surface
  - Associate field's range of values with a colour scale
  - Colour each point on surface according to its field value
  - Colour scales can also be mapped onto other visualization icons



Colour Mapping (cont'd)

- Advantages
  - Easy to implement
  - Gives overall impression of distribution of a scalar field
  - Can be mixed with other visualization icons
  - Can use discontinuous colour scale for accurate information along contours



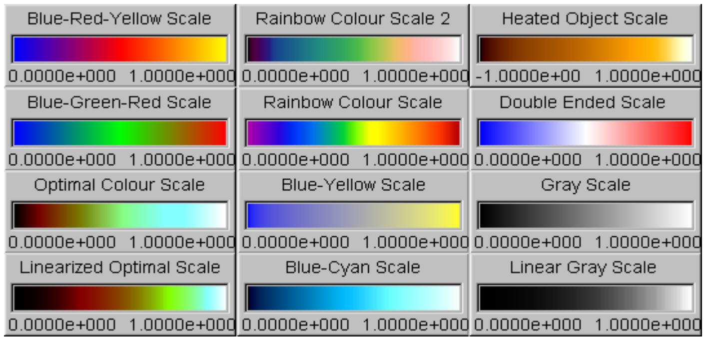
- Disadvantages
  - Quantitative information displayed by colour can not be perceived accurately for a continuous colour map
  - Effectiveness of colour map depends on the colour scale used and perceptual issues
  - No information about scalar field values outside the mapped surface

Colour Mapping (cont'd)

- Desirable properties of a good colour scale
  - Colours should be perceived as preserving the order of the scalar values they represent.
  - Colours should convey the distances between values they represent and should associate related values and separate unrelated values.
  - Colours should be continuous for a continuous range.
  - Accentuates important features.
- NOTE:
  - If colour mapping an illuminated surface choose colour scale with hue variations only if.
  - If we want to maximising the range of differentiable values then we choose a colour scale with hue and intensity/brightness variations.

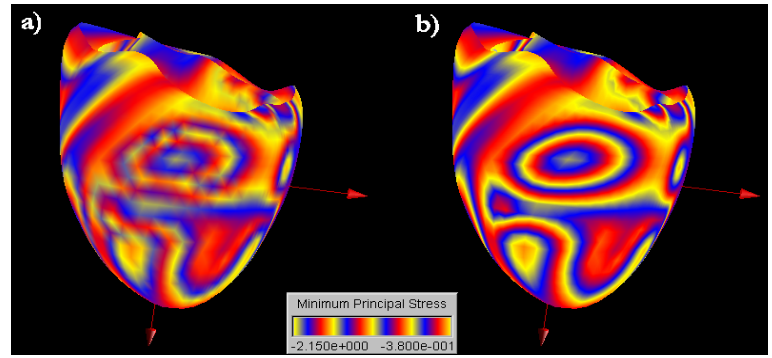
Colour Mapping (cont'd)

- Common colour scales
  - E.g. H. Levkowitz and G. T. Herman. Colour scales for image data. IEEE Computer Graphics & Applicatios, 12(1):72-80, January 1992.



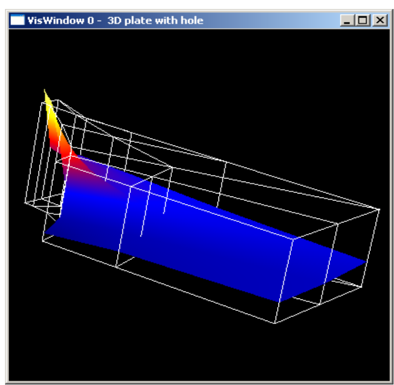
Colour Mapping (cont'd)

- Implemented using Gouraud shaded polygons (a) or 1D texture maps (b).

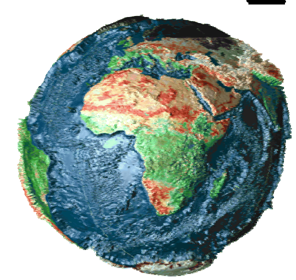


5.3 Height Fields

- Used to visualize a field over a (planar) surface
  - Visualize field's values over the surface by constructing an offset surface.
  - Height of offset surface at each point proportional to the field value at that point.
  - Can colour map the height field to encode additional information.



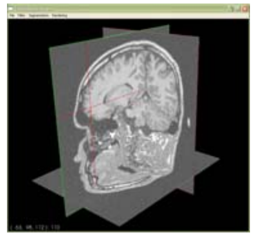
Height Fields (cont'd)

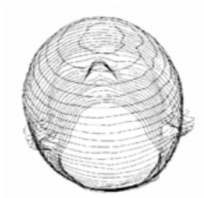


- Advantages
  - Accurate display of quantitative information
  - Can be colour mapped to display several scalar fields simultaneously
    – good for displaying correlation
- Disadvantages
  - Works best for planar surfaces.
  - For curved surfaces height values difficult to perceive and offset surface might self-intersect
  - Requires a large amount of screen space - might interfere with other visualization icons
  - Often not obvious for which surface the scalar field is visualized
  - No information about scalar field values outside the mapped surface

5.4 Quick-look Techniques

- Slicing
  - May just display as images slices along coordinate axes
  - Better to allow arbitrary slicing plane
  - Perhaps animate motion of slicing plane to improve visualization
- Wire-frame contours
  - Take the sampled data in slices
  - Compute iso-value contours in the slice planes
  - Display those contours as lines in 3 space
  - Possibly do on more than one axis
  - Is a simple example of "surface fitting"



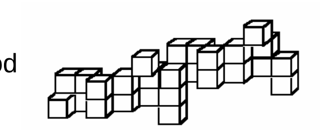


Surface Fitting

- Is just contouring in 3D
  - Contours are now surfaces
- Easiest method – "Opaque Cubes"

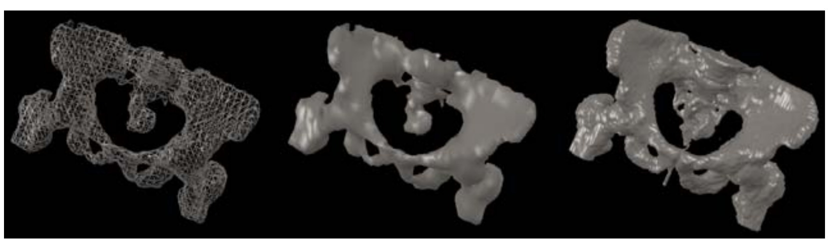
```
For each cell in the volume
   If cell's voxel values encompass the
    isolevel then Display the cell as a solid
    cube
```

  - Is really another "quick look" method
  - Builds "Lego" approximation to object



5.5 “Marching Cubes” Algorithm

- Approximates isosurface through each cell with a set of polygons



Low resolution mesh | Low resolution mesh rendered as Gouraud shaded surface | High resolution mesh rendered as Gouraud shaded surface

“Marching Cubes” Algorithm (cont’d)

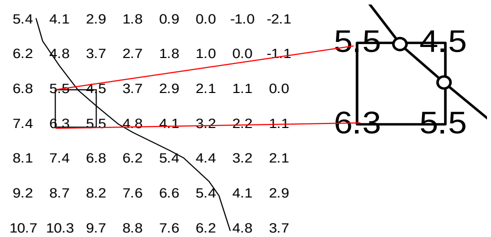
- Easy in principle

```
For each cubical cell
  For each edge of cell
    If endpoint voxel values encompass the
    isosurface
      value determine the intersection point
  Connect all intersection points in cell to give
  one or more polygons representing surface
  through cell
```

- Can make it fast by building look-up table of all possible cell configurations
- Easier to understand by doing 2D case first

Contour Computation in 2D

- Picture shows contour for isosurface level = 5



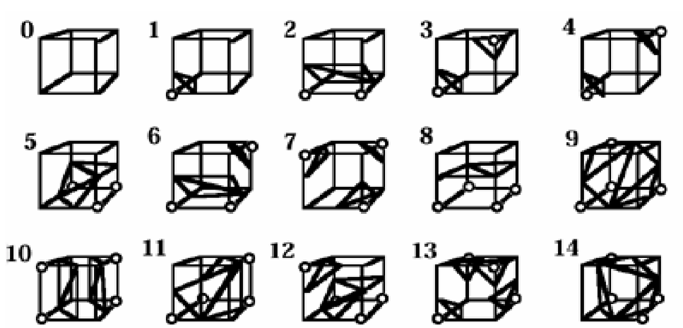
- Compute where contour crosses each square cell. Connect intersection points
- Ambiguous if 4 intersection points – see assignment!

3D Case

- Categorise each voxel of cell as *above* (1) or *below* (0) the iso-value.
  - Forget equality. Floating point numbers are never equal!
- Encode each voxel into one bit. 8 voxels ⇒ 8-bit code ⇒ 256 configurations
- Excluding rotations, reflections and complements, only 15 topologically distinct cases
  - See next slide
- As in 2D, have some ambiguous cases. UDOO: which ones?
  - See assignment for method of resolving these

The 15 Cases

- In figure below, a circled vertex = 1, uncircled = 0
  … or vice-versa!!



Marching Cubes Algorithm

```
// Build look-up table [Table is constructed only once during initialisation]
For config = 0 to 255
    intersectingEdges = set of all intersecting edges computed from bit pattern of config
    LUTable[config].polygons = [];
    While unused intersectingEdges remain
        currentIntersectingEdge = any unused edge from intersectingEdges
        Initialise new outputPolygon
        firstIntersectingEdge = currentIntersectingEdge
        repeat
            outputPolygon.add(currentIntersectingEdge)
            Choose face to right of currentIntersectingEdge (if going 0→1)
            currentIntersectingEdge = first intersecting edge clockwise around face
              from currentIntersectingEdge
        until currentIntersectingEdge = firstIntersectingEdge
        LUTable[config].polygons += outputPolygon
```

Marching Cubes Algorithm (cont'd)

```
// "March" through volume, outputing all polygons
For each cell in volume
    Classify voxels at the 8 vertices as 0 or 1 to get 8-bit config value
    For each entry in LUTable[config].polygons
      For each "edge" stored in polygon
        Compute actual isosurface intersection point given the sample
        values at the edge endpoints – this is a vertex of the new polygon
        Compute isosurface normal at that vertex from the gradient (or its
        inverse)
      Output the polygon
```


Notes on Marching Cubes

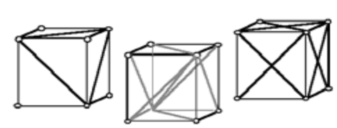
- The normal to the isosurface at polygon vertices is given by the direction of the field gradient or its negation (careful!)
  - Either trilinearly interpolate the central difference estimates at sample points or directly evaluate the original (unsampled) field function at the vertex, if that's possible.
- Although only 15 distinct topologies, it's not worth compressing the 256-element table
- Often subdivide all polygons into triangles (since generally non-planar)
  - But renderer (e.g. OpenGL) usually does that, so why bother?
- Can get huge number of polygons. Sometimes follow MC with a mesh-optimization algorithm that combines near-planar adjacent faces (see Wünsche & Lobb paper).

MC Notes (cont'd)

- Can have multiple isosurfaces
  - make outermost one partially transparent
  - use different colours for different surfaces
- Handling ambiguities complicates the algorithm
  - Probably not important for performance, since these cases are relatively rare (mainly confined to regions of rapid change)
- Term *Marching Cubes* comes from paper by Lorensen and Cline
  - Patent for the algorithm has expired – now free to use
  - Wyvill and McPheeters came up with a similar (and in some ways better) algorithm the previous year.
  - Because of the patent some authors avoided the term *Marching Cubes*, and didn't reference Lorensen and Cline.

MC Notes (cont'd)

- Above algorithm is $O(n^3)$ where n is number of samples in each direction.
  - Alternative is to track surface starting from given "seed" points.
  - Is then $O(n^2)$.
  - But more complicated, and need that seed point!
- Tetrahedral subdivision of space is also possible ("Marching Tetrahedra")
  - Simple table with no ambiguities
  - Cubical cell can be subdivided into 5, 6 or 24 tetrahedra



  - 5-tetrahedron case requires flipping adjacent cells for continuity across faces
  - Tends to give excessive fragmentation and "ripply" surfaces

Dividing Cubes

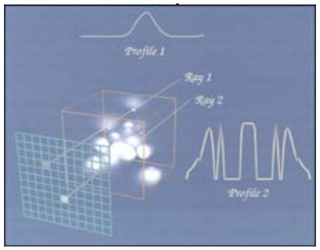
- Marching cubes can give huge number of polygons.
- Can be very slow to render without special-purpose hardware – poor interactivity
- A faster method in such cases is *Dividing Cubes*. Not widely known/used.
- Simple idea – like opaque cubes but
  - recursively subdivide each cube that contains isosurface until its projection area is pixel-sized.
  - then colour the pixel(s) it projects onto with a shade computed using a standard illumination model. Use the gradient at the centre of the cube as the surface normal.
- Can get real-time frame rates on modern PCs if you're sufficiently cunning.

5.6 Direct Volume Rendering

- Regard scalar field values as densities of a gas-like material
- Gas emits light, and also attenuates light coming from behind.
- Let ε_λ be the emission per unit length along a ray for some wavelength λ
- Let β_λ be the attenuation coefficient the ray, defined by

$$\frac{dI_\lambda}{dt} = -\beta_\lambda I_\lambda$$

where I_λ is intensity.



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The Emission-Absorption Model

- Then can easily derive the *emission-absorption model*:

$$I_\lambda = \int_{t=0}^{t_{\max}} \varepsilon_\lambda(t) e^{-\int_{s=0}^{t} \beta_\lambda(s)\,ds} dt$$

where $\varepsilon_\lambda dt$ is the light emitted by an element of the ray path, and $e^{-thingo}$ is the attenuation factor of the medium between the eye and the element. I_λ is just the integral over the whole ray path.

- Good reference: Nelson Max "Optical Models for Direct Volume Rendering", IEEE Trans. Vis. and Computer Graphics", 1(2) June 1995.

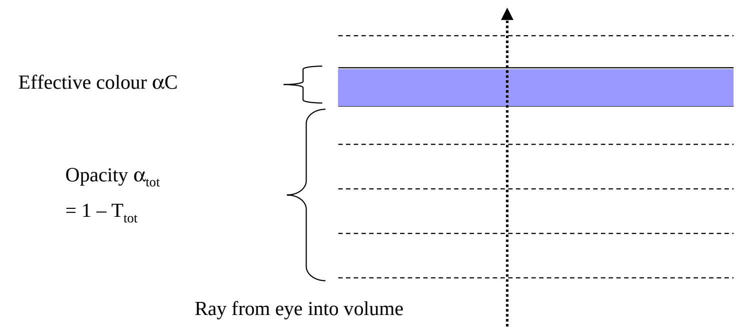
Opacity, Transparency and Colour

- Papers often talk about *opacity* or *transparency* of the medium.
- Confusing. Defined only for a fixed distance through the medium
  - Usually a "slab" of the medium, i.e. the spacing between voxel slices
- Transparency of a slab = Intensity Out / Intensity In
  - So transparency of two consecutive slabs with transparencies T_1 and T_2 is just $T_1 T_2$
- Opacity α = 1 – Transparency

Opacity, Transparency and Colour (cont'd)

- UDOO: If the opacity of a 2 mm thick section of tissue in 0.8, what is the opacity of a 1mm thick section?
  - No, it is *not* 0.4.
  - Should get 1 – Sqrt(1– 0.8) ≈ 0.55
- The simple optical model assumes medium is populated with small opaque particles with emissive colour *C*
- For a thin slab, α represents the probability that a photon will *not* pass through the slab.
- α *C* then represents the colour emitted by the slab (since α is a measure of "coverage")

Solving the Emission-Absorption Equation



Contribution of shaded slab = $\alpha\, CT_{tot} = \alpha\, C(1-\alpha_{tot})$

Solving the E-A Equation (cont'd)

- Accumulate colour and opacity working through slabs from front to back
- At each step,
  - $T_{tot}' = T_{tot}\, T_{thisLayer}$
  - $C'_{tot} = C_{tot} + T_{tot}\, (\alpha_{thisLayer}\, C_{thisLayer})$

    $= C_{tot} + T_{tot}\, (1 - T_{thisLayer})\, C_{thisLayer}$

Notes

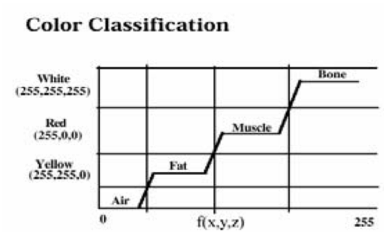
- In a simple minded model, α is proportional to "density" *f*(*x*,*y*,*z*), and $C_{thisLayer}$ is constant
- When viewing from arbitrary angles, "slabs" aren't really slabs at all – just steps along ray path
- For efficiency, should ideally vary step size according to magnitude of contribution to C_{tot}
- Can cut off calculation along ray when T_{tot} falls below some small minimum
- Slow in software but fast in hardware (use fragment program – best on NVIDIA GeForce 6800 or higher)

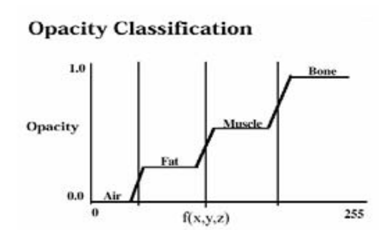
Notes (cont'd)

- Method as described so far just tends to produce a foggy mess. So:
  - Compute $C_{thisLayer}$ using a pseudo-surface reflection model, e.g. Lambert or Phong illumination
  - Assume some lighting configuration
  - Take –**grad** *f* as the surface normal
  - Also possibly weight colour by | **grad** *f* | to emphasise high gradient regions, representing e.g. transitions between tissue types
- Even with above, may still be a foggy mess unless pre-process dataset as in next slide

Classification

- If different density ranges represent different physical properties (e.g. different tissues, with CT scan), want different colours for those different ranges
- So now α and *C* are more complex functions of density *f*(x,y,z)





Results

Maximum Intensity projection Composite (unshaded) Composite (shaded)

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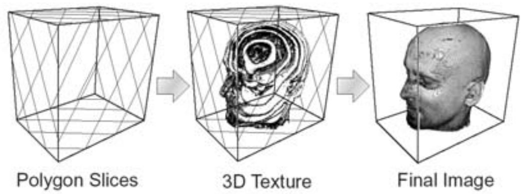
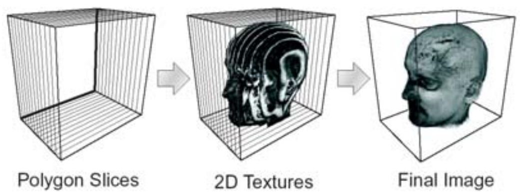
Direct Projection Methods

- Ray tracing technique described above is an exact solution to Emission-Absorption equation.
- Called an "image order" method, since traverse volume one ray at a time, i.e. in an order determined by image
- Also have a range of "object order" methods, where we attempt to determine the contribution to the final image of each cell or voxel in turn.
- Can do exactly ("Vbuffer algorithm" or similar) or approximately ("Splatting" algorithms).
- Nowadays hardware implemented methods are most common
  - Use 2D or 3D texture mapping

GPU Based Volume Rendering



View-aligned slices used as proxy geometry with 3D texture mapping.



Object-aligned slices used as proxy geometry with 2D texture mapping.

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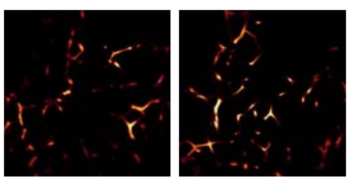
Volume Visualization in VTK

Three steps:

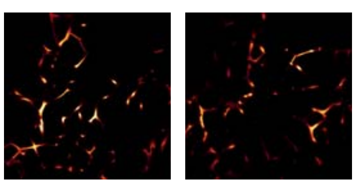
1. Classification
   - Opacity transfer function (use vtkPiecewiseFunction)
   - Colour transfer function (use vtkColorTransferFunction)
   - Add to volume properties
2. Define a mapper (rendering technique)
   - Ray casting (vtkVolumeRayCastMapper and vtkVolumeRayCastCompositeFunction)
   - Texture mapping (vtkVolumeTextureMapper2D)
3. Render
   - Add properties and mapper to the volume and add it to the render

Example – Microscopy images of a sea sponge



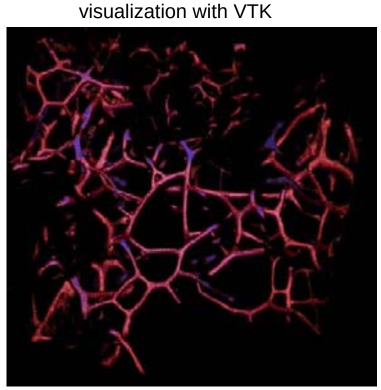
Slice 0 Slice 1

(slices 2-37 not shown)



Slice 38 Slice 39

The resulting volume visualization with VTK



Data obtained with kind permission from the Biomedical Imaging Research Unit (BIRU), University of Auckland, New Zealand

5.7 References

Marching Cubes references

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Volume Visualization references

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