COMPSCI 716 S2 T - Assignment 4
Due Date: Saturday 21st October 2006, 9pm

Computer Science

This assignment is worth 10% of your final mark for COMPSCI 716 S2 T.

1. Data Transformation and Reconstruction   (14 Marks)

Download the zip-file `Ass4Q1.zip` from the assignment webpage. The enclosed program `InterpolatedSurface.cpp` draws the function

$$f(x, y) = -0.5xy + 0.2e^x - 0.1\sin(2\pi y)$$

within the interval $[0,1] \times [0,1]$ using a Gouraud shaded surface. You can toggle the visibility of the surface by pressing the '1' key.

Assume the function $f(x,y)$ is sampled such that $f_{ij} = f(0.25i, 0.25j)$.

(a) [3 marks] Compute the bilinear interpolant of this surface within the interval $[0,1] \times [0,1]$ and draw it as a wire frame mesh. Please represent each bilinear surface patch with 3x3 polygons, i.e. the bilinearly interpolated surface is represented by 12 x 12 polygons. Modify the program such that the visibility of the bilinearly interpolated surface is toggled with the '2' key.

(b) [3 marks] Compute the surface gradient of $f(x,y)$ at each sample points analytically and visualize it using thin blue cylinders. Modify the program such that the visibility of these cylinders is toggled with the '3' key. What is the value for the gradient at the point (0.5, 0.5)? Write your answer into the document `Ass4Answers.doc`.

(c) [4 marks] Approximate the surface gradient of the bilinear interpolant of $f(x,y)$ at each sample points by computing the surface gradients of all bilinear patches sharing that point and averaging these value. Visualize these gradients using thin green cylinders. Modify the program such that the visibility of these cylinders is toggled with the '4' key. What is the value for the gradient computed with this method at the point (0.5, 0.5)? Write your answer into the document `Ass4Answers.doc`.

(d) [4 marks] Approximate the surface gradient of the bilinear interpolant of $f(x,y)$ at the sample points using central differences and visualize it using thin green cylinders. Modify the program such that the visibility of these cylinders is toggled with the '5' key. What is the value for the gradient computed with this method at the point (0.5, 0.5)? Write your answer into the document `Ass4Answers.doc`.

NOTE: The surface gradient is a 2D function. In order to display it in your visualization you have to represent it by a 3D icon. Find a suitable representation which can be perceived easily, e.g if you draw the gradient icons in the xy-plane then it's difficult to see to which points if the surface they belong. If you choose translate them in z-direction to the corresponding surface point then some icons will lie below and some above the surface and some will intersect it. Find a visually more pleasing and informative display.

2. The "Marching Cubes" Algorithm   (9 Marks)

(a) [2 Marks] A Marching Cubes algorithm is being used to output the 0-isosurface of a scalar field. The figure below shows on the left one cube configuration. The numbers written beside the vertices are the field values at those vertices. The figure below on the right shows the coordinates of the cube on the left.
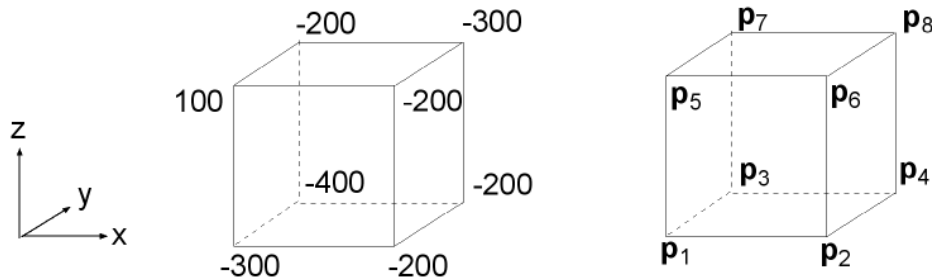
What are the vertices of the polygon(s) generated for the cube configuration on the left? Express the polygon vertices in terms of the cube vertices $p_1,…,p_8$.

(b) [4 Marks] The marching cubes algorithm is used to output the 0-isosurface in a scalar field. The figure below shows a face and the field values at the vertices of the face.
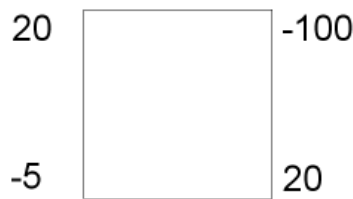
As explained in the lectures there are two possible topologies for the 0-isocontour of within this face. **Compute** which topology is that of the bilinear interpolant.

(c) [3 Marks] The lecture notes introduced an algorithm to build the look-up table for the Marching Cubes algorithm.

(i) Which polygon(s) would the algorithm produce for the configuration below if you use the cube table from slide 17 in handout 5? Please draw your answer (using a drawing program or by scanning a hand drawn image) into the document `Ass4Answers.doc`.

● High vertex (1)

○ Low vertex (0)
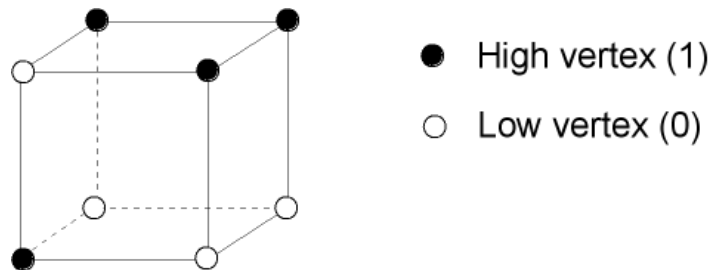
(ii) How many different ways are there to form topological polygons from the edge intersections in the above configuration?

3. *Volume Rendering*   *(7 Marks)*

Download the zip-file `Ass4Q3.zip` from the assignment webpage. The enclosed program `Pelvis.cpp` reads a Computed Tomography data set of a pelvis. All values within the volume are between 0 and 255 and are of type unsigned char.

(a) [1Marks] Find a nice picture of a pelvis bone (use a drawing or an image) and add it together with a reference to your answer sheet.

(b) [6 Marks] Complete the file `Pelvis.cpp` so that it renders the pelvis data set using **direct volume rendering**. Define opacity and colour transfer functions so that the skin, muscle, and bone layer are all visible using anatomically realistic colours.

4. *Vector Field Visualization*   *(8 Marks)*

Given is a vector field

$$\mathbf{v}(x) = \begin{pmatrix} 2y \\ -2x \\ 0.1 \end{pmatrix}$$

A streamline $\mathbf{s}(t)$ is defined by

$$\frac{d\mathbf{s}}{dt} = \mathbf{v}(\mathbf{s}(t)), \qquad \mathbf{s}(0) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(a) [2 marks] Proof that

$$\mathbf{s}(t) = \begin{pmatrix} \sin 2t \\ \cos 2t \\ 0.1t \end{pmatrix}$$

is the closed form of the streamline defined above.

(b) [6 marks] Download the zip-file `Ass4Q4.zip` from the assignment webpage. The enclosed OpenGL program `Streamlines.cpp` draws the closed form of the streamline above in yellow. Complete the program so that it draws additionally two approximations of the above streamline computed by numerically solving the ordinary differential equations describing the streamline. The first approximation should be computed using 100 Euler steps of size $\Delta t = 0.1$ and be drawn in blue. The second approximation should be computed using 100 steps of size $\Delta t = 0.1$ with the Mid-point method and be drawn in red.
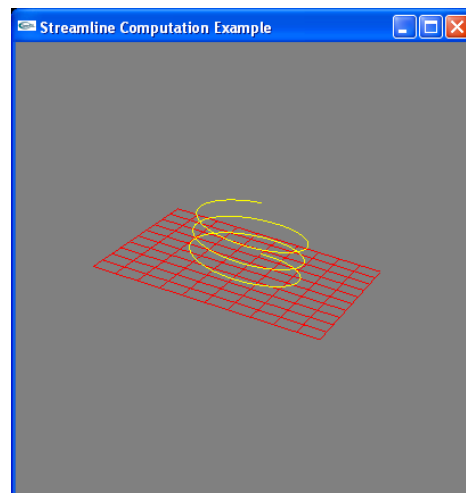
5. Tensor Field Visualization   *(7 Marks)*

(a) [4 Marks] What are the eigenvalues and eigenvectors of the 3D symmetric second-order tensor

$$\mathbf{T} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 2 \end{pmatrix}$$

(b) [3 Marks] Assumed you are given a unit sphere and the eigenvalues λ_1, λ_2, and λ_3 and the corresponding eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ of a 3D symmetric second-order tensor. Give the transformation matrix which scales and rotates the unit sphere into a tensor ellipsoid representing the given tensor.

☺ *Enjoy!*

Assignment submission

All files should include your name, UPI and ID in a comment at the beginning of the file. All your files should be able to be compiled under .NET 2005 without requiring any editing. In particular they should include the correct VTK and/or OpenGL libraries. All your files should include adequate documentation.

The assignment due date is Saturday the 21st October 2006, 9pm.

Please submit your assignments using the assignment dropbox. If the dropbox server is down please create a zip-file which contains all the files you want to submit and email it to me. The name of the zip-file must be "Ass4_<your_UPI>.zip" (e.g. `Ass4_bwue001.zip`).
No assignments are accepted after the due date.

Please Submit:
- The file `InterpolatedSurface.cpp` with the missing code completed.
- The file `Pelvis.cpp` with the missing code completed.
- The file `Streamlines.cpp` with the missing code completed.
- A Word document `Ass4Answers.doc` with the answers to all questions except of the programming tasks. Please use an equation editor such as Microsoft Equation or MathType to write mathematical formulas.