Basic recurrence: $\mathrm{T}(n) = 2\,\mathrm{T}\left(\frac{n}{2}\right) + cn$; $\mathrm{T}(1) = 0$.

Its meaning: running time for processing n data items is equal to running time of separate processing of two halves (of size $n/2$ each) plus linear time for merging the processed halves into the whole processed collection of n items. This scheme (processing of halves and linear merging of results) is recursively repeating for each subcollection

Telescoping to solve the recurrence: for simplicity, let us restrict the derivation to data sizes that are powers of 2, that is, $n = 2^m$ where $m = \log_2 n$. After substituting 2^m for n, the recurrence becomes as follows:

$$\mathrm{T}(2^m) = 2\cdot\mathrm{T}(2^{m-1}) + c\cdot 2^m$$

The same relationship holds for each value of n, in particular, 2^m, 2^{m-1}, 2^{m-2}, …, 2, that is:

$$\mathrm{T}(2^{m-1}) = 2\cdot\mathrm{T}(2^{m-2}) + c\cdot 2^{m-1}$$

$$\mathrm{T}(2^{m-2}) = 2\cdot\mathrm{T}(2^{m-3}) + c\cdot 2^{m-2}$$

$$\dots$$

$$\mathrm{T}(2) = 2\cdot\mathrm{T}(1) + c\cdot 2$$

To make telescoping more transparent, the recurrence should have such a form that its left side depends on 2^m in the same fashion as the right side depends on 2^{m-1}. To obtain such a feature, both sides of the basic recurrence should be divided by $n = 2^m$:

$$\frac{\mathrm{T}(n)}{n} = \frac{\mathrm{T}\left(\frac{n}{2}\right)}{\frac{n}{2}} + c, \qquad \text{or} \qquad \frac{\mathrm{T}(2^m)}{2^m} = \frac{\mathrm{T}(2^{m-1})}{2^{m-1}} + c$$

For this latter representation, telescoping becomes very simple: to sequentially substitute the fractional term in the right side of the recurrence with the equivalent relationship:

$$\frac{\mathrm{T}(2^m)}{2^m} = \left[\frac{\mathrm{T}(2^{m-2})}{2^{m-2}} + c\right] + c = \left[\left[\frac{\mathrm{T}(2^{m-3})}{2^{m-3}} + c\right] + c\right] + c$$

$$= \dots = \left[\left[\dots\left[\frac{\mathrm{T}(2^0)}{2^0} + c\right]\dots + c\right] + c\right] + c = mc$$

To make the substitution easier, the relationships for gradually decreasing arguments are written in column, left and right parts of the column are summed together, and the similar terms in the both sums are reduced:

$$\begin{array}{c} \frac{\mathrm{T}(2^m)}{2^m} = \frac{\mathrm{T}(2^{m-1})}{2^{m-1}} + c \\ \frac{\mathrm{T}(2^{m-1})}{2^{m-1}} = \frac{\mathrm{T}(2^{m-2})}{2^{m-2}} + c \\ \frac{\mathrm{T}(2^{m-2})}{2^{m-2}} = \frac{\mathrm{T}(2^{m-3})}{2^{m-3}} + c \\ \dots \\ \frac{\mathrm{T}(2^1)}{2^1} = \frac{\mathrm{T}(2^0)}{2^0} + c \end{array} \Rightarrow \left\{\begin{array}{l} \frac{\mathrm{T}(2^m)}{2^m} \\ + \frac{\mathrm{T}(2^{m-1})}{2^{m-1}} \\ + \frac{\mathrm{T}(2^{m-2})}{2^{m-2}} \\ \dots \\ + \frac{\mathrm{T}(2^1)}{2^1} \end{array}\right\} = \left\{\begin{array}{l} \frac{\mathrm{T}(2^{m-1})}{2^{m-1}} + c \\ + \frac{\mathrm{T}(2^{m-2})}{2^{m-2}} + c \\ + \frac{\mathrm{T}(2^{m-3})}{2^{m-3}} + c \\ \dots \\ + \frac{\mathrm{T}(2^0)}{2^0} + c \end{array}\right\} \Rightarrow \frac{\mathrm{T}(2^m)}{2^m} = \frac{\mathrm{T}(2^0)}{2^0} + cm$$

Reduction of the similar terms and substitution of T(1) = 0 results in the following close form formula:

$$\frac{\mathrm{T}(2^m)}{2^m} = cm \quad \text{or} \quad \mathrm{T}(2^m) = cm\cdot 2^m\text{, that is, } \mathrm{T}(n) = cn\log_2 n$$