

TUTORIAL-2

Example-1: Suppose we are analyzing two types of algorithms (say algorithm-A & algorithm-B). The running time of algorithm-A is $0.01 * (\log_2 n)^2$ ms and algorithm-B is 2 ms. We want to find out for what value of n (here n is input size) the algorithm-B is faster.

Faster algorithm means the algorithm which takes less amount of time to execute. In this case to become the algorithm faster,

$$2 < 0.01 * (\log_2 n)^2$$

$$0.01 * (\log_2 n)^2 > 2$$

$$\log_2 n > \sqrt{2} * 10$$

$$n > 2^{\sqrt{2} * 10}$$

$$n > 18080$$

So, the algorithm-B is faster for the values greater than 18080.

Example-2: Suppose we have an algorithm with a complexity of $O(\log_{10} \log_{10} n)$. The running time of the algorithm is 10 ms for input size $n=10^3$. What should be the running time of that algorithm for $n=10^{17}$?

We can calculate the running time from the complexity of the algorithm using the function, $T(n) = k * \log_{10} \log_{10} n$

$$\text{For } T(n) = 10 \text{ ms and } n = 10^3.$$

$$k = 10 / (\log_{10} \log_{10} 10^3) = 10/3$$

$$\text{When } n=10^{17}$$

$$T(n) = k * \log_{10} \log_{10} n = (10/3) * \log_{10} \log_{10} 10^{17} = 25.76 \text{ ms.}$$

Example-3: Suppose we have an algorithm of running time $T(n) = \log_m n$. We have to select the value of m either 5 or 7. What value of m should we select to get lowest running time of the algorithm ?

We can write $\log_m n = \ln n / \ln m$

$$T(n) = \ln n / \ln m$$

$$\text{For } m=5, T(n) = 0.6062 * \ln n$$

For $m=7$, $T(n) = 0.5139 \cdot \ln n$

So we will select $m=7$ to get lowest running time.

Example-4: We want to calculate the running time and Big-oh of the following algorithm.

```
for(int i= n; i > 0: i= i/2)
    for(int j= 0; j <i: j++)
        System.out.println("415.220");
```

Let us assume that $n = 2^m$ where m is a constant.

In this example we should calculate the number of execution of `println()` method considering both the loop at a time because the inner loop variable j is dependent on outer loop variable i .

For $i= n$ the inner loop will execute for $n (=2^m)$ times.
For $i= n/2$ the inner loop will execute for $n/2 (=2^{m-1})$ times.
For $i= n/4$ the inner loop will execute for $n/4 (=2^{m-2})$ times.
.....

For $i= 1$ the inner loop will execute for $1 (=2^0)$ times.

Total number of execution of `println()` method is

$$2^0 + 2^1 + 2^2 + \dots + 2^{m-1} + 2^m = 2^{m+1} - 1 = 2 * 2^m - 1 = 2n - 1$$

Running time of the above algorithm $T(n) = k(2n-1)$ where k is the constant time required to execute `println()` method one time .

$T(n)$ is $O(n)$.