Exercises[1]

For Chapters 01 to 05

See Exercise Sections in
Reinhard Klette: Concise Computer Vision
Springer-Verlag, London, 2014

[1]See last slide for copyright information.

Agenda

1.1-1.3

Exercise

Show the correctness of

$$\sigma_I^2 = \left[\frac{1}{|\Omega|} \sum_{(x,y)\in\Omega} I(x,y)^2 \right] - \mu_I^2$$

Exercise

Who was Fourier? When was the Fast Fourier Transform designed for the first time? How is the Fourier transform related to optical lens systems?

Exercise

Assume an $N \times N$ image, for even N. Prove that a multiplication with $(-1)^{x+y}$ in the spatial domain causes a shift by $N/2$ (both in row and column direction) in the frequency domain.

1.4-1.5

Exercise

In extension of the given example in the lectures, transform a few more (easy) RGB values manually into corresponding HSI values.

Exercise

Let (δ, S, M) *be the color representation in the HSI space. Justify the following steps for recovering the RGB components in the following special cases:*

- *If* $\delta \in [0, 2\pi/3]$ *then* $B = (1 - S)M$.
- *If* $\delta \in [2\pi/3, 4\pi/3]$ *then* $R = (1 - S)M$.
- *If* $\delta \in [4\pi/3, 2\pi]$ *then* $G = (1 - S)M$.

How can we compute the remaining components in each of the above cases?

1.7

Exercise

In the CIE's RGB color space (which models human color perception), scalars R, G, or B may also be negative. Provide a physical interpretation (obviously, we cannot subtract light from a given spectrum).

Agenda

2.1-2.2

Exercise

Linear local operators are those which can be defined by a convolution. Classify the following whether they are linear operators or not: box, median, histogram equalization, sigma-filter, Gauss filter, and LoG.

Exercise

Equalization of color pictures is an interesting area of research. Discuss why the following approach is expected to be imperfect: do histogram equalization for all three color (e.g., RGB) channels separately; use the resulting scalar pictures as color channels for the resulting image.

2.3-2.5

Exercise

Prove that conditional scaling correctly generated an image J which has mean and variance identical to those corresponding values of image I used for normalization.

Exercise

Specify exactly how the integral image can be used for minimizing run time for a box filter of large kernel size.

Exercise

Following the derivation of a Laplace mask in the lecture, what could be a filter kernel for the quadratic variation (instead of the one derived for the Laplace operator)?

2.6-2.7

Exercise

Prove that Sobel masks are of the form $\mathbf{d}\mathbf{s}^\top$ *and* $\mathbf{s}\mathbf{d}^\top$ *for 3D vectors* $\mathbf{s}$ *and* $\mathbf{d}$ *which satisfy the assumptions of the Meer-Georgescu algorithm for edge detection.*

Exercise

The sigma-filter replaces $I(p)$ *by* $J(p)$ *as defined in the lecture. The procedure uses the histogram* $H(u)$ *computed for values* u *in the window* $W_p(I)$ *which belong to the interval* $[I(p)-\sigma, I(p)+\sigma]$. *Alternatively, a direct computation can be applied:*

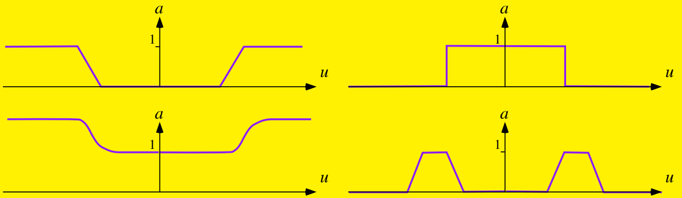
$$J(p) = \frac{\sum_{q \in Z_{p,\sigma}} I(q)}{|Z_{p,\sigma}|}$$

where $Z_{p,\sigma} = \{q \in W_p(I) : \; I(p)-\sigma \leq I(q) \leq I(p)+\sigma\}$. *Analyze possible advantages of this approach for small windows.*

2.8

Exercise

Sketch, as in the figure



filter curves in the frequency domain which might be called "exponential low-emphasis filter" and "ideal band-pass filter".

Agenda

3.1

Exercise

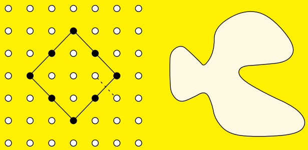
Consider 6-adjacency in images, for pixel location $p = (x, y)$: In an even row (i.e. y is even) use

$$A_6^{even} = A_4 \cup \{(x+1, y+1), (x+1, y-1)\}$$

and in an odd row use

$$A_6^{odd} = A_4 \cup \{(x-1, y+1), (x-1, y-1)\}$$

Consider the case on the left in the figure. Discuss the result. Consider a chess-board type of binary image. What is the result?



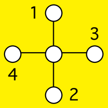
3.2–3.3

Exercise

K-adjacency requires a test about ownership of the central corner of a 2×2 pixel configuration in some cases. Specify the condition when such a test becomes necessary. For example, if all four pixels have identical values, no test is needed; and if three pixels have identical values, no test is needed either.

Exercise

What will happen if you use the local circular order



in the Voss algorithm, rather than a clockwise or counter-clockwise local circular order?

3.4–3.5

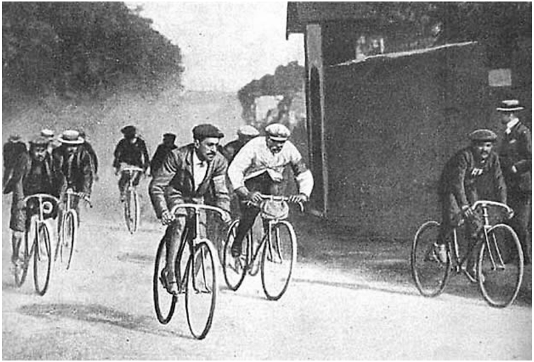
Exercise

Discuss similarities and differences for the eccentricity measure and the shape factor. Use examples of binary shapes in your discussion.

Exercise

Explain how image gradient information can be used to enhance accuracy and speed of circle detection using the concept of the Hough transform.

3.6



Exercise

Design a Hough transform method for detecting parabolas, for example as present in an image as shown in the figure.

Agenda

4.1–4.2

Exercise

Calculate the coefficients of the Taylor expansion of the function $I(x, y, t) = 2x^2 + xy^2 + xyt + 5t^3$ up to derivatives of the 3rd order.

Exercise

Verify the equations

$$
\begin{aligned}
\frac{\partial E_{data}}{\partial u_{xy}}(u, v) &= 2\left[I_x(x, y)\, u_{xy} + I_y(x, y)\, v_{xy} + I_t(x, y)\right] I_x(x, y) \\
\frac{\partial E_{data}}{\partial v_{xy}}(u, v) &= 2\left[I_x(x, y)\, u_{xy} + I_y(x, y)\, v_{xy} + I_t(x, y)\right] I_y(x, y) \\
\frac{\partial E_{smooth}}{\partial u_{xy}}(u, v) &= 2\ \left[(u_{xy} - u_{x+1,y}) + (u_{xy} - u_{x,y+1})\right. \\
&\quad \left. + (u_{xy} - u_{x-1,y}) + (u_{xy} - u_{x,y-1})\right]
\end{aligned}
$$

4.3–4.4

Exercise

An initialization by zero in the Horn-Schunck algorithm would not be possible if the resulting initial u- and v-values would also be zero. Verify that this is not happening in general (i.e., if there exists motion) at the start of the iteration of this algorithm.

Exercise

Use the provided simple data example for manual calculations of the first 3 iterations of the Horn-Schunck algorithm, using zero as initialization, and simple two-pixel approximation schemes for I_x, I_y, *and* I_t.

4.5–4.6

Exercise

Verify the equation

$$(\mathbf{G}^\top \mathbf{G})^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Exercise

Show for the Lucas-Kanade algorithm that the matrix inversion in the equation

$$\mathbf{u} = (\mathbf{G}^\top \mathbf{G})^{-1} \mathbf{G}^\top \mathbf{B}$$

fails if image gradients in the selected neighborhood are parallel to each other. Is this possible to happen for real image data? Check a linear algebra book about how to tackle such singularities in order to get a least-square solution.

Agenda

5.1

Exercise

Suppose that the mean-shift algorithm for features clustering uses a window of K grid points in a feature space; the histogram table includes C nonempty cells. Exactly $M \geq C$ different grid points are visited as the result of all mean calculations.
(1) Show that the total time of mean-shift algorithm is of complexity $\mathcal{O}(MK + M^2)$ where power two comes from multiple visits over the same path from a grid point to the stable mean.
(2) In order to avoid multiple visits apply the ideas of the UNION-FIND algorithm, i.e. when you visit the grid point $v = u + m_g(u)$ as the result of mean shifting to grid point u then assign to q the segment $\mathrm{SEG}(v) = \mathrm{UNION}(\mathrm{SEG}(u), \mathrm{FIND}(q))$ where v is rounded to the nearest grid point, and FIND(q) returns the segment q belongs to.
Show that the use of UNION-FIND data structures reduces the time for mean-shift clustering from $\mathcal{O}(MK + M^2)$ to $\mathcal{O}(MK)$.

5.2

Exercise

If we like to use the mean-shift idea to identify local maxima in a histogram in the feature space then we find all grid points in feature space for which shift $m_g(u)$ is sufficiently small:

$$\overline{M} = \{u : \|m_g(u)\|_2 < \Delta u\}$$

where Δu is a grid step in the feature space.
Show that using a window with K grid points for the feature histogram with C nonempty cells requires $\mathcal{O}(CK)$ time to identify the set of grid points $\overline{M}$ approximating local maxima.

5.3

Exercise

We define the recovery rate, which is useful when comparing different segmentation or clustering techniques.
We consider clustering of vectors $\mathbf{x} \in \mathbb{R}^d$*, for* $d > 0$*. For example, consider vectors* $\mathbf{x} = [x, y, R, G, B]^\top$*, with* $d = 5$*, for segmenting a color image.*
Our general definition is: A clustering algorithm $\mathcal{A}$ *maps a finite set* S *of points in* $\mathbb{R}^d$ *into a family of pairwise-disjoint clusters. A segmentation algorithm is an example for this more general definition.*
Assume that we have an algorithm A *which maps* S *into* $m > 0$ *pairwise disjoint clusters* C_i *(e.g. segments), for* $i = 1, 2, \ldots, m$*, containing* $m_i = |C_i|$ *vectors* $\mathbf{x}_{ij} \in \mathbb{R}^d$*. When segmenting an image, the sum of all* m_i*'s is equal to the cardinality* $|\Omega|$*. We call the* C_i*'s the old clusters.*

5.3 - Continued

Exercise

*Now consider another clustering algorithm B which maps the same set S into $n > 0$ pairwise disjoint clusters G_k, for $k = 1, 2, \ldots, n$. We call the G_k's the new clusters. A new cluster G_k contains vectors **x** which were assigned by A to old clusters. Let*

$$G_k = \cup_{j=1}^{s_k} G_{k_j}$$

where each G_{k_j} is a non-empty subset of exactly one old cluster C_i, for $j = 1, 2, \ldots, s_k$. Indices or names i and k of old and new clusters are not related to each other, and in general we can expect that $n \neq m$. Let us assume that $n \leq m$ (i.e. the number of new clusters is upper bounded by the the number of old ones).
An ideal recovery would be if each old cluster is equal to one of the new clusters, i.e. the two sets of clusters are just permutations by names, and $n = m$.

5.3 - Continued

Exercise

Both algorithms A and B, would, for example, lead to the same image segmentation result; segments might be just labeled by different colors. We select contributing sets $G_{1_{j_1}}, G_{2_{j_2}}, \ldots, G_{n_{j_n}}$, *one for each new cluster, which optimize the following two properties:*

1. *For each pair* a_{j_a} *and* b_{j_b} *of two different indices in the set* $\{1_{j_1}, 2_{j_2}, \ldots, n_{j_n}\}$, *there exist two different old clusters* C_a *and* C_b *such that* $G_{a_{j_a}} \subseteq C_a$ *and* $G_{b_{j_b}} \subseteq C_b$.
2. *Let* C_k *be the old cluster assigned to subset* $G_{k_{j_k}}$ *of the new cluster* G_k *in the sense of the previous item such that the sum*

$$\sum_{k=1}^{m} \frac{|G_{k_{j_k}}|}{|C_k|}$$

is maximized; and this maximization is achieved for all possible index sets $\{1_{j_1}, 2_{j_2}, \ldots, n_{j_n}\}$.

5.3 - Continued

Exercise

The selected contributing sets $G_{1_{j_1}}, G_{2_{j_2}}, \ldots, G_{n_{j_n}}$ are thus assigning each new cluster G_k to exactly one old cluster C_k by maximizing the given sum. In particular, a chosen subset $G_{k_{j_k}}$ might be not the one of maximum cardinality in the partition of G_k; the selected contributing sets have been selected by maximizing the total sum. Then, value

$$\mathcal{R}_A(B) = \sum_{k=1}^{n} \frac{|G_{k_{j_k}}|}{|C_k|} \times \frac{100\%}{n}$$

is called the recovery rate for clustering algorithm B with respect to algorithm A for input set S.
Discuss the time complexity of the proposed measure for a recovery rate for clustering (and segmentation in particular).

5.4

Exercise

The lectures suggested to replace the simple Potts smoothness term by an alternative smoothness term in BP segmentation. For example, if μ_1 and μ_2 are the intensity means in adjacent segments, then constant c in equation

$$E_{smooth}(l-h) = E_{smooth}(a) = \begin{cases} 0 & \text{if } a = 0 \\ c & \text{otherwise} \end{cases}$$

can be replaced by a term where c is scaled in dependency of difference $|\mu_1 - \mu_2|$.
Specify and discuss modified smoothness functions based on this equation which include data characteristics into the smoothness term.

5.5

Exercise

Consider the dissimilarity measure $\mathcal{D}$ defined by equation

$$\mathcal{D}(A, B) = \frac{|(A \cup B) \setminus (A \cap B)|}{|A \cup B|}$$

We recall: The general axioms of a metric d on a base set S are as follows:

1. $f = g$ *iff* $d(f, g) = 0$,
2. $d(f, g) = d(g, f)$ *(symmetry), and*
3. $d(f, g) \leq d(f, h) + d(h, g)$, *for a third element h (triangular inequality).*

for all $f, g, h \in S$. Show that $\mathcal{D}$ is a metric on the family of sets of pixels. Each set of pixels has a defined cardinality (i.e. the number of pixels in this set).

Copyright Information