Exercises

See also exercises to discussed sections in Reinhard Klette: Concise Computer Vision Springer-Verlag, London, 2014

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Agenda

1 Lecture 1: Images, Windows, Histograms
2 Lecture 2: Fourier Transform, Frequency Domain, Fourier Filtering
3 Lecture 3: Colour, RGB, HSI
4 Lecture 4: Point, Local, and Global Operators
5 Lecture 5: Smoothing, Sharpening, Edges
6 Lecture 6: Adjacency, Regions, and Their Borders
7 Lecture 7: Area, Length, and Curvature
8 Lecture 8: Cameras, Central Projection, Binocular Vision
9 Lecture 9: Calibration, Rectification, Stereo Vision
10 Lecture 10: Stereo Matching, Depth Maps, and an Application
Exercise (1.1)

*Explain the processes of sampling and integrating as discussed for the recording of a digital image.*

Exercise (1.2)

*How are 4-adjacency and 8-adjacency defined in the grid cell model, and how in the grid point model?*

Exercise (1.3)

*How follows the definition of connectedness from a given pixel adjacency?*

Exercise (1.4)

*Which of the coordinate systems A to F on the next page are left-hand, and which are right-hand?*
Examples of Coordinate Systems for Exercise (1.4)
1.5-1.7

Exercise (1.5)

*How to generate a binary image from a grey-level image by using thresholding?*

Exercise (1.6)

*Assume an image \( I \) which has a constant value \( u \) at all of its \(|\Omega|\) pixel locations. What are mean and standard deviation of image \( I \) ? Now consider an image \( J \) where 50% of its pixels have value 0, and the other 50% have value 10. What are mean and standard deviation of image \( J \)?*

Exercise (1.7, optional, not examinable)

*Show that*

\[
\sigma^2_I = \left[ \frac{1}{|\Omega|} \sum_{(x,y) \in \Omega} I(x, y)^2 \right] - \mu_I^2
\]
Exercise (1.8)

Assume a $200 \times 200$ image $I$ with values $I(x, y) = x$. Draw the histogram and the cumulative histogram of image $I$. Specify the relative histogram and the relative cumulative histogram of $I$.

Exercise (1.9)

Assume a $200 \times 200$ image $I$ with values $I(x, y) = 0$ if $x \leq 100$ and $y \leq 100$, and $I(x, y) = 10$ otherwise. Draw the histogram and the cumulative histogram of image $I$. Specify the relative histogram and the relative cumulative histogram of $I$. 
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Exercise (2.1)

For a complex number \( z = a + ib \), what is the meaning of \( i \), how to define the absolute value (or amplitude) of \( z \), and what is the phase of \( z \)?

Exercise (2.2)

Specify the equation of the 2D DFT of an image \( I \). Name all the used symbols in your equation, such as \( x \) and \( y \), \( u \) and \( v \), \( N_{\text{rows}} \) and \( N_{\text{cols}} \), and so forth.

Exercise (2.3)

List the steps of Fourier filtering of an image and explain each of those steps briefly.

Exercise (2.4)

Provide the Eulerian formula for \( \exp(i\alpha) \).
2.5-2.7

Exercise (2.5)

*Why are all the powers of $W = \exp\left(\frac{i2\pi}{N}\right)$ located on the unit circle, for any $N \geq 1$?*

Exercise (2.6)

*How are low frequencies and high frequencies related to short wavelengths and long wavelengths, and to the origin in frequency space?*

Exercise (2.7)

*How do we take the* symmetry property

$$I(N_{cols} - u, N_{rows} - v) = I(-u, -v) = I(u, v)^*$$

*into account when defining filter functions?*
Exercise (2.8)

Sketch, as in the figure

Filter curves in the frequency domain which might be called “exponential low-emphasis filter” and “ideal band-pass filter”.

Exercise (2.9)

List tasks which may be solved by using a high-pass filter, a low-pass filter, a high-emphasis filter, and a band-pass filter.
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Exercise (3.1)

*What is the approximate range of frequencies or wavelengths of visible light within the electromagnetic spectrum? Where is Red, and where is Blue?*

Exercise (3.2)

*Let X, Y, and Z be tristimulus values of a monochromatic energy distribution function \(L(\lambda)\). How are the \(x\) and \(y\) coordinates of \(L(\lambda)\) defined by the CIE?*

Exercise (3.3)

*What is the gamut when representing or perceiving colors? How can a gamut be characterised in the \(xy\) CIE color space?*
Exercise (3.4)

_Hue identifies a color; intensity and saturation are properties of a color. Why is a grey-level not a color?_

Exercise (3.5)

_The figure above shows on the right one color channel for the RGB image shown on the left; the shown channel is either Red, Green, or Blue. Which one? Explain your answer._
Exercise (3.6)

What is the difference between an additive and a subtractive color model?

Exercise (3.7)

In extension of the given examples in the lectures, transform a few more (easy) RGB values manually into corresponding HSI values, where

\[
H = \begin{cases} 
\delta, & \text{if } B \leq G \\
2\pi - \delta, & \text{if } B > G 
\end{cases} 
\]

with

\[
\delta = \arccos \frac{(R - G) + (R - B)}{2\sqrt{(R - G)^2 + (R - B)(G - B)}} \quad \text{in } [0, \pi)
\]

\[
S = 1 - 3 \cdot \min\{R, G, B\} \cdot \frac{1}{R + G + B}
\]

You are supposed to know the definition of the intensity value yourself.
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Exercise (4.1)

Explain how a point operator is defined by a gradation function \( g \), with \( v = g(u) \) for \( 0 \leq u \leq G_{\text{max}} \).

Exercise (4.2)

Which gradation function can be used for achieving an approximate histogram equalisation of an image? Explain symbols used in your formula.

Exercise (4.3)

Derive a gradation function for linear scaling, assuming that the given image only has positive histogram values in the interval \( [u_{\text{min}}, u_{\text{max}}] \), and the gradation function should map those values linearly onto the whole scale \( [0, G_{\text{max}}] \).
Exercise (4.4)

Define the general concept of a local parallel operator for transforming an image $I$ into a new image $J$.

Exercise (4.5)

Linear local operators are those which can be defined by a convolution. Classify the following whether they are linear operators or not: box, median, histogram equalization, sigma-filter, Gauss filter, and LoG.

Exercise (4.6)

Specify the filter kernel of a $3 \times 3$ box filter (also called mean operator).

Exercise (4.7)

What is an example for a global image operator?
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Exercise (5.1)

What is the median of the set \{2, 8, 2, 3, 6, 1, 2, 4, 1, 6, 5\}? 

Exercise (5.2)

Provide the defining formula of a zero-mean 2D Gauss function. Name all the involved symbols in your formula.

Exercise (5.3)

Explain the steps involved for defining a filter kernel for a discrete Gauss filter for smoothing images.

Exercise (5.4)

What is a possible way for sharpening an image?
Exercise (5.5)

Explain how to detect step-edges in images by using noise removal, approximations of first-order derivatives, or approximations of second order derivatives. In your answer you may refer to the figure above.
5.6 - 5.7

Exercise (5.6)

*What are the results of the Sobel operator for the shown two $3 \times 3$ image windows?*

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Exercise (5.7)

*Apply a discrete Laplacian defined by the following filter kernel to the two input windows of the previous exercise. Discuss your results.*

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<tr>
<td>0</td>
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Exercise (6.1)

Let $p = (4, 10)$. What is the set $A_4(p) = p + A_4$ for this particular pixel location? What is the 4-neighborhood of this pixel location $p$?

Exercise (6.2)

How many black 8-regions and how many white 4-regions in the figure above?
Exercise (6.3)

Assume the key “gray > white > black” for K-adjacency. How many gray components in the figure above? How many white components?

Exercise (6.4)

Will borders of regions change in a binary image if we use either “black < white” or “white < black”? Explain.
Exercise (6.5)

Assume 8-adjacency for black pixels and a top-to-bottom, left-to-right image scan. Draw into the figure above the steps of border tracing when assuming counter-clockwise local circular orders. Indicate the start pixel of your border tracing sequence.
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Exercise (7.1)

*Explain by means of an example why the length of a 4-path is not a way to ensure accurate length measurements in an image.*

Exercise (7.2)

*How to improve length estimation by using different weights for diagonal or isothetic steps along an 8-path in the image grid? Discuss limitations of this approach.*

Exercise (7.3)

*How to estimate the area of a region in an image? Consider the case that the same planar object has been digitised in two different images of smaller and higher image resolution (see example on next page). Can you expect that this leads to identical area estimates for both images?*
Exercise (7.4)

What kind of theoretical model can be applied for digitising a planar object (as shown on the left) into binary images as shown in the middle or on the right? (Hint: Gauss had a digitisation model in mind when discussing area estimation)
Exercise (7.5)

Describe how curvature can be calculated in the Euclidean plane (i.e. continuous curves) by considering the rate of change of the angle of a tangential line at a point p moving along the given curve.

Exercise (7.6)

Consider samples $x(0) = x_i$, $x(1) = x_{i-k}$, $x(2) = x_{i+k}$, and $y(0) = x_i$, $y(1) = x_{i-k}$, $y(2) = x_{i+k}$ along a digital curve, for $k \geq 1$, to be used for calculating parameters $a_0$, $a_1$, $a_2$, $b_0$, $b_1$ and $b_2$ of a 2nd order curve

$$x(t) = a_0 + a_1 t + a_2 t^2 \quad \text{and} \quad y(t) = b_0 + b_1 t + b_2 t^2$$

for interpolating the given digital curve at $p_i = (x_i, y_i)$. Calculate

$$\kappa_{\text{tan}}(t) = \frac{\dot{x}(t) \cdot \ddot{y}(t) - \dot{y}(t) \cdot \ddot{x}(t)}{\left[ \dot{x}(t)^2 + \dot{y}(t)^2 \right]^{1.5}}$$
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Exercise (8.1)

Specify the purpose of the Bayer pattern shown above.

Exercise (8.2)

List five properties which are of importance for high-quality cameras used for computer vision solutions.

Exercise (8.3)

What is the meaning of units pps, Hz, and fps?
Exercise (8.4)

*How to measure color accuracy of a camera?*

Exercise (8.5)

*Provide graphical sketches for the barrel and the pincushion transform as known from lens distortion.*

Exercise (8.6)

*How is linearity defined for a monochromatic camera?*

Exercise (8.7)

*What is the principal point for a camera?*
Exercise (8.8)

*What are the equations of central projection of points given in the camera (or sensor) coordinate system into undistorted image coordinates. Name all the used symbols.*

![Diagram of central projection](image)

Exercise (8.9)

*Characterise canonical stereo geometry of two cameras, as illustrated above (Hint: Which geometric constraints need to be satisfied?).*
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Exercise (9.1)

How to map a point $P_w = (X_w, Y_w, Z_w)$ in 3D world coordinates into a representation $P_s = (X_s, Y_s, Z_s)$ in 3D camera coordinates?

Exercise (9.2)

When defining an affine transform in 3D space by a rotation and a translation, does it matter in which order we perform both operations? Explain your answer.

Exercise (9.4)

List 4 examples of intrinsic camera parameters. What are extrinsic camera parameters?
Exercise (9.4)

List a sequence of steps to be performed when doing camera calibration, and specify each step briefly.

Exercise (9.5)

For what purpose do we perform image rectification based on calculated (by camera calibration) intrinsic and extrinsic parameters? What is the benefit when using rectified images for stereo matching?

Exercise (9.6)

Projection centres $O_1$ and $O_2$ and a pixel location $p_1$ in one of the two images define an epipolar plane. How does this epipolar plane define an epipolar line in the second image?
**Exercise (9.7)**

*What is a disparity in canonical stereo geometry? Are smaller or larger disparities defining 3D points closer or further away from the given pair of cameras?*

**Exercise (9.8)**

*Solve the projection equations*  

\[
p_{uL} = (x_{uL}, y_u) = \left( \frac{f \cdot X_s}{Z_s}, \frac{f \cdot Y_s}{Z_s} \right) \\  
p_{uR} = (x_{uR}, y_u) = \left( \frac{f \cdot (X_s - b)}{Z_s}, \frac{f \cdot Y_s}{Z_s} \right) 
\]

*of standard stereo geometry for the unknowns $X_s$, $Z_s$, and $Y_s$.***
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Exercise (10.1)

What is the stereo matching problem of binocular vision? Use a base image $B$ and a match image $M$ in your explanation, and also the technical term “epipolar line”.

Exercise (10.2)

Explain what a depth map as shown above (on the right) is illustrating at differently colored positions.
Exercise (10.3)

For stereo matching, consider the case that $B = R$ and $M = L$. We initiate a correspondence search by selecting $p = (x, y)$ in $B$. What is the search interval in this case in $M$ for a corresponding pixel location $q$?

Exercise (10.4)

Consider the SSD data cost measure

$$E_{SSD}(p, d) = \sum_{i=-l}^{l} \sum_{j=-k}^{k} [B(x + i, y + j) - M(x + d + i, y + j)]^2$$

where $d$ takes all the possible values for the defined search interval for a pixel location $q$ in $M$ which corresponds to $p$ in $B$. If a stereo matcher only uses those $E_{SSD}$ data cost values for selecting the "best match", how to select the best match? What are expected difficulties (i.e. at particular input stereo pairs) for such a simple stereo matcher?
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