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In the following table, bold entries represent optimal (Δ, t) broadcast networks. The $B(3,6) = 40$ entry is a vertex-symmectric non-Cayley graph. If $B(3,7) > 64$ then it is via a non-vertex-symmetric graph. The lower bound for $B(9,12)$ is the double of the $(8,11)$ entry.

Table 1: Orders of the largest known broadcast networks with degree $\leq \Delta$ and broadcast time $\leq T$.

Δ \ T	3	4	5	6	7	8	9	10	11	12
3	**8**	**14**	**24**	**40***	*64*	*96*	*144*	*216*	*324*	*506*
4		**16**	**30**	**56**	*96*	*162*	*272*	*465*	*776*	*1360*
5			**32**	**62**	*116*	*210*	*390*	*686*	*1204*	*2176*
6				**64**	**126**	*234*	*440*	*840*	*1518*	*2808*
7					**128**	**254**	*486*	*952*	*1752*	*3164*
8						**256**	**510**	*1000*	*1971*	*3612*
9							**512**	**1022**	*2020*	*3942*†
10								**1024**	**2046**	*4060*

Table 2: Upper bounds on $B(\Delta, T)$.

Δ \ T	3	4	5	6	7	8	9	10	11	12
3	8	14	24	40	66	108	176	286	464	752
4		16	30	56	104	192	354	652	1200	2208
5			32	62	120	232	448	864	1666	3212
6				64	126	248	488	960	1888	3712
7					128	254	504	1000	1984	3936
8						256	510	1016	2024	4032
9							512	1022	2040	4072
10								1024	2046	4088

Most of the Cayley graphs are based on semidirect products of two cyclic groups $\mathbb{Z}_m \rtimes \mathbb{Z}_n$. In Table 3, the triple (m, n, k) indicates that the homomorphism from $\mathbb{Z}_n$ into $\mathbb{Z}_m^* \cong \mathrm{Aut}(Z_m)$ is determined by the requirement that it map the generator 1 of $\mathbb{Z}_n$ to an element $k \in \mathbb{Z}_m^*$ such that $k^n = 1$. The ordered pairs represent the Δ generators of the Cayley graph in the usual way as elements of the set $\mathbb{Z}_m \times \mathbb{Z}_n$. The group multiplication in $\mathbb{Z}_m \rtimes \mathbb{Z}_n$, which is usually non-commutative, of the elements (m_1, n_1) and (m_2, n_2) is the element $(m_1 + k^{n_1} \cdot m_2, n_1 + n_2)$.

Table 3: Data for Cayley graphs of semidirect products of cyclic groups.

(Δ, T)	(m,n,k)	generators
(4,7)	(24,4,5)	(23,2), (1,2), (17,3), (11,1)
(4,8)	(27,6,2)	(10,3), (15,4), (12,2), (0,3)
(4,9)	(17,16,3)	(5,7), (15,9), (7,11), (16,5)
(4,10)	(31,15,3)	(4,8), (22,7), (5,4), (23,11)
(4,11)	(97,8,5)	(86,5), (65,3), (79,4), (0,4)
(4,12)	(85,16,3)	(39,11), (43,5), (80,14), (45,2)
(5,7)	(29,4,2)	(20,2), (13,2), (26,1), (22,3), (0,2)
(5,8)	(35,6,2)	(12,3), (2,3), (19,2), (1,4), (0,3)
(5,9)	(13,30,2)	(10,22), (9,8), (12,6), (1,24), (0,15)
(5,10)	(49,14,3)	(6,1), (27,13), (46,12), (31,2), (0,7)
(5,11)	(86,14,3)	(7,2), (25,12), (5,6), (49,8), (0,7)
(5,12)	(136,16,3)	(131,15), (15,1), (54,9), (86,7), (0,8)
(6,8)	(13,18,2)	(1,6), (12,12), (10,4), (9,14), (8,7), (11,11)
(6,9)	(44,10,3)	(39,6), (9,4), (41,9), (9,1), (22,6), (22,4)
(6,10)	(35,24,2)	(25,14), (20,10), (32,18), (17,6), (11,1), (12,23)
(6,11)	(69,22,2)	(44,2), (58,20), (60,5), (24,17), (43,16), (8,6)
(6,12)	(117,24,2)	(42,23), (33,1), (59,5), (53,19), (29,20), (4,4)
(7,9)	(27,18,2)	(8,13), (14,5), (7,7), (1,11), (9,8), (18,10), (0,9)
(7,10)	(17,56,3)	(9,34), (15,22), (16,51), (8,5), (10,10), (11,46), (0,28)
(7,11)	(73,24,5)	(27,17), (52,7), (52,11), (3,13), (49,10), (1,14), (0,12)
(7,12)	(113,28,3)	(75,5), (35,23), (29,4), (34,24), (83,20), (49,8), (0,14)
(8,10)	(50,20,3)	(23,3), (1,17), (9,6), (29,14), (34,8), (6,12), (5,15), (35,5)
(8,11)	(73,27,5)	(8,2), (71,25), (6,20), (35,7), (2,3), (18,24), (10,22), (45,5)
(8,12)	(43,84,3)	(29,13), (37,71), (34,5), (8,79), (20,29), (18,55), (34,14), (11,70)
(9,11)	(101,20,2)	(66,6), (8,14), (29,9), (19,11), (35,3), (52,17), (51,9), (16,11), (0,10)
(10,12)	(145,28,2)	(121,2), (6,26), (24,25), (98,3), (77,6), (137,22), (140,23), (15,5), (113,23), (9,5)

For a description of the largest (3,t) broadcast graphs see the following paper.

References

[1] Michael J. Dinneen, Geoffrey Pritchard, and Mark C. Wilson. Degree- and time- constrained broadcast networks. Report CDMTCS-080, Centre for Discrete Mathematics and Theoretical Computer Science, University of Auckland, New Zealand, April 1998. Submitted to *Networks*.