Hierarchies, Ties and Power in Organisational Networks: Model and Analysis

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***Abstract*—An organisational structure consists of a network where employees are connected by working and social ties. Analysing this network, one can discover valuable insights into information flow within the organisation. Moreover, properly defined centrality measures reveal the distribution of power and, therefore, important individuals in the network. We develop this idea and propose a model that is consistent with management theory, and that captures main traits of large corporations. The carcass of the model is an organisational hierarchy. We extend it by allowing additional types of connections such as collaboration, consultation, and friendship. Having both reporting and non-reporting interpersonal ties, our model supports a multilevel approach to social networks. We then formally define power and stability in organisations. These notions enable us to analyse a range of organisational phenomena such as limited hierarchy height, restructuring through flattening, and impact of non-reporting ties. We support our framework with examples and case studies.**

***Keywords*—Organisational network, strong and weak ties, power, flattening**

I. Introduction

Rapid development of information technologies in the last few decades enabled effective communication and management in large corporations, which facilitated their growth. Clearly, the bigger the company, the more incentive there is to identify hidden information inside its structure – the more sense it makes to study how decisions pass from the top levels to the bottom, how individuals interact with each other, and which are the most important positions. Chaotic growth can lead to inefficient management, and, hence, loss of money. This is why corporations are willing to pay large sums of money not only on hiring talented managers, who define the firm's direction, but also on costly business intelligence software that guarantees all the layers are on the same wave length [7].

Behavioral management theories usually consider how different tenuous aspects – such as motivation, expectation or conflicts – define the productivity of an individual. In this work, we move our focus onto structural properties and formal characteristics of organizations, which allow automated and rigorous analysis. An organisational structure can be naturally represented as a network where employees are connected with each other by working relationships. Using this network, one can analyse how the flow of information circulates within the organism of the firm, and uncover problems invisible at first glance. Moreover, centrality measures are capable of detecting the most influential positions in the network, and hence, define the distribution of *power* in organisations.

Power is indeed a multiplex concept. It is often interpreted as potential influence [20], i.e. how much an individual can affect others. Clearly, the strongest relationships in a company are reporting; they define the formal organisational hierarchy. Power in organisations is frequently associated with hierarchical authority; however, sociology and management studies show that informal social communication also grants power. Brass in his work [6] suggested that individual power in organisations comes from a structural perspective, which includes both formal and informal communication.

We follow this idea and propose a network model that is consistent with management theory, and that captures main traits of large corporations. We define the structure of a firm as a network where employees are connected to their managers and each other by working ties. The carcass of the model is an organisational hierarchy. We extend it by allowing additional types of connections between two employees (e.g. collaboration, friendship, family relations and others), and introduce the notion of an *organisational network*. Having both reporting and non-reporting relationships, our model supports a multiplex approach to organisation structures.

The meaning of the model is three-fold. Firstly, by integrating different interpersonal relations, we suggest a uniform approach to social network analysis (SNA) on organisation structures; thus the model extends works on organisational network analysis [8], [13], [19]. Secondly, we provide a centrality measure for organisational networks that enhances established works in social sciences [2], [3]. This measure enables formal analysis of concepts specific to organisations such as capacity and spans of control. Thirdly, our model can be used to derive a novel business intelligence tool, which extracts hidden information from inside the structure; hence providing more accurate decision support.

Our contribution. In this work, we apply social network analysis to our multilevel model. We use the Bonacich power [3] to define power of individuals in the network. Comparing to existing centrality notions, our definition of power is novel in the following aspects: 1) the adjacency matrix of the network takes into account essentially three types of directed links: the influence from a manager to her subordinates, the mutual influence between two employees connected by a non-reporting relation, and the *backflow* influence from a subordinate to her manager. 2) the original formulation of Bonacich power involves a real-valued parameter β. In our model, β has a natural interpretation that captures the "loss of control" of a manager: the more connections a manager maintains, the less her power depends on each of her neighbours' power [21].

As an application of our model, we introduce a notion of *stability*: intuitively, as supported by a number of researches [6], a hierarchical network is stable if positions of individuals truthfully translate to their power. This definition enables us to study three phenomena:

1) *Bounded height*: A management hierarchy typically involves a bounded number of levels, regardless of the individual capabilities. A common belief is that a tall hierarchy reduces the effectiveness of communication. We provide an alternative explanation: as a company creates more and more levels in its hierarchy, it will eventually become unstable. See Section IV.
2) *Flattening as empowerment*: Flattening is a well-known phenomenon of organizational change when a company acquires a new structure with fewer hierarchical levels. The alleged benefits of flattening include empowering employees, increasing flexibility, pushing down decision making, and improving information flow [18]. We provide a somewhat paradoxical view on flattening through computation: although flattening reduces average power in the company, the majority of employees gain more power. See Section V
3) *Impact of Social links*: As argued by numerous studies, social links significantly impact on organisations [8]. We analyse this phenomenon from the point of view of stability: a network is more likely to be destablised by social links in taller hierarchies than in flatter hierarchies. See Section VI

Finally in Section VII, we demonstrate that our model is capable of revealing power and structural properties of real-world organisations by considering a case study (Krackhardt and Hanson's network [17]).

II. Organisational Networks

An organisational structure is often defined as a set of positions, groups of positions, reporting relationships, and interaction patterns [1]. We use the network approach and propose a model that captures main traits of a company. On the one hand, our model delineates the organisational hierarchy of a firm by featuring reporting relationship. On the other hand, we enrich the model by including also non-reporting relations. Indeed, as we will show later, these non-reporting relations can significantly affect a company as a whole.

Definition 1: An *organisational network* is a structure $\mathcal{G} = (V, r, E_{\mathsf{s}}, E_{\mathsf{w}})$, where V is a set of *nodes*, $E_{\mathsf{s}}, E_{\mathsf{w}} \subseteq V^2$ are edge relations such that

1) $r \in V$ is called the *root* and $(r, r) \in E_{\mathsf{s}}$;
2) the pair (V, E_{s}) forms a directed acyclic graph (ignoring the edge (r, r)), where every node apart from r has an incoming edge from another node;
3) the pair (V, E_{w}) forms an undirected graph.

Informally, the set V above denotes the work positions (or people) in the network. The root r is the top manager, i.e. r does not report to anyone else. The edge set E_{s} represents the *reporting relation* on members of the network; if $(u, v) \in E_{\mathsf{s}}$ then v reports to u and is called a *subordinate* of u. The edge set E_{w} represents the undirected dyadic *non-reporting relation*. This could be collaborations, advice relations or friendship between employees, etc. We will refer to edges in E_{s} as *strong ties* since reporting relations are usually more important. We will call undirected edges in E_{w} *weak ties*. For simplicity, we assume that any two nodes (u, v) can be connected either by a strong tie or a weak tie, but not both. In fact, this can be justified intuitively: any reporting relation presumes some social interaction between a manager and her subordinates.

We remark that the requirement that there is a single root of the network is too restrictive. Indeed, large corporations tend to have a board of directors. Nevertheless, we argue that this simplified model is still reasonable as the board of directors normally perform as a whole by hiring a CEO. The loop $(r, r) \in E_{\mathsf{s}}$ indicates that the root makes decisions by herself. Another reason why we need this loop is technical – as we will show later it makes capacity of nodes uniform.

To define a "well-built" structure, we accompany the definition above with two principles. Firstly, if a person has several sources of instructions, which in real life happens sometimes, a head-on collision may occur, while the structure where each employee has exactly one manager seems to be more natural and effective. Hence we require:

Principle 1: One Manager. Each node has exactly one incoming directed edge, which represents relationship with its manager, i.e., for all $u \in V$ there is a unique $v \in V$ with $(v, u) \in E_{\mathsf{s}}$.

Note that Principle 1 requires the directed graph (V, E_{s}) to form a tree, which we call the *reporting hierarchy* of $\mathcal{G}$. The top (level 0) of the hierarchy contains only the root r.

Definition 2: The *level* of any node v in $\mathcal{G}$ is the length of the path from r to v in the reporting hierarchy. The *height* of the hierarchy is the maximal level of any node $v \in V$.

Secondly, one may notice that a person can maintain only a limited number of interpersonal relations, due to limited time and effort. In fact, all social networks emerge under the constraint of limited resources. For example, in the context of online social networks, the number of strong ties (mutual communication during some period of time) for networks of more than 500 nodes on Facebook varies from 10 to 20 [11].

In defining the notion of *capacity* of individuals, we distinguish the strong and weak ties in terms of how much resource each of them consumes. Let Δ be an abstract quantity that defines the maximum amount of resources (working hours, for instance) that a person can distribute between his or her ties. For simplicity, we assume that each person in the network has the same amount of resources Δ. We also assume that a person needs s resources and w resources to maintain a strong and a weak tie, respectively. The root node also spends s resource on some exogenous factors, which are represented by the loop (r, r). Therefore, for any node v, if $S(v)$ is the number of directed edges (including self-loop), and $W(v)$ is the number of undirected edges, then A spends $S(v) \times s + W(v) \times w \leq \Delta$

resources to maintain all his connections. Let $\delta := \frac{w}{s}$ be called the *correlation coefficient*.

Definition 3: The *relative degree* of a node $v \in V$ is defined as $d(v) = E_{\mathsf{s}}(v) + E_{\mathsf{w}}(v) * \delta$, where $E_{\mathsf{s}}(v)$, $E_{\mathsf{w}}(v)$ are the numbers of strong ties (including both incoming and outgoing edges) and weak ties v maintains, respectively.

Clearly, if $\delta = 1$, then the relative degree is the conventional degree notion in graph theory. The *relative capacity* of a node is a given number that defines the upper bound on its relative degree. In other words, it defines the total available resources for a person to maintain all ties.

Principle 2: Maximal Relative Capacity. There is a constant relative capacity c for any node $v \in V$.

Management theory defines the *span of control* of a manager as the number of her direct subordinates. If we only consider the reporting relation, Principle 2 guarantees that the span of control of every individual is limited, and, thus, refers to the "limited managerial attention", a phenomenon in hierarchy theory [15]. The loop (r, r) guarantees that the root must not have more direct subordinates than all the other managers and, hence, make our approach uniform.

Definition 4: An organisational network is called *well-built* if it satisfies the principles 1 and 2.

In the rest of the paper we assume that all organisational networks are well-built without explicit mention.

III. A Measure of Power

In this section, we measure the *power* of a node in an organisational network. Analogously to the concept of *social capitals* (see e.g. [11]), we stipulate that power in an organisational network is derived from positions. We point out three intuitive factors affecting the power of a node: The first is the node's proximity to the root in the reporting hierarchy. The second is the number of ties the node maintains – more connections provide more sources of information. Finally, the span of control indicates how many subordinates a manager has, and, hence, how much involved he or she is in making decisions over the network.

Example 1. Even in relatively small networks, it is often not easy to define 'good' positions at first glance. Consider an example as in Figure 1. Three nodes, A, B and C have directed incoming edges from the root r, i.e. their hierarchy level is $L(A) = L(B) = L(C) = 1$. However, these three nodes are quite different: A has two direct subordinates, but he does not maintain any weak tie; C has three subordinates, but only one of them is a child, while B does not have any outgoing directed edges; nevertheless, she is connected to all the nodes except A by undirected edges. Several natural questions arise: which position is the 'best' among A, B, and C? How much power does each node has? Does the link between B and C affects B's power the same way as the link between B and D does?

To answer the questions posed above, we need a centrality measure that takes into account a node's span of control, its level, ties with others, and size of the sub-network "below"

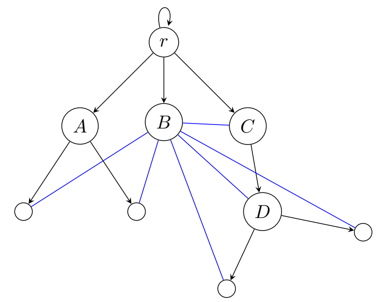


Fig. 1: Defining power of A, B and C

this node. We differentiate the effects of strong and weak ties using a parameter $\kappa \in [0, 1]$: every edge (u, v) is seen as direct influence from u to v; a strong tie has a weight of 1, and a weak tie a weight of κ. There are two more technical concerns: Firstly, it is natural to assume that the influence between an employee and her manager is not one-way: while the manager influences the employee through a strong tie, the employee also influences her manager through social interaction, and hence can be regarded as a weak tie, *backflow*. Secondly, the root has a self-loop, which can have any non-negative weight. We adopt for simplicity a weight of 0. The above has resulted in a weighted *influence graph*. An example is in Fig. 2.

Bonacich power, introduced in [2], is a widely-adopted eigenvalue centrality measure in social networks. The basic idea is that the power of any individual depends on the power of those it is connected to; the difference between Bonacich power and the usual eigenvalue centrality is the inclusion of a parameter β, which affects the meaning of centrality. Let R be the adjacency matrix of the network (here we implicitly mean there is an indexing of all nodes in the matrix as natural numbers $1, \ldots, n$) and $R_{i,j}$ denote the (i, j)-entry of R. The Bonacich power of $i = 1, \ldots, n$ is

$$p_i = \sum_{j=1}^{n} (\alpha + \beta p_j) R_{i,j} \tag{1}$$

where α, β are scalar constants. In matrix form, the vector of Bonacich power $\vec{p} = (p_1, \ldots, p_n)$ is

$$\vec{p} = \alpha (I_n - \beta R)^{-1} R \vec{e}_n \tag{2}$$

where I is the $n \times n$ identity matrix, and $\vec{e}_n$ is the column vector of ones with length n. It is clear that different values of α and β result in different centrality measures. Here α only serves as a normalising factor; it is selected such that the norm $||\vec{p}||$ equals $\sqrt{n}$. Thus the most "evenly distributed" case is when $p_i = 1$ for every $i = 1, \ldots, n$. The parameter β can be any value on the interval $[-\frac{1}{\lambda}, \frac{1}{\lambda}]$ where λ is the dominating eigenvalue of R. In some sense it captures the contribution of ties of a node to its power [1].

To derive a measure of power in an organisational network, we propose to adopt Bonacich power on the influence graph of the network. The parameter β specifies the loss of control of the managers: the more connections an individual has, the less her power depends on each of her neighbour's power. Capacity

[1]A negative value of β implies a negative exchange power where connections to nodes with smaller power results in a bigger power

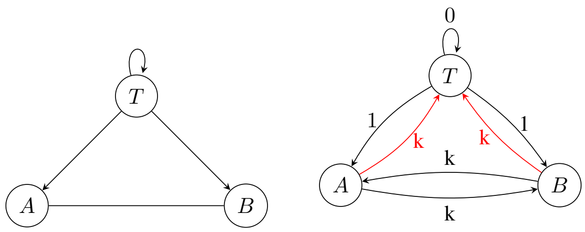


Fig. 2: An organisational network (on the left) and its weighted influence graph (on the right)

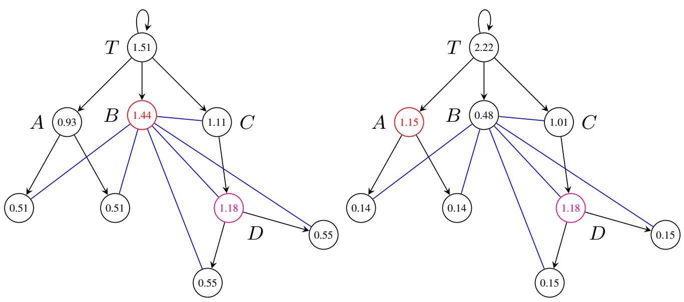


Fig. 3: Individual power with $\kappa = 0.5$ (left) and $\kappa = 0.1$ (right)

indicates how many resources a worker spends on keeping each tie. Therefore, we require β to be inversely proportional to the capacity minus one (the "minus one" is for the relation with its manager). Hence, we fix a value of β such that $\beta < \frac{1}{\lambda}$ if $\lambda > 1$, and $\beta < \frac{1}{c-1}$, otherwise.

Definition 5: Let $i = 1, \ldots, n$ be a node in $\mathcal{G}$. Let $S_i = \{j \mid 1 \leq j \leq n, (i,j) \in E_{\mathsf{s}}\}$, $W_i = \{j \mid 1 \leq j \leq n, (i,j) \in E_{\mathsf{w}}\}$, and μ_i be the node such that $(\mu_i, i) \in E_{\mathsf{s}}$. We define the *power* p_i of i as discussed above, i.e., by (1) it is

$$p_i = \sum_{s \in S_i} (\alpha + \beta p_s) + \kappa \sum_{w \in W_i \cup \{\mu_i\}} (\alpha + \beta p_w) \qquad (3)$$

Now we can answer questions stated in Example 1 (Fig. 1). Let the correlation coefficient $\delta = 0.5$. Assume that capacity of each node is 4, and $\beta = 0.3 < \frac{1}{3}$. Fig. 3 shows the resulting power of each node when $\kappa = 0.5$ (left) and $\kappa = 0.1$ (right). When $\kappa = 0.5$, even though B does not have a single subordinate, she is almost as powerful as the top manager while A and C possess similar power. However, when $\kappa = 0.1$, A and C are much more powerful than B. Hence κ captures in some sense the "importance" of weak ties.

Note that the power of D, who has two direct subordinates and collaborates with B, exceeds her manager C in both cases above. We view that D's influence is not fully represented by her level in the hierarchy, which serves as her "nominal" authority. This may imply a form of "instability" within the structure, as the employee D seeks more formal recognition (or promotion). Furthermore, C may experience certain loss of control over D's subordinates, as communication may not effectively pass down from C to these nodes. Hence, in a *stable* network, the position should truthfully reflect the actual influence (i.e. power) of an individual, which, in other words, means that the power of nodes are consistent with their respective levels.

Definition 6: An organisational network $\mathcal{G}$ is *stable* if for any nodes $i, j \in V$, $\mathsf{lev}(i) < \mathsf{lev}(j)$ implies that $p_i > p_j$ where $\mathsf{lev} : V \to \mathbb{N}$ maps every node to its level in the hierarchy of E_{s}. We say that $\mathcal{G}$ is *unstable* if it is not stable.

This definition allows us to formally analyse several phenomena, which we elaborate in subsequent section.

IV. Stability and Height

In this section, we aim to study the relation between stability and height of an organisational network. Throughout this section, we assume $E_{\mathsf{w}} = \varnothing$. Consider a network $\mathcal{C}_n$ consisting of n nodes $1, \ldots, n$ such that $E_{\mathsf{s}} = \{(i, i+1) \mid 1 \leq i \leq n-1\}$; this is a *chain* of n nodes. Intuitively, a chain is not a good form of organizational hierarchy: large $n$leads to ineffective communication. We argue that the notion of stability can give us a formal evidence for the ineffectiveness of the chain network. By (3), the power of node i is

$$p_i = \begin{cases} \alpha + \beta p_2 & \text{if } i = 1 \\ \kappa(\alpha + \beta p_{n-1}) & \text{if } i = n \\ \alpha + \beta p_{i+1} + (\alpha + \beta p_{i-1})\kappa & \text{if } 2 \leq i \leq n-1 \end{cases} \qquad (4)$$

Lemma 7: If $0 < \beta \leq \kappa < 1$, $\mathcal{C}_n$ is unstable for any $n > 2$.

Proof. By (4) we derive

$$\begin{aligned} p_1 &= \alpha + \beta p_2 \\ &= \alpha + \beta(\alpha + \beta p_3 + (\alpha + \beta p_1)\kappa) \\ &= \alpha + \alpha\beta + \alpha\beta k\kappa + \beta^2 p_3 + \beta^2 \kappa p_1, \end{aligned}$$

or, $p_1 = \dfrac{\alpha + \alpha\beta + \alpha\beta\kappa + \beta^2 p_3}{1 - \beta^2\kappa}$. Similarly,

$$\begin{aligned} p_2 &= \alpha + \beta p_3 + (\alpha + \beta p_1)\kappa \\ &= \alpha + \alpha\beta\kappa + \alpha\kappa + \beta p_3 + \beta^2 \kappa p_2, \end{aligned}$$

or, $p_2 = \dfrac{\alpha + \alpha k + \alpha\beta\kappa + \beta p_3}{1 - \beta^2\kappa}$. Then,

$$p_1 - p_2 = \frac{\alpha\beta + \beta^2 p_3 - \alpha\kappa - \beta p_3}{1 - \beta^2\kappa}$$

Since $1 - \beta^2\kappa > 0$ for any positive $\beta, \kappa < 1$, $p_1 - p_2$ is negative whenever $\alpha\beta + \beta^2 p_3 < \alpha\kappa + \beta p_3$. Clearly, since α is positive and $\beta^2 p_3 < \beta p_3$, $\beta \leq \kappa$ implies $p_1 < p_2$ for any $n > 2$ □

Lemma 8: The chain $\mathcal{C}_n$ is stable if and only if $p_1 > p_2$.

Proof. We only need to prove the "only if" direction. Suppose $p_1 > p_2$. Then by (4), $\beta p_2 > \beta p_3 + (\alpha + \beta p_2)\kappa$. Since $(\alpha + \beta p_2)\kappa > 0$, $p_2 > p_3$. Consequently, we have $\beta p_2 > \beta p_4 + (\alpha + \beta p_2)\kappa$, and hence $p_2 > p_4$. Inductively we may show that $p_2 > p_i$ for any $i = 3, \ldots, n$.

We now prove that $p_i > p_{i+1}$ for any $i = 3, \ldots, n-1$. Suppose on the contrary that $i > 2$ is the smallest such that $p_{i+1} \geq p_i$. Then by (4) $p_{i+2} + \kappa p_i \geq p_{i+1} + \kappa p_{i-1}$. Since $p_{i-1} > p_i$, it must be that $p_{i+2} \geq p_{i+1} \geq p_i$. Iterate the same argument we conclude $p_n \geq p_i$. However, by (4) again this would mean that $\alpha + \kappa\beta p_{n-1} \geq \alpha + \beta p_{i+1} + \kappa(\alpha + \beta p_{i-1}) > \alpha + \kappa\beta p_{n-1}$. A clear contradiction. Hence such an i does not exist and we conclude $p_1 > p_2 > \cdots > p_n$. □

Theorem 9: Fix κ and β such that $0 < k < 1$ and $0 \leq \beta < 1$. There is some $n \geq 1$ such that $\mathcal{C}_m$ is unstable for any $m \geq n$.

Proof. Lemma 7 shows the statement holds when $\beta \leq \kappa$ (where $n = 3$). Suppose $\beta > \kappa$, by Lemma 8 we need to find n such that $p_1 < p_2$ holds in $\mathcal{C}_n$. Iteratively applying (4), we get that

$$\begin{aligned} p_1 &= \alpha + \alpha\beta + \cdots + \alpha\beta^{n-2} + \kappa(\beta(\alpha + \beta p_1) + \beta^2(\alpha + \beta p_2) + \\ &\quad \cdots \beta^{n-1}(\alpha + \beta p_{n-1})) \\ p_2 &= \alpha + \alpha\beta + \cdots + \alpha\beta^{n-3} + \kappa((\alpha + \beta p_1) + \beta(\alpha + \beta p_2) + \\ &\quad \cdots \beta^{n-2}(\alpha + \beta p_{n-1})) \\ \cdots &\quad \cdots\cdots \end{aligned}$$

In general, $p_i = \alpha \sum_{j=0}^{n-i-1} \beta^i + \kappa \sum_{r=0}^{n-i} \beta^r(\alpha + \beta p_{r+i-1})$. Thus we have

$$\begin{aligned} p_1 - p_2 &= \alpha\beta^{n-2} - \alpha\kappa\left(1 + \beta + \beta^2 + \cdots + \beta^{n-2}\right)(1-\beta) - \\ &\quad \beta\kappa\left(p_1 + \beta p_2 + \beta^2 p_3 + \cdots + \beta^{n-2} p_{n-1}\right)(1-\beta) \\ &= \alpha\beta^{n-1} - \alpha\kappa(1-\beta^{n-1}) - (1-\beta)\beta\kappa\sum_{i=0}^{n-2}\beta^i p_{i+1} \end{aligned}$$

Since $0 \leq \beta < 1$, $p_1 < p_2$ if $\alpha\beta^{n-2} \leq \alpha\kappa(1 - \beta^{n-1})$. Solve this inequality to get

$$n \geq \left\lceil \log_\beta \frac{\kappa}{1 + \kappa\beta} \right\rceil + 2$$

Thus the theorem is proved. □

Theorem 9 justifies that the "chain-like" hierarchies are not suitable for organisations from the point of view of stability: the structure will become unstable as the number of people (and thus levels) increases. With a similar but more involved technical analysis, the above results can be generalised to *perfect d-ary tree networks* (where $d \geq 1$), i.e., the E_s hierarchy form a tree in which every non-leaf node has exactly d children and all leaves are at the same level in the tree. We use $\mathcal{D}_n^d$ to denote a perfect d-ary tree network of height n; note that $\mathcal{D}_n^1 = \mathcal{C}_n$ for any $n \in \mathbb{N}$. The arity d in the perfect tree network equals to the capacity c minus one, and therefore we get $d\beta < 1$ by our earlier assumption that $\beta < \frac{1}{c-1}$. The following is a lemma generalising Lem. 7.

Lemma 10: If $\beta \leq \frac{k}{d^2}$, then any perfect d-ary tree network $\mathcal{D}_n^d$, with $d \geq 1$ of any height $n > 2$, is unstable.

We omit the proof as it is similar to the proof of Lem. 7. The next theorem generalises Thm. 9 to d-ary perfect trees. Lem 10 handles the case when $\beta \leq \frac{k}{d^2}$. The case when $\beta > \frac{k}{d^2}$ can be proved similarly to Thm. 9.

Theorem 11: For any arity $d \geq 1$, there is a constant $c_d \in \mathbb{R}$ such that any perfect tree network $\mathcal{D}_n^d$ is unstable if

$$n \geq c_d + \log_{d\beta}(1/d). \tag{5}$$

Remark. The proof of Thm.11 gives us an upperbound for the constant c_d: $c_d \leq \log_{d\beta} \kappa/(1+\kappa\beta) + 2$. Note that this bound may be much larger than the actual bound. For example, using UCINET [5], we computed the actual limits on numbers of hierarchy levels with $k = 0.5$: for $d = 2$, it is 5; for $d = 3$, it is 8 (the theoretical bounds are 18 and 21, respectively.) Furthermore, by increasing the span of control (i.e., d) of nodes, the theorem implies an logarithmic growth on the bounds on the number of levels.

We now interpret the main result (Theorem 11) of the section. A general and significant organizational change trend in the last 50 years is the shift from *tall hierarchies* with many levels to *flat hierarchies*, where the number of levels is kept bounded. Research has found that most large companies changed their structures to the flattened ones in the past 3-4 decades [25], e.g., back in 1950s companies had up to twenty layers of hierarchy while by the end of the twentieth century they were trimmed to five or six. We conjecture that this delayering process implies some fundamental truth regarding organisational networks. The well-known theory of "six degree of separation" has been extensively studied and verified in the social network analysis community [22]. This theory states that six is a natural bound in the acquaintance relation on the distance between two people in the world. Analogously, it seems that for organisational networks, a bound on the number of levels of the hierarchy also exists. Moreover, this bound is natural as it allows the top manager to maintain control over the hierarchy. Theorem 11 provides an evidence of the existence of such a bound: *As the arity d is bounded (by capacity of individuals), the maximum height for a perfect tree network to maintain stability is bounded.* It will be an interesting future work to study the exact value of such a bound.

V. Flattening as an Empowerment Strategy

Flattening happens when a company acquires a new structure by increasing the span of control of individuals. For instance, the average number of employees who report directly to the CEOs in large companies grows from 4.7 in 1980 to 9.8 in 1999. The alleged benefits of flattening include empowering employees, increasing flexibility, pushing down decision making and improving information flow and, thus, enabling faster decision. However, some researchers argue that flattening leads to the opposite effect – more control and decision making is concentrated on the top in the flattened organization [23], [25].

In this section, we analyse the flattening process in a formal approach. Based on our organisational network model, we argue that most employees indeed obtain more power through flattening, although the average power decreases. Moreover, the root is always the one whose power changes most considerably. Therefore, we speak about flattening as the strategy of empowering.

Consider two organisational networks $\mathcal{A}, \mathcal{B}$ with the same number of nodes; $\mathcal{A}$ has a taller structure (height 3) than $\mathcal{B}$ (height 2). Assume again that no weak tie exists in the networks; see Fig. 4 and Fig. 5. We ran several tests computing the individual power with different parameters and list results in the Table I, where n is the number of nodes and ℓ is a hierarchy level. One can see that the average power in $\mathcal{A}$ is strictly greater than the average power in $\mathcal{B}$. Similarly, the figures for the case $\kappa = 0.1$ shows that flattening negatively impact power of individuals in the network: only *four* nodes increase their power while 11 others become less powerful and 16 stay the same. However, when we even slightly increase κ to 0.15, a majority of nodes increase their power. Moreover, when $k = 0.8$, the network $\mathcal{A}$ becomes unstable while $\mathcal{B}$ is still stable.

In [18], the authors carried out a survey in a company after introducing a new flat structure. The survey showed that

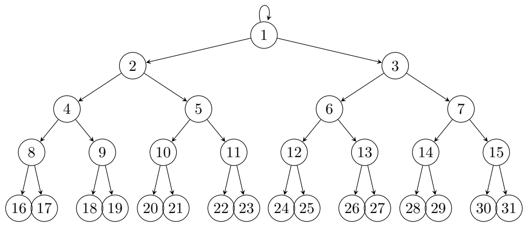


Fig. 4: Network $\mathcal{A}$ with 31 nodes

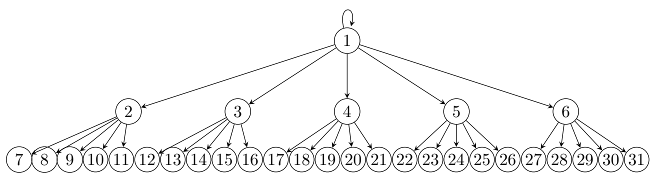


Fig. 5: Network $\mathcal{B}$ with 31 nodes

65.9% of employees were very happy, 26.3% were not happy, and 7.8% were not concerned about the change. This correlates very well with the results we obtain: the computation reveals that 64.5% (20 out of 31) of nodes when $\kappa = 0.15$ become more powerful.

Through this example, we argue the following rather paradoxical aspect of flattening in an organisational network *(flattening paradox): Flattening decreases the average power in the company, but empowers most employees.*

VI. The Role of Social Links

So far our analysis only covered pure hierarchies, where no undirected weak tie is maintained. However, it has been long argued in management studies that informal social connections, such as collaboration, advice or friendship, play important roles [8], [17].

In this section, we analyse a theoretical example in the hope of finding how social links affect individual power. As the test methodology, for each network we first compute individual power in the *pure hierarchy*, i.e., without taking into consideration the undirected weak-ties. We then add a randomly generated weak-ties to these networks, and compute power again. The results are then compared.

	$\mathcal{A}$: $d = 2$, $\beta = 0.45$				
	n	k=0.1	k=0.15	k=0.5	k=0.8
$l = 0$	1	2.613	2.553	2.193	1.949
$l = 1$	2	2.175	2.181	2.181	2.149
$l = 2$	4	1.522	1.527	1.556	1.587
$l = 3$	8	0.819	0.828	0.864	0.882
$l = 4$	16	0.070	0.100	0.252	0.323
max		2.613	2.533	2.193	2.149
min		0.070	0.100	0.252	0.323
avrg		0.668	0.685	0.765	0.800
	$\mathcal{B}$: $d = 5$, $\beta = 0.18$				
$l = 0$	1	3.503	3.450	3.056	2.733
$l = 1$	5	1.929	1.941	1.966	1.936
$l = 2$	25	0.070	0.103	0.306	0.437
max		3.503	3.450	3.056	2.733
min		0.070	0.103	0.306	0.437
avrg		0.481	0.507	0.662	0.753

TABLE I: Comparing individual power in networks $\mathcal{A}$ and $\mathcal{B}$ (tests performed using UCINET [5])

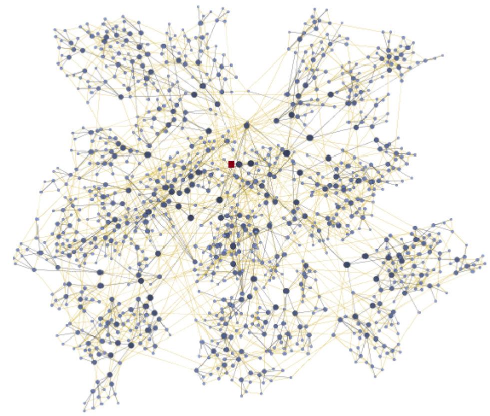


Fig. 6: A randomly generated network for $d = 3$ and 7 levels. Blue and yellow lines are strong and weak ties, resp. The root is the brown square. Sizes of nodes indicates their power.

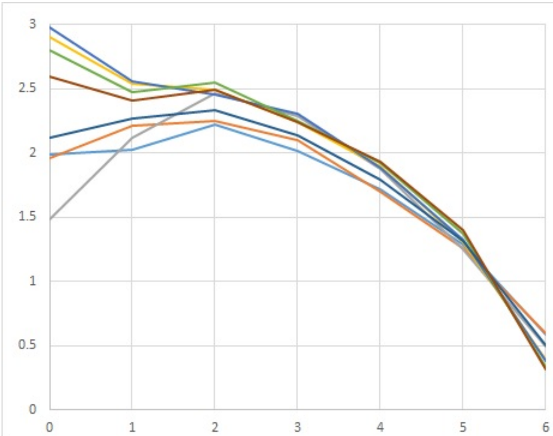
β	0.3		0.07	
d	3		10	
n	1093		1111	
Level:	k = 0.1	k =0.5	k =0.1	k=0.5
0	5.03	4.8	6.16	5.66
1	4.53	4.79	4.86	4.7
2	3.77	3.9	2.89	2.85
3	2.95	2.91	0.05	0.22
4	2.06	1.96	-	-
5	1.1	1.07	-	-
6	0.06	0.24	-	-

TABLE II: Power Distribution in Two Perfect Tree Hierarchies

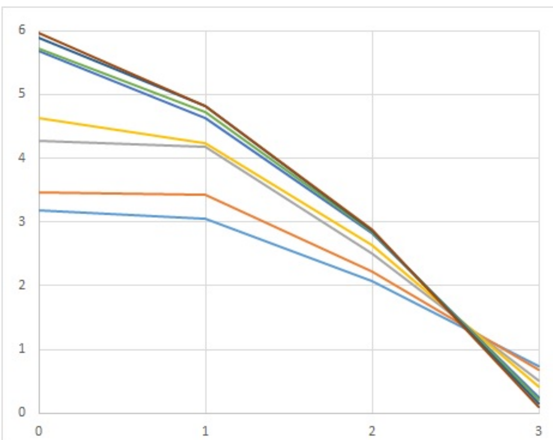
To correctly predict the impacts of weak ties on the network, it is crucial to adopt a reasonable benchmark for generating random social links. Existing benchmarks such as planted ℓ-partition, relaxed caveman graphs and the LFR graphs [12] are not suitable as they do not take into account the hierarchical positions of nodes; indeed it is widely observed that positions affect the likelihood of weak-ties between employees in a company. For example, employees who work in the same department, or are on a similar level, are more likely to set up more informal links [24]. We adopt a distributed approach where each node randomly chooses to set up weak-ties with other, in such a way that closer nodes (in distance) enjoy a higher "probability" of a weak tie. The result is a random network that not only captures main characteristics of social networks (such as community structure), but also entails reporting hierarchy of the network; see Fig. 6 for a generated network visualised using a force-directed method. The community structure clearly resembles departments and reflect hierarchical levels in an organisation.

We consider two perfect trees – one has the span of control $d = 3$ and 7 levels, the other has the span of control 10 and 4 levels. The resulting power in both networks are listed in Table II. Note that both hierarchies are stable. We then generate random weak-tie networks with different parameters over these hierarchies. In Figure 7, we plot the distribution of average values of power at each level for eight randomly weak-tie networks. In 7a only two out of eight generated networks are stable. However in Fig. 7b, one can see that the non-reporting relations does not change the stability of the network.

As the result shows, the taller hierarchy's stability is very



(a) Distribution of power in the tall organisation



(b) Distribution of power in the flat organisation

Fig. 7: Average values of power at each hierarchy level in randomly generated social networks (a) in the tall organisation, and (b) in the flat organisation. The different lines indicate differences in "density" of the weak-ties; in general a denser weak-tie relation causes a more even distribution of power across levels, hence a "flatter" (less-steep) curve.

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fragile – adding weak ties in all experiments makes the network unstable. On the other hand, the flatter hierarchy stays stable in most of our experiments with $k = 0.1$ and the probability $p = 0.5$ of existing friendship between two nodes who have the same direct manger. When the probability is small, corporate networks stay stable even with $k = 0.5$. Thus, this experiments justify that: *As an organisational hierarchy has more levels, it is much more likely to be destabilised by non-reporting connections*.

VII. Case Study: Krackhardt and Hanson's Network

Krackhardt and Hanson in [17] studied a high-tech company with 21 managers. They analysed the formal hierarchy in the company, as well as reconstructed two types of social links on the same group of employees through a series of interviews – one type of social link is the advice relation (based on the interview question "*To whom do you go for advice?*") and

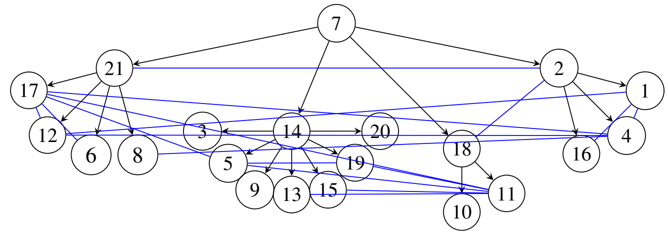

Fig. 8: Krackhardt and Hanson's hierarchy with 21 nodes.

the other is friendship (based on the question "*Who are your friends?*"). Hence this data provides a real world case study for testing our model. In [17], the friendship links are directed; to fit our model we make them undirected by keeping only mutual friendship connections. The formal hierarchy of the network is depicted in Figure 8: there is one top manager (7), four departments, managed by 2, 14, 17, and 21, respectively.

We considered separately a formal hierarchy and a "hybrid" corporate network that contains both strong and weak ties. The results are listed in Table III. From the results we draw two conclusions:

(1) There is no correlation between power and the age, nor years of service of employees.

(2) By taking into consideration the weak ties, the power of individuals on the bottom (leaves) increases while those on higher levels lose some of their power.

Note further that this network is unstable by our definition as 14 has more power than 7 in all the cases. We suggest two possible ways to interpret this fact:

- A high power of a manager may suggest high capability and performance, as well as a high workload. This could be used as a rigorous basis for certain rewards to the particular employee in the form of, for instance, bonuses or promotion. Such bonuses would increase the loyalty of the employee, and, as a result, decrease possible risks.

- The node 14 is overwhelmed, as it has too many direct subordinates. To reduce this number and, therefore, to "stabilise" the structure, certain structural changes can be done. One of the possible solutions is to promote two of 14's most powerful direct subordinates (5 and 19) and distribute the rest (3, 9, 13, 15, 20) between them.

VIII. Related Works

Organisational network analysis (ONA) amounts to a collection of tools in business management [9], [13]. Existing works apply SNA to study organisational processes and problems. Among them are identifying important individuals [4], improving awareness about informal networks [19], improving collaboration [8], building a new business [10].

The importance of non-reporting links within a business hierarchy has also been intensively studied. Firstly, in management studies, Krackhardt and Hanson [17] noted that much of the work in a company happens despite the formal organisation. They draw an analogy between a company and a human body: the formal structure of a company is the skeleton, while the informal structure is the central nervous system.

Attribute				Hierarchy			Hybrid		
ID	Dept	Age	YoS	$\kappa=0.1$	0.5	0.75	$\kappa=0.1$	0.5	0.75
1	4	33	9	0.06	0.25	0.34	0.18	0.68	0.84
2	4	42	20	1.35	1.41	1.42	1.43	1.55	1.48
3	2	40	13	0.07	0.32	0.45	0.11	0.41	0.5
4	4	33	8	0.06	0.25	0.34	0.22	0.85	1.06
5	2	32	3	0.07	0.32	0.45	0.2	0.77	0.97
6	1	59	28	0.06	0.27	0.37	0.1	0.39	0.49
7	-	55	30	2.39	2.13	1.96	2.33	1.67	1.33
8	1	34	11	0.06	0.27	0.37	0.1	0.38	0.47
9	2	62	5	0.07	0.32	0.45	0.07	0.24	0.28
10	3	37	9	0.05	0.23	0.31	0.05	0.18	0.21
11	3	46	27	0.05	0.23	0.31	0.26	1	1.24
12	1	34	9	0.06	0.27	0.37	0.19	0.72	0.9
13	2	48	0	0.07	0.32	0.45	0.11	0.42	0.51
14	2	43	10	3.06	2.98	2.88	2.99	2.34	1.96
15	2	40	8	0.07	0.32	0.45	0.16	0.59	0.73
16	4	27	5	0.06	0.25	0.34	0.1	0.36	0.45
17	1	30	12	0.06	0.27	0.37	0.27	1.03	1.29
18	3	33	9	0.92	1.03	1.07	0.96	1.04	1
19	2	32	5	0.07	0.32	0.45	0.24	0.89	1.09
20	2	38	12	0.07	0.32	0.45	0.07	0.24	0.28
21	1	36	13	1.77	1.8	1.78	1.9	1.68	1.53

TABLE III: Power in Krackhardt and Hanson's network, $\beta = 0.1$

Informal networks are more flexible and adaptive; the formal structure is static. Secondly, Cross et al. in [8] adopted a computational approach and argued that even though informal networks are invisible, they are more reflective than the formal organisations. The authors defined scenarios where SNA is useful to assess informal networks and facilitate effective collaboration. Finally, a number of works show how informal networks can be used to reveal the reporting hierarchy [14], [24].

IX. Conclusion and Future Work

Compared to the mentioned works on ONA above, our approach is novel in the fact that we work on a two-tier model of organisational network taking into account both the strong, hierarchical ties, as well as the weak, mutual ties between individuals. By integrating both types of ties in the same framework, one is able to rigorously define and study complex organisational behaviors. The mathematical and computational analysis reveals some new insights on important phenomena: bounded hierarchical height, empowerment through flattening, and impacts of social links.

There are several obvious ways in which the model can be extended: 1) As argued above a company may be lead by a board of directors rather than a single person. Hence, one may allow several nodes in the network making the reporting hierarchy a forest rather than a single tree. 2) Weak-ties are heterogeneous; and different types of weak-ties (such as friendship, and collaborations) may result in different influences on power. Hence one may allow several weak-tie (undirected or directed) edges with different correlation coefficients and κ. 3) The capacity of individuals are different, and therefore, one may assign different capacities to different individuals.

We conclude that the overall goal is to develop accurate decision support for business intelligence. The desired technology would facilitate answering queries of the form: *What are the influential/power positions in a company? Is there a hidden structural hole?* or *How would the company restructure in order to optimise effectiveness?*, and we hope formalisms, suggested in this paper, may lay a promising foundation for such technology.

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