Formal Proofs for AIT

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November 21, 2019

Formal proofs will dominate mathematics in the near future [9, 4]. The first formal proofs in AIT have been obtained in [2, 10]; they include a sharper form of the Kraft-Chaitin Theorem. The project consists in using the proof-assistants Isabelle [11] or Coq [12] to formally proving important theorem in AIT. References [1, 6, 5, 7, 3, 8] contain many results which can be formalised and formally proved.

References

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