

Supplementary material to: On the Use of Dictionary Learning in Time Series Imputation

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Abstract—We present in this document experimental results that have not been included in [1] because of the limited space.

The results shown in Sec. I are for the evaluation of DLU when the missing data are simulated by sampling without replacement (see also [1, Sec. IV]). Each table in Sec. I contains the normalized errors computed for a time series from the dataset of $K = 20$ time series, when various imputation methods are applied.

In the caption of the table, we mention the acronym of the time series as well as the transformations that have been applied for the methods Trend, Trend+DL, Trend+imputeTS and Trend+MTSDI. For example, for the first time series in the dataset (TS₁), we write the following in the caption of the Table I: $p = 3, \omega_1 = 1/24, \omega_2 = 1/12, \omega_3 = 1/8$. This means that three periodic components have been removed, and their frequencies are $\omega_1, \omega_2, \omega_3$. Similarly, for TS₅, we write $\theta_1 = 16$ in the caption of Table V for indicating that the time series z was transformed by computing the differences $z_t - z_{t-\theta_1}$ for all the time points t . We emphasize that the linear trend is removed from *all* time series and this transformation is not mentioned in the captions of the tables.

The symbol ρ stands for the percentage of the missing data. For each value of ρ , the smallest normalized error is written in red and the second smallest is written in bold. For ease of comparison, we plot in Fig. 1 the scores of the imputation methods, which are computed according to the rules given [1, Sec. IV]. In Fig. 2, we show the best and the second best result for each time series. Each panel of the figure corresponds to a particular value of ρ .

The results in Sec. II are also for the evaluation of the DLU imputation method, but this time the missing data are produced by using the Polya urn model (see again [1, Sec. IV]). All the conventions for the tables within this section are the same as the conventions for the tables in Sec. I. The graphical settings in Fig. 3 are the same as in Fig. 1; Fig. 4 is similar to Fig. 2.

The results for DLM are presented in Sec. III, where some information about the experimental settings is provided (see also [1, Sec. V]).

I. DLU: EXPERIMENTAL RESULTS OBTAINED WHEN THE MISSING DATA ARE SIMULATED BY SAMPLING WITHOUT REPLACEMENT

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0150	0.0203	0.0188	0.0252
Trend	-	0.0132	0.0113	0.0116	0.0119
Trend+DLU	Oracle	0.0032	0.0036	0.0028	0.0047
	BIC	0.0035	0.0042	0.0028	0.0052
	EBIC	0.0035	0.0042	0.0034	0.0052
imputeTS	-	0.0150	0.0140	0.0124	0.0166
Trend+imputeTS	-	0.0066	0.0075	0.0070	0.0084
MTSDI	-	0.0110	0.0122	0.0102	0.0127
Trend+MTSDI	-	0.0077	0.0087	0.0078	0.0091

TABLE I: TS_1 : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0080	0.0142	0.0216	0.0172
Trend	-	0.0237	0.0231	0.0217	0.0224
Trend+DLU	Oracle	0.0038	0.0042	0.0036	0.0039
	BIC	0.0041	0.0044	0.0040	0.0050
	EBIC	0.0041	0.0049	0.0040	0.0050
imputeTS	-	0.0122	0.0122	0.0141	0.0142
Trend+imputeTS	-	0.0087	0.0079	0.0072	0.0079
MTSDI	-	0.0107	0.0102	0.0113	0.0124
Trend+MTSDI	-	0.0090	0.0097	0.0091	0.0106

TABLE II: TS_2 : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0168	0.0245	0.0287	0.0231
Trend	-	0.0190	0.0173	0.0167	0.0174
Trend+DLU	Oracle	0.0045	0.0047	0.0047	0.0054
	BIC	0.0048	0.0048	0.0048	0.0058
	EBIC	0.0048	0.0048	0.0061	0.0061
imputeTS	-	0.0173	0.0208	0.0211	0.0202
Trend+imputeTS	-	0.0110	0.0110	0.0105	0.0110
MTSDI	-	0.0128	0.0123	0.0121	0.0134
Trend+MTSDI	-	0.0105	0.0098	0.0106	0.0119

TABLE III: TS_3 : $p = 4$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$, $\omega_4 = 1/6$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0127	0.0176	0.0156	0.0183
Trend	-	0.0265	0.0257	0.0281	0.0281
Trend+DLU	Oracle	0.0034	0.0044	0.0058	0.0049
	BIC	0.0048	0.0052	0.0078	0.0059
	EBIC	0.0048	0.0052	0.0078	0.0059
imputeTS	-	0.0103	0.0100	0.0124	0.0129
Trend+imputeTS	-	0.0041	0.0048	0.0067	0.0057
MTSDI	-	0.0231	0.0265	0.0347	0.0312
Trend+MTSDI	-	0.0114	0.0134	0.0163	0.0135

TABLE IV: TS_4 : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0123	0.0305	0.0250	0.0294
Trend	-	0.1277	0.1233	0.1258	0.1233
Trend+DLU	Oracle	0.0041	0.0069	0.0062	0.0044
	BIC	0.0044	0.0073	0.0073	0.0051
	EBIC	0.0044	0.0073	0.0073	0.0051
imputeTS	-	0.0339	0.0512	0.0509	0.0451
Trend+imputeTS	-	0.0046	0.0058	0.0043	0.0046
MTSDI	-	0.1299	0.1250	0.1257	0.1221
Trend+MTSDI	-	0.0047	0.0072	0.0067	0.0058

TABLE V: TS_5 : $\theta_1 = 16$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0168	0.0134	0.0219	0.0240
Trend	-	0.1710	0.1746	0.1757	0.1612
Trend+DLU	Oracle	0.0050	0.0031	0.0043	0.0054
	BIC	0.0073	0.0032	0.0060	0.0056
	EBIC	0.0073	0.0032	0.0060	0.0056
imputeTS	-	0.0118	0.0146	0.0132	0.0141
Trend+imputeTS	-	0.0045	0.0038	0.0038	0.0059
MTSDI	-	0.0086	0.0093	0.0090	0.0097
Trend+MTSDI	-	0.0061	0.0053	0.0057	0.0063

TABLE VI: TS_6 : $\theta_1 = 24$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0185	0.0193	0.0297	0.0248
Trend	-	0.2050	0.1918	0.2111	0.2030
Trend+DLU	Oracle	0.0042	0.0062	0.0102	0.0067
	BIC	0.0049	0.0064	0.0112	0.0079
	EBIC	0.0049	0.0064	0.0112	0.0079
imputeTS	-	0.0170	0.0261	0.0244	0.0359
Trend+imputeTS	-	0.0038	0.0061	0.0104	0.0065
MTSDI	-	0.0246	0.0238	0.0247	0.0245
Trend+MTSDI	-	0.0069	0.0094	0.0124	0.0095

TABLE VII: TS_7 : $\theta_1 = 24$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0158	0.0219	0.0187	0.0258
Trend	-	0.0341	0.0352	0.0317	0.0333
Trend+DLU	Oracle	0.0033	0.0052	0.0059	0.0069
	BIC	0.0041	0.0057	0.0064	0.0075
	EBIC	0.0053	0.0062	0.0064	0.0075
imputeTS	-	0.0139	0.0177	0.0203	0.0213
Trend+imputeTS	-	0.0090	0.0108	0.0101	0.0110
MTSDI	-	0.0158	0.0162	0.0160	0.0181
Trend+MTSDI	-	0.0146	0.0156	0.0148	0.0169

TABLE VIII: TS_8 : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0097	0.0143	0.0145	0.0180
Trend	-	0.0024	0.0027	0.0025	0.0037
Trend+DLU	Oracle	0.0019	0.0023	0.0020	0.0030
	BIC	0.0020	0.0024	0.0021	0.0032
	EBIC	0.0020	0.0024	0.0021	0.0032
imputeTS	-	0.0109	0.0082	0.0106	0.0140
Trend+imputeTS	-	0.0035	0.0036	0.0044	0.0050
MTSDI	-	0.0101	0.0125	0.0114	0.0117
Trend+MTSDI	-	0.0028	0.0036	0.0043	0.0049

TABLE IX: TS_9 : $\theta_1 = 24$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0111	0.0112	0.0168	0.0192
Trend	-	0.0266	0.0275	0.0260	0.0268
Trend+DLU	Oracle	0.0033	0.0037	0.0043	0.0045
	BIC	0.0038	0.0038	0.0043	0.0051
	EBIC	0.0039	0.0045	0.0049	0.0051
imputeTS	-	0.0099	0.0095	0.0110	0.0130
Trend+imputeTS	-	0.0056	0.0043	0.0058	0.0056
MTSDI	-	0.0285	0.0253	0.0351	0.0329
Trend+MTSDI	-	0.0109	0.0119	0.0128	0.0124

TABLE X: TS_{10} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0116	0.0158	0.0161	0.0353
Trend	-	0.0120	0.0111	0.0108	0.0118
Trend+DLU	Oracle	0.0036	0.0039	0.0040	0.0047
	BIC	0.0045	0.0045	0.0047	0.0052
	EBIC	0.0045	0.0045	0.0047	0.0052
imputeTS	-	0.0137	0.0141	0.0157	0.0204
Trend+imputeTS	-	0.0053	0.0053	0.0052	0.0057
MTSDI	-	0.0147	0.0169	0.0175	0.0175
Trend+MTSDI	-	0.0087	0.0095	0.0085	0.0091

TABLE XI: TS_{11} : $p = 4$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$, $\omega_4 = 1/6$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0108	0.0172	0.0266	0.0252
Trend	-	0.0166	0.0178	0.0155	0.0157
Trend+DLU	Oracle	0.0041	0.0047	0.0040	0.0049
	BIC	0.0043	0.0048	0.0043	0.0056
	EBIC	0.0043	0.0048	0.0043	0.0056
imputeTS	-	0.0124	0.0158	0.0140	0.0147
Trend+imputeTS	-	0.0082	0.0087	0.0081	0.0088
MTSDI	-	0.0092	0.0104	0.0116	0.0113
Trend+MTSDI	-	0.0072	0.0076	0.0077	0.0088

TABLE XII: TS_{12} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0081	0.0169	0.0245	0.0150
Trend	-	0.0137	0.0138	0.0123	0.0126
Trend+DLU	Oracle	0.0033	0.0047	0.0044	0.0044
	BIC	0.0035	0.0050	0.0054	0.0048
	EBIC	0.0044	0.0050	0.0054	0.0048
imputeTS	-	0.0098	0.0147	0.0152	0.0134
Trend+imputeTS	-	0.0071	0.0073	0.0075	0.0069
MTSDI	-	0.0137	0.0123	0.0128	0.0116
Trend+MTSDI	-	0.0105	0.0106	0.0094	0.0093

TABLE XIII: TS_{13} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.1841	0.1845	0.1750	0.1687
Trend	-	0.3039	0.3200	0.2772	0.3292
Trend+DLU	Oracle	0.1847	0.1844	0.1727	0.1818
	BIC	0.1961	0.2006	0.1770	0.1870
	EBIC	0.1961	0.2006	0.1770	0.1870
imputeTS	-	0.2147	0.2146	0.1931	0.1876
Trend+imputeTS	-	0.2152	0.2139	0.1912	0.1849
MTSDI	-	0.3074	0.2793	0.2580	0.2970
Trend+MTSDI	-	0.3017	0.3065	0.2542	0.3104

TABLE XIV: TS_{14} : $p = 1$, $\omega_1 = 1/24$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.1361	0.1136	0.1372	0.1791
Trend	-	0.3323	0.3433	0.3889	0.3384
Trend+DLU	Oracle	0.1306	0.1095	0.1484	0.1323
	BIC	0.1359	0.1134	0.1649	0.1335
	EBIC	0.1432	0.1190	0.1649	0.1335
imputeTS	-	0.1763	0.1458	0.1835	0.2032
Trend+imputeTS	-	0.1765	0.1467	0.1821	0.1984
MTSDI	-	0.2897	0.2869	0.3392	0.3158
Trend+MTSDI	-	0.3029	0.3255	0.3614	0.3374

TABLE XV: TS_{15} : $p = 1$, $\omega_1 = 1/24$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.1170	0.1557	0.1865	0.1701
Trend	-	0.3618	0.3141	0.3525	0.3248
Trend+DLU	Oracle	0.1323	0.1425	0.1434	0.1451
	BIC	0.1389	0.1484	0.1458	0.1451
	EBIC	0.1389	0.1484	0.1458	0.1451
imputeTS	-	0.2701	0.2402	0.2728	0.2474
Trend+imputeTS	-	0.2756	0.2478	0.2509	0.2317
MTSDI	-	0.3274	0.3352	0.3348	0.3641
Trend+MTSDI	-	0.3385	0.3128	0.3134	0.3273

TABLE XVI: TS_{16} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.1341	0.1386	0.1425	0.1763
Trend	-	0.3929	0.3888	0.4036	0.3511
Trend+DLU	Oracle	0.1274	0.1283	0.1270	0.1295
	BIC	0.1356	0.1364	0.1365	0.1295
	EBIC	0.1356	0.1364	0.1365	0.1295
imputeTS	-	0.2134	0.1784	0.2055	0.2614
Trend+imputeTS	-	0.2148	0.1743	0.2044	0.2476
MTSDI	-	0.2757	0.3072	0.2993	0.2305
Trend+MTSDI	-	0.3255	0.3366	0.3394	0.2783

TABLE XVII: TS_{17} : $p = 2$, $\omega_1 = 1/24$, $\omega_2 = 1/12$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.0710	0.1503	0.1107	0.1322
Trend	-	0.2633	0.2798	0.3073	0.2861
Trend+DLU	Oracle	0.0692	0.1007	0.0971	0.1074
	BIC	0.0888	0.1030	0.1045	0.1080
	EBIC	0.0888	0.1030	0.1045	0.1080
imputeTS	-	0.1457	0.1841	0.1649	0.1867
Trend+imputeTS	-	0.1431	0.1598	0.1548	0.1590
MTSDI	-	0.1778	0.2602	0.2434	0.2459
Trend+MTSDI	-	0.1921	0.2365	0.2347	0.2346

TABLE XVIII: TS_{18} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.1009	0.1342	0.1405	0.1519
Trend	-	0.4314	0.4809	0.4312	0.4426
Trend+DLU	Oracle	0.1044	0.1292	0.1361	0.1320
	BIC	0.1124	0.1401	0.1513	0.1337
	EBIC	0.1126	0.1401	0.1513	0.1337
imputeTS	-	0.1551	0.2161	0.1790	0.1966
Trend+imputeTS	-	0.1551	0.2161	0.1790	0.1966
MTSDI	-	0.2183	0.2103	0.2313	0.2308
Trend+MTSDI	-	0.3802	0.4133	0.3826	0.4078

TABLE XIX: TS_{19} :

Method	Model Selection	Percentage Missing Data (ρ)			
		5%	10%	15%	20%
DLU	Oracle	0.1764	0.1652	0.1791	0.1567
Trend	-	0.2407	0.2510	0.2086	0.2162
Trend+DLU	Oracle	0.1645	0.1719	0.1373	0.1481
	BIC	0.1774	0.1804	0.1559	0.1535
	EBIC	0.1774	0.1804	0.1559	0.1535
imputeTS	-	0.1771	0.1699	0.1443	0.1498
Trend+imputeTS	-	0.1762	0.1706	0.1401	0.1519
MTSDI	-	0.2634	0.2738	0.2290	0.2535
Trend+MTSDI	-	0.2202	0.2228	0.1912	0.2024

TABLE XX: TS_{20} : $p = 4$, $\omega_1 = 1/168$, $\omega_2 = 1/28$, $\omega_3 = 1/24$, $\omega_4 = 1/12$.

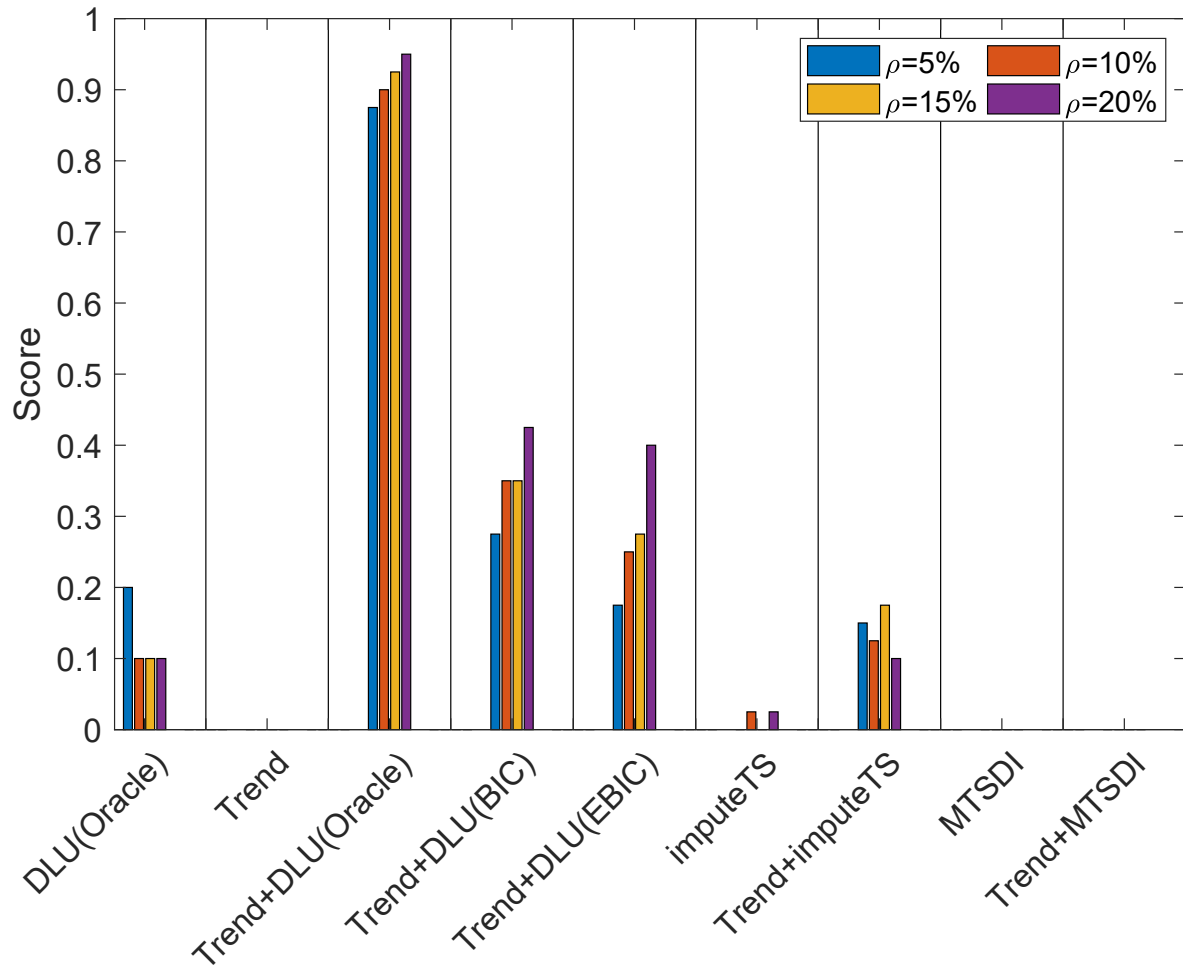


Fig. 1: Missing data generated by sampling without replacement: Scores for the imputation methods are based on the results reported in Tables I-XX. The percentages of the missing data are given in the legend.

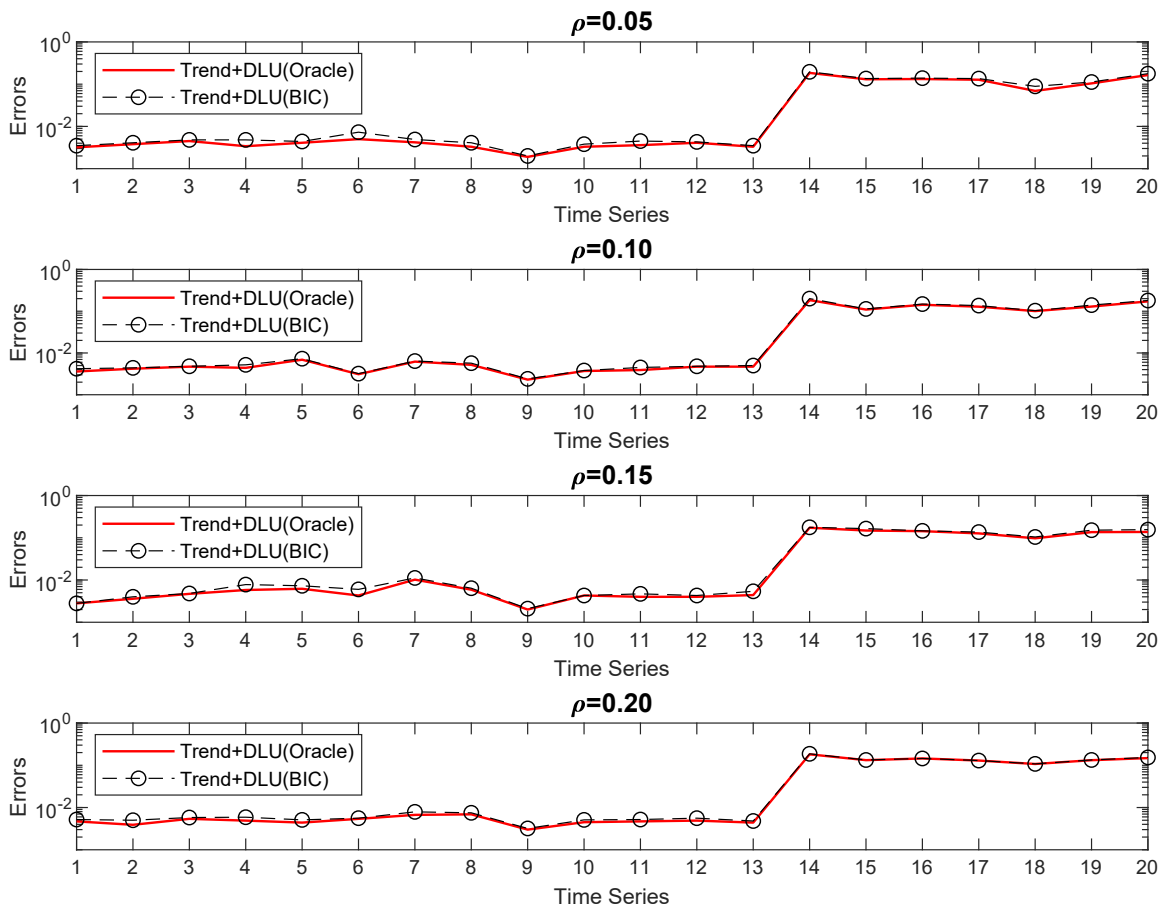


Fig. 2: Missing data generated by sampling without replacement: For each time series, we plot the normalized errors corresponding to the best and to the second best imputation methods. For each panel, the percentage of the missing data is written on top of the plot. Note that the logarithmic scale is used for the vertical axis.

II. DLU: EXPERIMENTAL RESULTS OBTAINED WHEN THE MISSING DATA ARE SIMULATED BY USING THE POLYA URN MODEL

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0200	0.0541	0.0594
Trend	-	0.0122	0.0184	0.0101
Trend+DLU	Oracle	0.0025	0.0088	0.0032
	BIC	0.0027	0.0109	0.0037
	EBIC	0.0027	0.0106	0.0037
imputeTS	-	0.0138	0.0291	0.0704
Trend+imputeTS	-	0.0076	0.0144	0.0104
MTSDI	-	0.0100	0.0155	0.0080
Trend+MTSDI	-	0.0090	0.0104	0.0089

TABLE XXI: TS_1 : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0189	0.4426	0.0433
Trend	-	0.0249	0.0325	0.0266
Trend+DLU	Oracle	0.0042	0.0090	0.0036
	BIC	0.0053	0.0126	0.0042
	EBIC	0.0053	0.0128	0.0036
imputeTS	-	0.0176	0.0921	0.0344
Trend+imputeTS	-	0.0083	0.0085	0.0072
MTSDI	-	0.0127	0.0127	0.0103
Trend+MTSDI	-	0.0132	0.0087	0.0075

TABLE XXII: TS_2 : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0309	0.0265	0.2017
Trend	-	0.0166	0.0201	0.0174
Trend+DLU	Oracle	0.0045	0.0052	0.0078
	BIC	0.0051	0.0060	0.0099
	EBIC	0.0051	0.0060	0.0099
imputeTS	-	0.0176	0.0384	0.0686
Trend+imputeTS	-	0.0116	0.0120	0.0169
MTSDI	-	0.0150	0.0117	0.0097
Trend+MTSDI	-	0.0105	0.0106	0.0126

TABLE XXIII: TS_3 : $p = 4$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$, $\omega_4 = 1/6$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0148	0.0383	0.1098
Trend	-	0.0247	0.0382	0.0590
Trend+DLU	Oracle	0.0042	0.0099	0.0140
	BIC	0.0045	0.0151	0.0140
	EBIC	0.0051	0.0151	0.0140
imputeTS	-	0.0106	0.0750	0.0267
Trend+imputeTS	-	0.0053	0.0136	0.0121
MTSDI	-	0.0203	0.0297	0.0272
Trend+MTSDI	-	0.0114	0.0168	0.0200

TABLE XXIV: TS_4 : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0221	0.0310	0.0996
Trend	-	0.0060	0.0049	0.0197
Trend+DLU	Oracle	0.0033	0.0044	0.0166
	BIC	0.0035	0.0047	0.0193
	EBIC	0.0035	0.0047	0.0193
imputeTS	-	0.0511	0.0625	0.0931
Trend+imputeTS	-	0.0034	0.0042	0.0177
MTSDI	-	0.1239	0.1294	0.1285
Trend+MTSDI	-	0.0049	0.0045	0.0163

TABLE XXV: TS_5 : $\theta_1 = 16$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0209	0.0377	0.0391
Trend	-	0.0107	0.0043	0.0152
Trend+DLU	Oracle	0.0064	0.0032	0.0097
	BIC	0.0081	0.0036	0.0120
	EBIC	0.0081	0.0036	0.0120
imputeTS	-	0.0205	0.0308	0.0377
Trend+imputeTS	-	0.0054	0.0039	0.0071
MTSDI	-	0.0119	0.0078	0.0107
Trend+MTSDI	-	0.0101	0.0053	0.0124

TABLE XXVI: TS_6 : $\theta_1 = 24$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0308	0.0399	0.0386
Trend	-	0.0088	0.0132	0.0223
Trend+DLU	Oracle	0.0033	0.0047	0.0129
	BIC	0.0037	0.0091	0.0138
	EBIC	0.0037	0.0091	0.0138
imputeTS	-	0.0185	0.1166	0.0464
Trend+imputeTS	-	0.0046	0.0046	0.0123
MTSDI	-	0.0187	0.0267	0.0253
Trend+MTSDI	-	0.0080	0.0078	0.0155

TABLE XXVII: TS_7 : $\theta_1 = 24$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0215	0.0391	0.0345
Trend	-	0.0330	0.0277	0.0373
Trend+DLU	Oracle	0.0036	0.0053	0.0064
	BIC	0.0062	0.0060	0.0065
	EBIC	0.0062	0.0062	0.0073
imputeTS	-	0.0181	0.0523	0.0699
Trend+imputeTS	-	0.0096	0.0136	0.0142
MTSDI	-	0.0174	0.0265	0.0163
Trend+MTSDI	-	0.0151	0.0213	0.0150

TABLE XXVIII: TS_8 : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0193	0.0169	0.0917
Trend	-	0.0024	0.0022	0.0023
Trend+DLU	Oracle	0.0019	0.0017	0.0020
	BIC	0.0021	0.0017	0.0022
	EBIC	0.0021	0.0017	0.0022
imputeTS	-	0.0191	0.0151	0.0748
Trend+imputeTS	-	1.0031	0.0032	0.0033
MTSDI	-	0.0115	0.0098	0.0129
Trend+MTSDI	-	1.0131	0.0031	0.0029

TABLE XXIX: TS_9 : $\theta_1 = 24$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0122	0.0292	0.0626
Trend	-	0.0308	0.0360	0.0265
Trend+DLU	Oracle	0.0045	0.0078	0.0049
	BIC	0.0057	0.0078	0.0056
	EBIC	0.0057	0.0100	0.0056
imputeTS	-	0.0185	0.0295	0.0430
Trend+imputeTS	-	0.0060	0.0074	0.0058
MTSDI	-	0.0203	0.0354	0.0231
Trend+MTSDI	-	0.0102	0.0110	0.0112

TABLE XXX: TS_{10} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0212	0.0207	0.0260
Trend	-	0.0121	0.0108	0.0114
Trend+DLU	Oracle	0.0054	0.0039	0.0036
	BIC	0.0061	0.0050	0.0043
	EBIC	0.0061	0.0050	0.0043
imputeTS	-	0.0260	0.0186	0.0193
Trend+imputeTS	-	0.0082	0.0047	0.0054
MTSDI	-	0.0170	0.0147	0.0150
Trend+MTSDI	-	0.0090	0.0083	0.0089

TABLE XXXI: TS_{11} : $p = 4$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$, $\omega_4 = 1/6$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0237	0.0550	0.1251
Trend	-	0.0217	0.0153	0.0176
Trend+DLU	Oracle	0.0057	0.0032	0.0082
	BIC	0.0069	0.0039	0.0113
	EBIC	0.0069	0.0039	0.0097
imputeTS	-	0.0228	0.0234	0.1991
Trend+imputeTS	-	0.0120	0.0072	0.0206
MTSDI	-	0.0105	0.0122	0.0144
Trend+MTSDI	-	0.0081	0.0079	0.0105

TABLE XXXII: TS_{12} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.0303	0.0236	0.3325
Trend	-	0.0151	0.0144	0.0134
Trend+DLU	Oracle	0.0045	0.0048	0.0051
	BIC	0.0048	0.0049	0.0062
	EBIC	0.0057	0.0073	0.0055
imputeTS	-	0.0161	0.0588	0.0619
Trend+imputeTS	-	0.0081	0.0091	0.0073
MTSDI	-	0.0123	0.0125	0.0090
Trend+MTSDI	-	0.0100	0.0111	0.0098

TABLE XXXIII: TS_{13} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.1696	0.1611	0.1908
Trend	-	0.2827	0.2881	0.3318
Trend+DLU	Oracle	0.1793	0.1643	0.1900
	BIC	0.1989	0.1663	0.2229
	EBIC	0.1989	0.1663	0.2229
imputeTS	-	0.2274	0.2082	0.2367
Trend+imputeTS	-	0.2343	0.2012	0.2432
MTSDI	-	0.2569	0.2363	0.2676
Trend+MTSDI	-	0.2921	0.2509	0.3056

TABLE XXXIV: TS_{14} : $p = 1$, $\omega_1 = 1/24$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.1678	0.1869	0.1921
Trend	-	0.3786	0.3429	0.3190
Trend+DLU	Oracle	0.1449	0.1815	0.1386
	BIC	0.1800	0.2065	0.1464
	EBIC	0.1800	0.2065	0.1464
imputeTS	-	0.2497	0.3247	0.2990
Trend+imputeTS	-	0.2345	0.2768	0.2751
MTSDI	-	0.3343	0.3227	0.2957
Trend+MTSDI	-	0.3707	0.3724	0.3196

TABLE XXXV: TS_{15} : $p = 1$, $\omega_1 = 1/24$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.1217	0.1518	0.2359
Trend	-	0.2992	0.3002	0.2378
Trend+DLU	Oracle	0.1002	0.0964	0.1271
	BIC	0.1079	0.1049	0.1371
	EBIC	0.1079	0.1049	0.1371
imputeTS	-	0.1900	0.3027	0.3141
Trend+imputeTS	-	0.1736	0.2181	0.2464
MTSDI	-	0.3267	0.2986	0.2316
Trend+MTSDI	-	0.3052	0.2649	0.2315

TABLE XXXVI: TS_{16} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.1882	0.3717	0.3289
Trend	-	0.3543	0.4563	0.5486
Trend+DLU	Oracle	0.1600	0.2246	0.3997
	BIC	0.1851	0.2472	0.4308
	EBIC	0.1851	0.2472	0.4308
imputeTS	-	0.3078	0.4820	0.5363
Trend+imputeTS	-	0.2627	0.4274	0.5441
MTSDI	-	0.2674	0.3201	0.4156
Trend+MTSDI	-	0.2826	0.3584	0.4850

TABLE XXXVII: TS_{17} : $p = 2$, $\omega_1 = 1/24$, $\omega_2 = 1/12$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.1303	0.0961	0.1766
Trend	-	0.3612	0.2891	0.2705
Trend+DLU	Oracle	0.1471	0.1176	0.1197
	BIC	0.1560	0.1241	0.1254
	EBIC	0.1560	0.1241	0.1254
imputeTS	-	0.1961	0.1478	0.3235
Trend+imputeTS	-	0.1992	0.1923	0.2420
MTSDI	-	0.2998	0.2274	0.2482
Trend+MTSDI	-	0.3152	0.2317	0.1968

TABLE XXXVIII: TS_{18} : $p = 3$, $\omega_1 = 1/24$, $\omega_2 = 1/12$, $\omega_3 = 1/8$.

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.1338	0.1591	0.1472
Trend	-	0.4651	0.3649	0.4342
Trend+DLU	Oracle	0.1418	0.1346	0.1441
	BIC	0.1445	0.2068	0.1739
	EBIC	0.1445	0.2068	0.1739
imputeTS	-	0.1710	0.1782	0.2742
Trend+imputeTS	-	0.1710	0.1782	0.2743
MTSDI	-	0.1940	0.2226	0.1773
Trend+MTSDI	-	0.2798	0.4011	0.3937

TABLE XXXIX: TS_{19} :

Method	Model Selection	Parameter Polya Urn (δ)		
		0.25	0.50	1.00
DLU	Oracle	0.1215	0.1598	0.2094
Trend	-	0.2076	0.1711	0.2296
Trend+DLU	Oracle	0.1123	0.1498	0.2018
	BIC	0.1321	0.1585	0.2299
	EBIC	0.1321	0.1585	0.2299
imputeTS	-	0.1465	0.1808	0.2396
Trend+imputeTS	-	0.1388	0.1557	0.2252
MTSDI	-	0.2367	0.1838	0.2598
Trend+MTSDI	-	0.1896	0.1705	0.2295

TABLE XL: TS_{20} : $p = 4$, $\omega_1 = 1/168$, $\omega_2 = 1/28$, $\omega_3 = 1/24$, $\omega_4 = 1/12$.

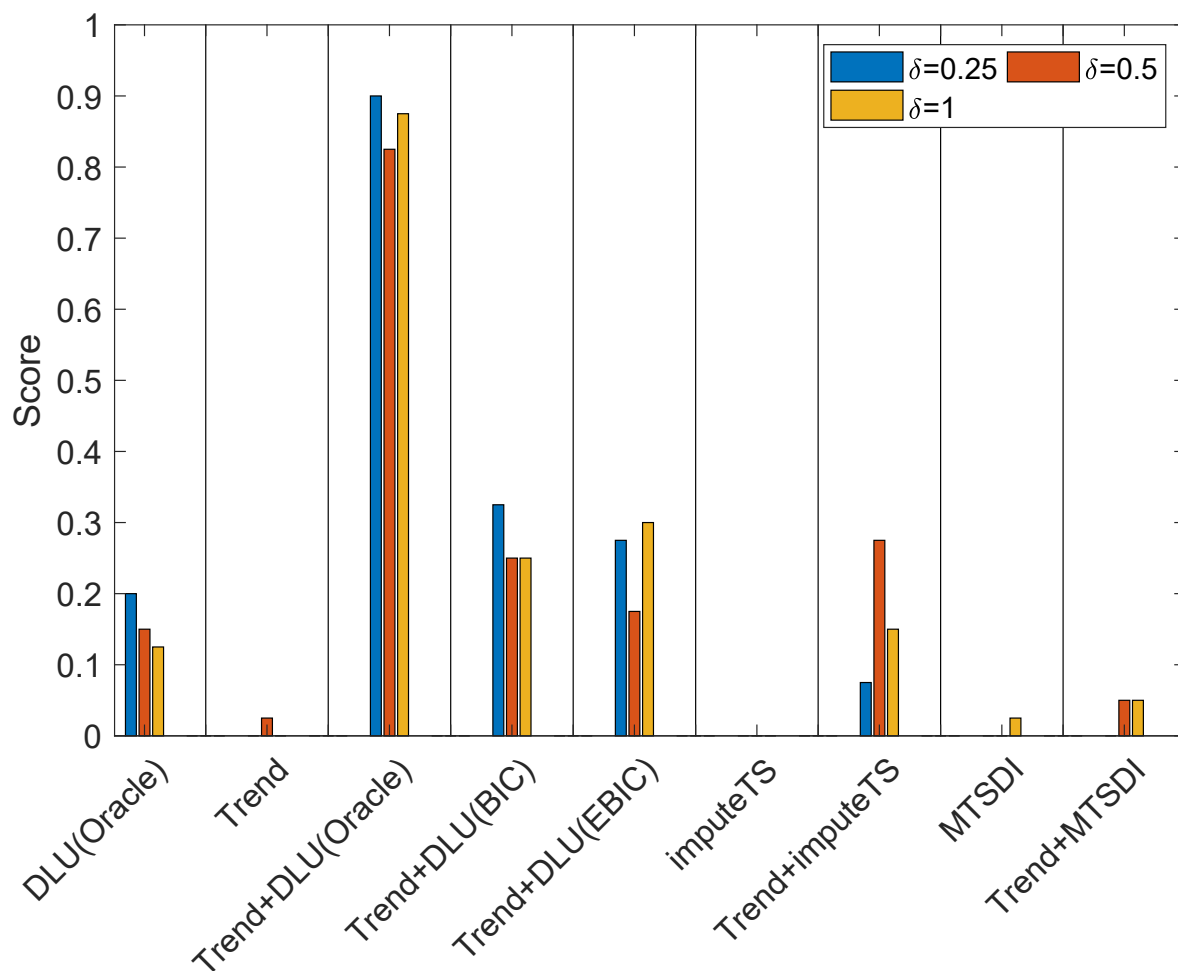


Fig. 3: Missing data generated by using the Polya urn model: Scores for the imputation methods are based on the results reported in Tables XXI-XL. The parameter M for the Polya urn model takes value 5; the values of the parameter δ are given in the legend. The percentage of the missing data is $\rho = 5\%$.

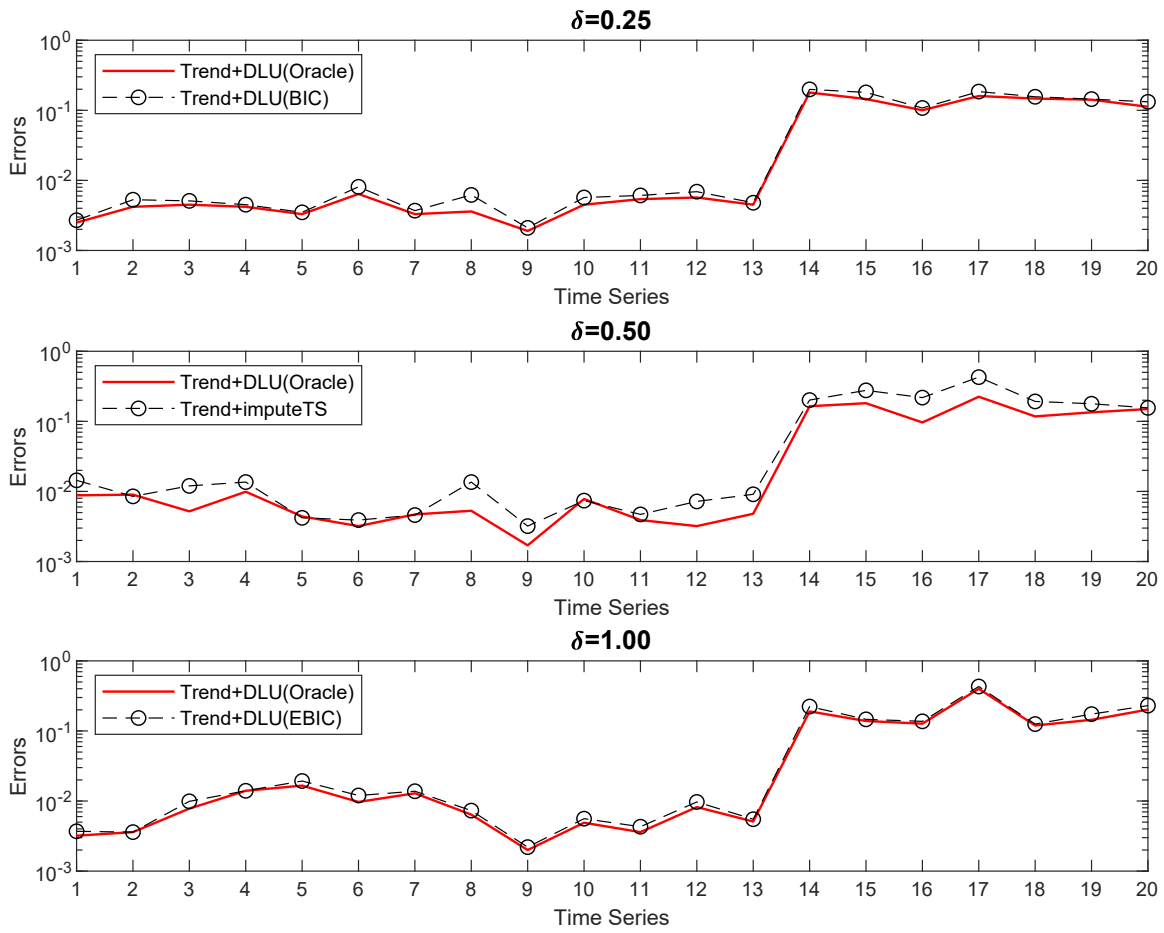


Fig. 4: Missing data generated by using the Polya urn model: For each time series, the normalized errors are represented with the same graphical conventions as in Fig. 2.

III. DLM: EXPERIMENTAL RESULTS

Some explanations about the normalized errors shown in Table XLI:

- The percentage of the missing data for each time series is $\rho = 5\%$, and the missing data are simulated by sampling without replacement.
- When applying imputation methods that contain the word “Trend” in the acronym, the linear trend is removed from all three time series. From the time series VIC and SA are also removed two periodic components with frequencies $\omega_1 = 1/12$ and $\omega_2 = 1/3$.
- The symbol n denotes the size of the dictionary and s stands for the sparsity of the dictionary. In the case of DLM, $n_4 = n_{K+1}$ because the number of time series is $K = 3$; n_4 denotes the size of the dictionary block that is common for all time series. The symbol n_u represents the number of atoms used in the representation of each time series.
- The imputation methods that involve Oracle are not included in the ranking. This is the main difference in comparison with the results presented in the previous sections of this document. For each time series, the smallest normalized error produced by a competing method is written in red; the second smallest normalized error produced by a competing method is written in bold.
- Avg. represents the average of the normalized errors for the time series NSW, VIC and SA.

Method	Model Selection	Time Series			Avg.
		NSW	VIC	SA	
DLM	Oracle	0.1170	0.1448	0.2221	0.1613
	$(n_u = 36; n_4 = 24; s = 2)$ BIC	0.0920	0.2831	0.2543	0.2098
	$(n_u = 24; n_4 = 12; s = 2)$ EBIC	0.0920	0.2831	0.2543	0.2098
Trend+DLM	Oracle	0.0932	0.0795	0.0970	0.0899
	$(n_u = 36; n_4 = 12; s = 2)$ BIC	0.0937	0.0785	0.1246	0.0989
	$(n_u = 36; n_4 = 24; s = 2)$ EBIC	0.0937	0.0785	0.1246	0.0989
DLU	Oracle	0.0931	0.1727	0.2797	0.1818
	$(n = 36; s = 2)$ BIC	0.1272	0.3091	0.3355	0.2573
	$(n = 24; s = 2)$ EBIC	0.1272	0.3091	0.3355	0.2573
Trend+DLU	Oracle	0.0837	0.0766	0.1104	0.0902
	$(n = 24; s = 3)$ BIC	0.0970	0.0896	0.1285	0.1050
	$(n = 24; s = 2)$ EBIC	0.0970	0.0896	0.1285	0.1050
imputeTS	–	0.2839	0.3699	0.4752	0.3763
Trend+imputeTS	–	0.2839	0.2949	0.3079	0.2956
MTSDI	–	0.1143	0.1016	0.1002	0.1054
Trend+MTSDI	–	1.0390	0.9755	0.9146	0.9764

TABLE XLI: Normalized errors computed for the dataset that contains the time series NSW, VIC and SA.

REFERENCES

- [1] X. Zheng, B. Dumitrescu, J. Liu, and C.D. Giurcăneanu, "On the use of dictionary learning in time series imputation," submitted, 2020.