Tsai Calibration and Noise

COMPSCI 773

What is calibration?

- **Given**: A mathematical model of a system with unknown parameters.
- **Given**: A set of measurements from an actual system.
- **Goal**: Find values for the unknown parameters so that the model best first the actual observations.
- **Example**: Hooke's Law for Springs: $F = -\kappa x$ where κ is the unknown
- **Problems**:
  - Unrealistic mathematical model
  - Error in measurements (noise)

Camera Calibration

- Many Types
  - Colour Calibration
  - Radiometric Calibration
- Our Focus: Tsai Calibration
  - What is the underlying model that we are fitting in Tsai Calibration?
  - Is this model realistic?
- Main Issue
  - Measurement Error (Noise)
  - Question: How do we deal with this noise?

Understanding Noise

- Different types of noise
- Noise is classified by the approach used to minimize it’s effect
- Three important types
  - **Systematic Error**: Consistently affects all measurements.
  - **Gross Error**: A blunder or mistake in the process. This includes human error, bugs in programs or sensors or malfunctions.
  - **Random Error**: Random events during the measurement process. Factors such as discretization and sensor accuracy prevent 100% accuracy.

Systematic Error

- In Tsai calibration: lens distortion
- Solution: A correction model
- $x_u = x_d(1 + k_1\rho^2)$ and $y_u = y_d(1 + k_1\rho^2)$
- Could be extended to allow for more radial and tangential coefficients.
- Could be extended to allow for decentring.
- How do we verify the quality of our distortion model?
- What problems could be associated with pre-applying a distortion model to measurements?

Harping on about distortion

- Zhang calibration uses:
  - $x_d = x_u(1 + k_1\rho^2)$ and $y_d = y_u(1 + k_1\rho^2)$
- What advantage does this have over the Tsai model?
  - How can we get distorted coordinates from undistorted coordinates using the Tsai model?

Reversing Tsai Distortion

- Given: Tsai model and a set of **undistorted** coordinates.
- Goal: Retrieve the **distorted** coordinates.
- Let $r_u = x_u^2 + y_u^2$ and $r_d = x_d^2 + y_d^2$
- $x_u^2 = x_d^2(1 + \kappa_1 r_d)^2$ and $y_u^2 = y_d^2(1 + \kappa_1 r_d)^2$
- Then $r_u = r_d(1 + \kappa_1 r_d)^2$ (cubic equation with unknown r_d)
- Newton's method: $x_1 = x_0 - \frac{f(x_0)}{f\prime(x_0)}$
- Closed Form: Cardano's Formula
- $x_d = (\frac{r_d x_u}{r_u})$ and $y_d = (\frac{r_d y_u}{r_u})$

Gross Error

- Good Lighting on the calibration cube! Why?
- Automated Corner extraction: See https://www.cs.auckland.ac.nz/courses/compsci773s1c/lectures/COMPSCI%20773%20feature%20point%20detection.pdf
- Automated detection of outliers: RANSAC
- Unit Testing your code: https://github.com/google/googletest

RANSAC

- Random Sample Consensus
- Could be applied to Tsai in phase 1 (linear part)
- Algorithm
  - Select a random subset of measurements (hypothetical inliers)
  - Fit the model to the inliers
  - Calculate reprojection error for all measurements
  - Form new hypothetical inliers based on error threshold
  - Repeat using the hypothetical inliers from the previous iteration
  - Quit when hypothetical inliers group stops growing or after x iterations.
- Warning: No guaranteed convergence, the algorithm might attempt to converge towards outliers.

Random Error

- Stochastic model of random noise
- Typically a normal distribution
- $\alpha e^{-\beta(\mu - x)^2}$
- Parameterized by a mean (μ) and a standard deviation (σ)
- The mean of a normal distribution is the value with maximum likelihood. This is known as the expected value.
- The standard deviation reflects the “spread” of the distribution.

Tsai Stochastic Model

- Use the normal distribution: $\alpha e^{-\beta(\mu-x)^2}$
- Goal is to find the expected value.
- This can be done be minimizing $(\mu - x)^2$.
- The term $(\mu - x)^2$ is known as the least squares error.
- The solution is Moore Penrose (also known as the pseudo inverse).

Moore Penrose

- $x = (A^T A)^{-1} A^T b$
- Similar to the vector projection equation: $\frac{A \cdot B}{A \cdot A} B$
- From a performance point of view, what problem do you see with Moore-Penrose?
- Things to research in your own time:
  - Cholesky Decomposition
  - QR Decomposition
  - Singular Value Decomposition (will be covered with fundamental matrices)

Rotation

- Linear least squares and rotation is problematic because rotation is not linear.
- Rotation is often approximated by it’s Taylor series expansion.
- After Moore Penrose rotation is not necessary orthonormal.
- Solution: Convert R into Euler angles and then find the corresponding orthonormal rotation.

Finding an orthonormal rotation

- Find the rotation matrix as per the notes
  - Extract the first two rows of the Rotation matrix from the L matrix
  - Use the outer product of the first two rows to get the last one.
- Find closest orthonormal matrix:
  - Convert to R to Euler from: heading(y) = atan2(-r20, r00), attitude(z) = asin(r10), bank(x) = atan2(-r12, r, r11)
  - Reform rotation matrix

[R] =

ch*ca	-ch*sa*cb + sh*sb	ch*sa*sb + sh*cb
sa	ca*cb	-ca*sb
-sh*ca	sh*sa*cb + ch*sb	-sh*sa*sb + ch*cb

where:

- sa = sin(attitude)
- ca = cos(attitude)
- sb = sin(bank)
- cb = cos(bank)
- sh = sin(heading)
- ch = cos(heading)

Further Refinements to least squares

- Normalize Errors by using negative log probabilities. This removes bias.
- Weighted Least Squares: $-\log\left(e^{-0.5r^2} + \tau\right)$
  - Removes the effects of outliers
  - Normal distribution tails do not model outliers well
  - Disadvantage is that this formulation must be iterative.
  - Best kept for the second phase of Tsai calibration
- In the case of iterative least squares such as Levenberg Marquadt or Gauss Newton, note that the order of parameters may influence the quality of convergence.

Zhang Calibration

- Solves the same problem as Tsai Calibration.
- Uses a single planar chess board as a calibration object
- Required to capture 8+ images for calibration (collectively cover FOV).
- Has more complex formulation: allows for decentring, radial and tangential distortion.
- Distortion equation solves for distorted coordinates rather than undistorted coordinates.
- Newer than Tsai and very popular (Matlab toolbox, OpenCV etc)
- http://opencv.org/

Summary: Trouble-Shooting my calibration

- Problem: The error associated with my model is beyond an acceptable range.
  - Is my program correct? (Gross Error)
    - Calculate relations between test image points and calibration points using an excel spreadsheet. Compare with your code to verify that no bugs exist.
  - If it is not the program, then it must be the measurements…
    - Find mean and variance of reprojection error. Noisy data has high variance.
    - If variance is high (gross error + random error)
      - Redo experiment and make sure lighting is good, images are not blurred, calibration object is close to ideal etc.
      - Use RANSAC to eliminate outliers for the linear part of Tsai calibration.
      - Refine parameters using weighted least squares optimization.
    - If variance is small but mean is consistently wrong or mean and variance seem to be dependant on position of the point (systematic error)
      - Improve model (more distortion parameters, rotation matrix not orthonormal etc)