THE UNIVERSITY OF AUCKLAND

FIRST SEMESTER, 2011
Campus: City

COMPUTER SCIENCE

COMPSCI 773: Intelligent Vision Systems

(Time allowed: TWO hours)

NOTE: Attempt **all** questions!

Write the answers in the boxes below the questions.

Marks for each question are shown just before each answer box.

This is an open book exam. Candidates **may** bring calculators, notes, reference books, or other written material into the examination room.

Section:	**A**	**B**	**C**	Total
Possible marks:	25	50	25	100
Awarded marks:				

SURNAME:

FORENAME(S):

STUDENT ID:

QUESTION/ANSWER SHEETS FOLLOW

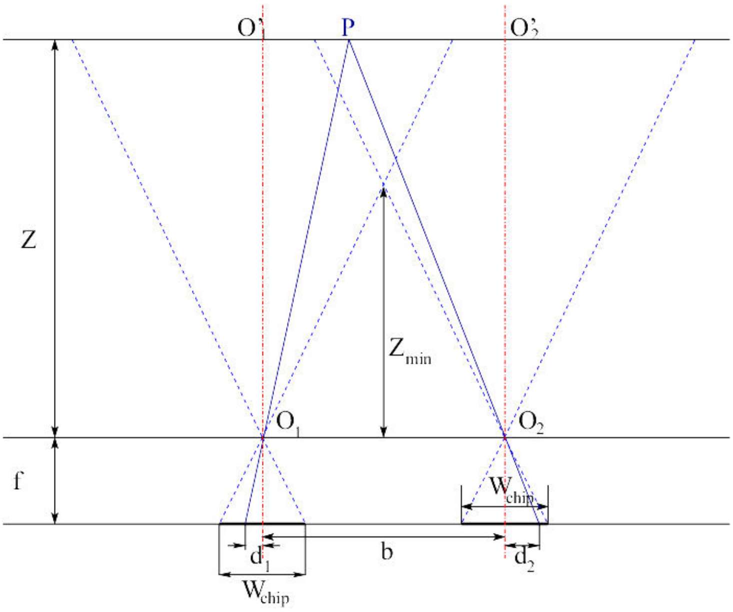
Student ID: ________________

Section A: Patrice: 3D geometry/calibration: 25 marks

1. [25 marks]

Student ID: ______________________________

Section B: Epipolar geometry and binary machine vision



Section B:.1 Calibration

2. Consider a 3 by 3 calibration matrix with r_1, r_2, and r_3 colum vectors. In both Tsai and Zhang's calibration, r_1 and r_2 are obtained first. Explain why and how r_3 can be inferred from r_1 and r_2? [3 marks]

3. **Application:** Consider q2, vector $r_1 = \begin{bmatrix} 1\sqrt{2} \\ 1\sqrt{2} \\ 0 \end{bmatrix}$ and $r_2 = \begin{bmatrix} 1\sqrt{2} \\ 0 \\ 1\sqrt{2} \end{bmatrix}$ [2 marks]

Student ID: ____________________

4. Consider the 3D world reference frame in which all cartesian points are written as $X_w = \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix}$.

   Consider the camera reference frame in which all cartesian points are written as $M_c = \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$.

   Consider the 3 by 3 rotation R and translation T which relates the camera reference frame to the world reference frame. Write the transform M such that $X_w = MX_c$ as a 4 by 4 homogeneous matrix. [3 marks]

5. Write the homogeneous matrix $\tilde{M}$ which relates X_c to X_w as a function of R and T ($X_c = \tilde{M}X_w$). [3 marks]

Student ID: ______________________

6. **Application:** Consider q4 and q5, Rotation $R = \begin{bmatrix} 1\sqrt{2} & 1\sqrt{2} & 0 \\ -1\sqrt{2} & 1\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and translation T $r_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Compute M and $\tilde{M}$. Show your working. [4 marks]

Section B:.2 Stereo vision

7. Two identical cameras, with optical centres O_1 and O_2, are placed in epipolar positions with parallel optical axes (O_1O_1 and O_2O_2) as displayed in the above figure. The distance between the optical centres is known as the baseline distance b and the focal length for both camera is given by f. p is the physical width of one pixel on the cameras sensor and n the number of pixels on

Student ID: ____________________

one scanline with W_{chip} the cameras sensor width, or scanline width. A point P at depth Z appears in each image at different position on a scanline. The disparity d_P of point P, is given by $d_P = d_2 - d_1$.
Exhibit the formula which links the disparity d_P to the depth Z at point P, the focal length f and the baseline b (Show your working). [3 marks]

8. Consider R_1, T_1 and R_2, T_2 rotation and translation matrices which relate camera 1, respectively camera 2, with the **same** calibration object. Find the matrix P which relates camera 1 and camera 2 optical centers ($O_2 = PO_1$) as a function of matrices R_1, T_1, R_2, and T_2. Show your working. [3 marks]

9. Consider the following values for R_1, T_1 and R_2, T_2. Compute the baseline b. [3 marks]

Student ID: ____________________

10. Consider point M_1 in camera 1 situated at the horizontal distance x_1 from the camera 1 optical center. M_1 is the projection of world point M into camera 1. Compute X_1 horizontal ccordinate of point M_1 as a function of x_1, camera 1 and 2 parameters and Z_M the shortest distance between point M and O_1O_2 . Show your working. [3 marks]

11. Knowing Z and the stereo system parameters f, and b, infer x_2. Consider point M_1 in camera 1 situated at the horizontal distance x_2 is the horizontal distance from the camera 1 optical center of point M_2 which is the projection of world point M into camera 2. [3 marks]

Student ID: ____________________

12. What is the disparity for point M in the cameta systsem defined in questionX [3 marks]

13. what is the disparity value when point P is at the horizon, respectively on the cameras scanline? [2 marks]

Student ID: ______________________________

Student ID: ______________________________

14. Practically, the camera resolution (that is its pixels width p and sensors width W_{chip}) will determine the minimum measurable depth Z_{min}. Give the formula which gives Z_{min} as a function of the sensors width, b and f. Compute Z_{min} when the cameras are 5 centimetres apart, the focal length is 25 mm, the number of pixel elements per line is 1000 and the pixel width is 5 μm.

[5 marks]

Student ID: ____________________

Section B:.3 Calibration

15. Explain what is the Radius of Ambiguity and what is it used for. [2 marks]

16. The world reference frame is situated on the cube and follows the clock-wise orientation rule (x-axis horizontal rightwards, y-axis vertical upwards). Patches P_1, P_2 and P_3 with respective centre at coordinates $[100, 100, 0]$, $[104, 100, 0]$, $[100, 104, 0]$ are circular targets on the plane used to calibrate a camera. After calibration, the rotation matrix R linking the camera optical centre to the world reference frame is equal to $[1, 0, 0; 0, -1, 0; 0, 0, 1]$, the translation vector T linking the WRF centre to the camera optical centre is given by $T = [0, 0, -1000]^T$ where v^T is the transpose of vector v. The focal length f is calculated equal to 5mm and the camera sensor specifications are:
Width 4.800 mm (640 elements)
Height 3.600 mm (480 elements)
A distortion-free lens is assumed. After some processing steps on the image of the calibration plane acquired for calibration purpose, the target P_1 is found to have its centre at position (389,308.333) in the **image**.

Draw the different reference frames as well as the patch centres and image point respecting the usual convention as well as the above-mentioned requirements. [3 marks]

Student ID: ____________________

17. Compute the corresponding Radius of Ambiguity using the L_2 Euclidean distance as a distance measure. [5 marks]

Student ID: ____________________

18. in your opinion, is the calibration satisfactory? [2 marks]

19. Assume a radial distortion effect with $\kappa_1 = -0.00066\ mm^{-2}$ and Tsai formulation between distorded and undistorded coordinates: $r_u = r_d \left(1 + \kappa_1 r^2\right)$
Compute the new radius of ambiguity. Comments? [5 marks]

Student ID: ______________________

Section C: Epipolar Geometry and Stereo Matching: 50 marks

20. Let the baseline of a two-camera system coincide with the X-axis of the world XYZ-coordinates. How are both the cameras placed one with respect to another if the epipole in the left image coincides with the principal point (trace of the optical axis) and the epipole in the right image is sitting infinitely far along the X-axis and has zero y-coordinate? [4 marks]

21. How are two cameras placed one with respect to another if epipoles in both images are sitting infinitely far along the Y-axis of the world co-ordinate frame and have the same x-coordinate? [4 marks]

22. Given a camera with the projection matrix $P_1 = \begin{bmatrix} 0.25 & 0 & 0 & -4 \\ 0 & 0.5 & 0 & -2 \\ 0 & 0 & 0.25 & -1 \end{bmatrix}$, determine the optical centre of this camera? [6 marks]

Student ID: ______________________________

23. Given the same camera as in Question 22 above, and the second camera with the projection matrix $P_2 = \begin{bmatrix} 0.5 & 0 & 0 & 4 \\ 0 & 0.25 & 0 & 2 \\ 0 & 0 & 0.5 & 1 \end{bmatrix}$, determine the point $\widetilde{\mathbf{D}}_1 = \begin{bmatrix} \mathbf{D}_1 \\ 0 \end{bmatrix}$ at the infinity of the projection ray, which projects the 3D point with homogeneous coordinates $[1, 1, 1, 1]^\mathsf{T}$ to the image plane of the first camera, and project $\widetilde{\mathbf{D}}_1$ to the image plane of the second camera. [6 marks]

24. What relationship does exist between the fundamental matrix $\mathbf{F} = [F_{i,j}]_{i,j=1}^{3}$ of a pair of cameras and the homogeneous coordinates $\widetilde{\mathbf{p}}_1$ and $\widetilde{\mathbf{p}}_2$ of corresponding points with the Cartesian coordinates $\mathbf{p}_1$ and $\mathbf{p}_2$ in the left and right images, respectively, of a stereo pair captured by the cameras. [5 marks]

25. In terms of the relationship in Question 24, specify the epipolar line $\mathbf{a}^\mathsf{T}\widetilde{\mathbf{p}}_1 = 0$ in the left image that corresponds to the point with Cartesian coordinates $\mathbf{p}_2 = (x_2, y_2)$ in the right image. [4 marks]

26. In which point(s) do all the epipolar lines of the right and left image of a stereo pair intersect? [3 marks]

Student ID: ______________________________

27. Describe, in brief, main reasons why stereo matching that searches for corresponding areas in a stereo pair of images is an ill-posed, in the math sense, problem. [4 marks]

28. 3D stereo reconstruction of human heads / faces typically uses stereo pairs captured with cameras having a vertically oriented baseline. Explain in brief why such pairs are more appropriate than the pairs with the conventional horizontal baseline. [4 marks]

29. Describe in brief which differences between the corresponding image signals are taken into account in the the correlation based matching.

    Hint: Consider math models of signals and noise that lead to the matching score. [6 marks]

30. Describe in brief which problem does the dynamic programming stereo (DPS) solve. [6 marks]

    Hint: Consider simplifications of a 3D surface model and the choice of the matching score leading to DPS.

Student ID: ______________________________

Section D: Patrice: PCA: 25 marks

31. the database A contains ten $2D$ points:

$$x_1 \begin{pmatrix} 3 \\ 3 \end{pmatrix}, x_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix}, x_3 \begin{pmatrix} 2 \\ 3 \end{pmatrix}, x_4 \begin{pmatrix} 2 \\ 1 \end{pmatrix}, x_5 \begin{pmatrix} 6 \\ 5 \end{pmatrix}, x_6 \begin{pmatrix} 7 \\ 6 \end{pmatrix}, x_7 \begin{pmatrix} 5 \\ 7 \end{pmatrix}, x_8 \begin{pmatrix} 7 \\ 7 \end{pmatrix},$$
$$x_9 \begin{pmatrix} 8 \\ 9 \end{pmatrix} \text{ and } x_{10} \begin{pmatrix} 9 \\ 8 \end{pmatrix}$$

Points x_1, x_2, x_3 and x_4 belong to class 1, x_5, x_6, x_7 belong to class 2, x_8, x_9 and x_10 belong to class 3.

Section D:.1 PCA

(a) Compute the covariance matrix of the centred database given by: $C = \sum_{i=1}^{i=10} y_i y_i^T$

[5 marks]

Student ID: ____________________

(b) Compute the eigenvalues (λ_1, λ_2) of the matrix C. [3 marks]

(c) Find the eigenvectors e_1, e_2 associated to the eigenvalues λ_1, λ_2.

[2 marks]

Student ID: ____________________

(d) Find the principal components of the database vectors along the direction of the largest variance. [2 marks]

(e) Express each vector of the database x_i as a weighted linear combination of eigenvectors e_1 and e_2. [3 marks]

Student ID: ______________________________

Compute the Euclidean distance between points x_6 and x_8 (d_{68}), x_5 and x_7 (d_{57}), x_8 and x_9 (d_{89}). Compute the same distances using the projected coordinates in the direction of the largest variance. Any comments? [5 marks]

Student ID: ____________________

Section D:.2 LDA

32. Use the same database as in question 31.

    (a) Compute the between-class scatter matrix S_B and the within-class scatter matrix S_W for the database A. [7 marks]

Student ID: ______________________________

(b) Find the direction (described by the vector e) which maximises the distance between the projected mean values of the classes of the database A while keeping the within class variances low. In other terms, do the LDA ! [6 marks]

Student ID: ______________________________

(c) Draw the database set and the directions of projections computed via PCA and LDA. [3 marks]

(d) Compare the distances obtained in question **??** with their values when the database points are projected along the direction obtained via LDA analysis. Comments? [3 marks]

Student ID: ______________________________

33. Compute for each class 1,2 and 3 the mean and standard deviation before and after projection on the main PCA and LDA axis. What do you think of the following two assertions:

    (a) PCA maximises the variance of the overall dataset

    (b) LDA maximises the between-class variance while minimizing the within-class variance [6 marks]

Student ID: ____________________

Overflow page 1

Student ID: ______________________________

Overflow page 2