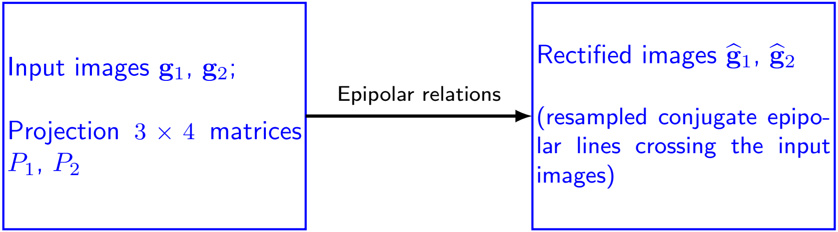
Rectification of a Stereo Pair: Calibrated Cameras

Input: two images of a stereo pair; projection matrices.
Output: the rectified stereo pair.

- Rectification – transforming to a canonical stereo geometry.



- Rectification via conjugate epipolar lines: Slides 30 – 31
- Epipolar relations for rectification: Slides 32 – 38
- Epipolar geometry (theory) leading to relations: Slides 20 – 29
- 3D reconstruction from a stereo pair: Slides 69 – 73

Rectification of a Stereo Pair: Uncalibrated Cameras

Evaluating the projection matrices P_1 and P_2:

Input: a set of n; $n \geq 8$, corresponding points in a stereo pair.

- Estimating the fundamental matrix F.
- Defining parametric projection matrices for the found F.

- Parametric projection matrices for a known F: Slides 80 – 85
- Numerical estimation of F: Slides 49 – 62
- Geometric theory justifying the rectification: Slides 63 – 68
- 3D reconstruction from a stereo pair: Slides 69, 74 – 79
  - Defining the projection matrices.
  - Returning to the calibrated triangulation (Slides 70 – 73).

Calibration Parameters: An Example

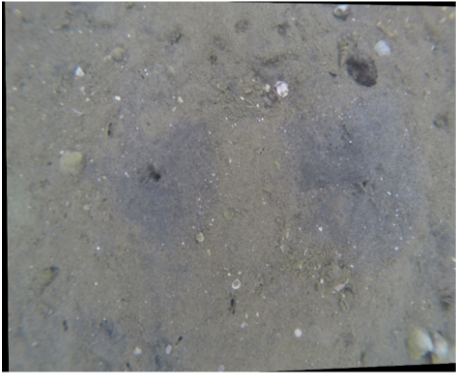
Underwater stereo pair captured for Leigh project by GoPro Hero 3+ cameras (black edition)

Initial left image

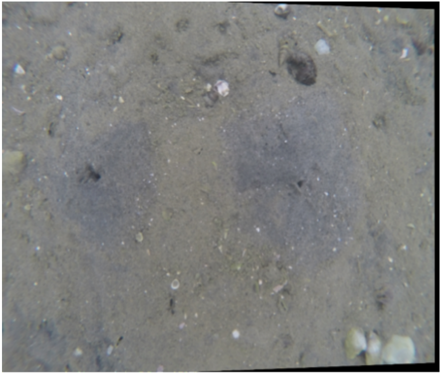
Initial right image

Calibration Parameters: An Example

Epipolar underwater stereo pair after distortion correction and rectification:



Left image



Right image

Calibration Parameters: An Example

After calibration, the rotation and translation between the left and right camera are represented by the extrinsic matrix for "idealised" 3D-to-2D projection by the left camera:

$$M_{\mathrm{e:l}} = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

and the extrinsic matrix for relative 3D-to-2D projection by the right camera:

$$M_{\mathrm{e:r}} = \begin{bmatrix} 0.9999 & 0.0103 & 0.0014 & -33.7193 \\ -0.0103 & 0.9999 & 0.0007 & -0.0319 \\ -0.0014 & -0.0007 & 1.0000 & 1.5822 \end{bmatrix}$$

Calibration Parameters: An Example

In other words, every 3D point $\mathbf{S} = [X, Y, Z]^\mathsf{T}$ with homogeneous coordinates $\widetilde{\mathbf{S}} = [X, Y, Z, 1]^\mathsf{T}$ is projected into two 2D points $\mathbf{s}_\mathrm{l} = [x_\mathrm{l}, y_\mathrm{l}]^\mathsf{T}$ and $\mathbf{s}_\mathrm{r} = [x_\mathrm{r}, y_\mathrm{r}]^\mathsf{T}$ with homogeneous coorddinates $\widetilde{\mathbf{s}}_\mathrm{l} = [x_\mathrm{l}, y_\mathrm{l}, 1]^\mathsf{T}$ and $\widetilde{\mathbf{s}}_\mathrm{r} = [x_\mathrm{r}, y_\mathrm{r}, 1]^\mathsf{T}$ as follows:

$$\begin{bmatrix} a_\mathrm{l} x_\mathrm{l} \\ a_\mathrm{l} y_\mathrm{l} \\ a_\mathrm{l} \end{bmatrix} = M_\mathrm{e:l} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

and

$$\begin{bmatrix} a_\mathrm{r} x_\mathrm{r} \\ a_\mathrm{r} y_\mathrm{r} \\ a_\mathrm{r} \end{bmatrix} = M_\mathrm{e:r} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9999 \cdot X + 0.0103 \cdot Y + 0.0014 \cdot Z - 33.7193 \\ -0.0103 \cdot X + 0.9999 \cdot Y + 0.0007 \cdot Z - 0.0319 \\ -0.0014 \cdot X - 0.0007 \cdot Y + 1.0000 \cdot Z + 1.5822 \end{bmatrix}$$

Calibration Parameters: An Example

3D-to-2D relations via extrinsic parameters obtained by calibration (i.e., via extrinsic matrices $M_{\mathrm{e:l}}$ and $M_{\mathrm{e:r}}$):

$$x_{\mathrm{l}} = \frac{X}{Z}$$

$$y_{\mathrm{l}} = \frac{Y}{Z}$$

$$x_{\mathrm{r}} = \frac{0.9999 \cdot X + 0.0103 \cdot Y + 0.0014 \cdot Z - 33.7193}{-0.0014 \cdot X - 0.0007 \cdot Y + 1.0000 \cdot Z + 1.5822}$$

$$y_{\mathrm{r}} = \frac{-0.0103 \cdot X + 0.9999 \cdot Y + 0.0007 \cdot Z - 0.0319}{-0.0014 \cdot X - 0.0007 \cdot Y + 1.0000 \cdot Z + 1.5822}$$

Calibration Parameters: An Example

3×3 left and right camera matrices:

$$M_{\mathrm{c:l}} = \left[m_{\mathrm{c:l}:i,j}\right]_{i,i=1,1}^{3,3} = \begin{bmatrix} 1725.2128 & 0.0000 & 1980.1533 \\ 0.0000 & 1720.6328 & 1497.4335 \\ 0.0000 & 0.0000 & 1.000 \end{bmatrix}$$

and

$$M_{\mathrm{c:r}} = \left[m_{\mathrm{c:r}:i,j}\right]_{i,i=1,1}^{3,3} = \begin{bmatrix} 1734.8389 & 0.0000 & 2051.2231 \\ 0.0000 & 1730.4694 & 1503.2441 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

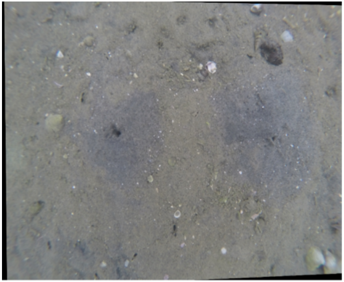
- Elements $m_{\mathrm{c:l(r)}:1,1}$ and $m_{\mathrm{c:l(r)}:2,2}$ of the camera matrices are the scaled focal length $\left(\gamma_{\mathrm{l(r)}:x(y)} f\right)$ divided by the pixel size (δ_{pix}).
- Elements $m_{\mathrm{c}:l(r):1,3}$ and $m_{\mathrm{c}:l(r):2,3}$ of the camera matrices are the scaled principal point $\left(\gamma_{\mathrm{l(r)}:x(y)} c_{\mathrm{l(r)}:x(y)}\right)$ divided by the pixel size.
- Matrices $P_{\mathrm{l}} = M_{\mathrm{c:l}} M_{\mathrm{e:l}}$ and $P_{\mathrm{r}} = M_{\mathrm{c:r}} M_{\mathrm{e:r}}$ – projections from 3D Cartesian coordinates to 2D pixel coordinates.

Calibration Parameters: An Example

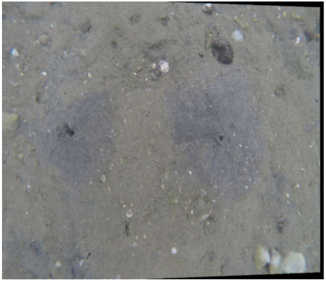
The calibration here uses 4 distortion coefficients:

Left image:	-0.2462	0.0610	-0.0006	-0.0000
Right image:	-0.2457	0.0598	-0.0007	-0.0000

This is a slight extension of the Tsai calibration model.



Left rectified image



Right rectified image

The pixel size of the image produced: 4000×3000.

Epipolar Relations: Calibrated Cameras ($\widetilde{\ldots}$ – homogeneous coordinates)

Given the 3×4 projection matrices $P_i = [Q_i \;\; \mathbf{q}_i]$; $i = 1, 2$, for 2 sensors, a 3D point $\mathbf{S}$ is projected to the corresponding image points $\widetilde{\mathbf{s}}_i = P_i \widetilde{\mathbf{S}}$

1. Find optical centres: $P_i \begin{bmatrix} \mathbf{O}_i \\ 1 \end{bmatrix} \equiv [Q_i \;\; \mathbf{q}_i] \begin{bmatrix} \mathbf{O}_i \\ 1 \end{bmatrix} = \widetilde{\mathbf{0}}$

   (i.e. the projected point with indefinite Cartesian coordinates $x, y, z = \frac{0}{0}$):
   $$Q_i \mathbf{O}_i + \mathbf{q}_i \cdot 1 = \mathbf{0} \;\Rightarrow\; \mathbf{O}_i = -Q_i^{-1} \mathbf{q}_i$$

2. Compute the epipoles (intersections of all epipolar lines) by projecting the optical centres $j \in \{1, 2\}$ ($i \in \{1, 2\}$; $j \neq i$):
   $$\widetilde{\mathbf{e}}_j = P_j \begin{bmatrix} \mathbf{O}_i \\ 1 \end{bmatrix} = [Q_j \;\; \mathbf{q}_j] \begin{bmatrix} -Q_i^{-1} \mathbf{q}_i \\ 1 \end{bmatrix} = -Q_j Q_i^{-1} \mathbf{q}_i + \mathbf{q}_j$$

3. Find the point $\mathbf{D}_i$ at the infinity of the projecting ray $\overline{\mathbf{O}_i \mathbf{s}_i}$:
   $$P_i \begin{bmatrix} \mathbf{D}_i \\ 0 \end{bmatrix} = Q_i \mathbf{D}_i + \mathbf{q}_i \cdot 0 = \widetilde{\mathbf{s}}_i \;\Rightarrow\; \mathbf{D}_i = Q_i^{-1} \widetilde{\mathbf{s}}_i$$

4. Project $\mathbf{D}_i$ to the other image j: $\widetilde{\mathbf{d}}_j = P_j \begin{bmatrix} \mathbf{D}_i \\ 0 \end{bmatrix} = Q_j Q_i^{-1} \widetilde{\mathbf{s}}_i$

Calibration Parameters: An Example

Simplified projection matrices in this example:

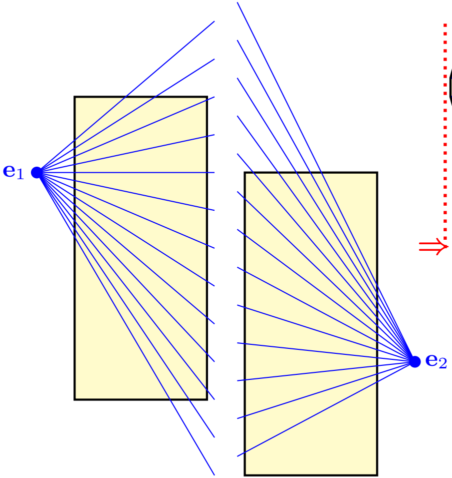
$$P_\mathrm{l} = \begin{bmatrix} f & 0 & c_{\mathrm{l:x}} \\ 0 & f & c_{\mathrm{l:y}} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} f_\mathrm{l} & 0 & c_{\mathrm{l:x}} & 0 \\ 0 & f_\mathrm{l} & c_{\mathrm{l:y}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$P_\mathrm{r} = \begin{bmatrix} f & 0 & c_{\mathrm{r:x}} \\ 0 & f & c_{\mathrm{r:y}} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -34 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} f & 0 & c_{\mathrm{r:x}} & -34f + 2c_{\mathrm{r:x}} \\ 0 & f & c_{\mathrm{r:y}} & 2c_{\mathrm{r:y}} \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

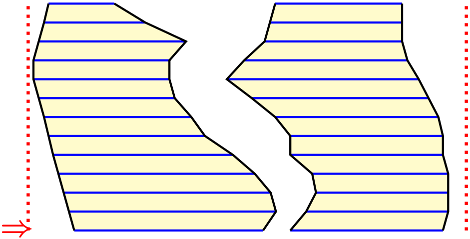
$f = 2000$; $\mathbf{c}_\mathrm{l} = \mathbf{c}_\mathrm{r} = [2000, 1500]^\mathsf{T}$:

- $\mathbf{O}_\mathrm{l} = -Q_\mathrm{l}^{-1}\mathbf{q}_\mathrm{l} = [0, 0, 0]^\mathsf{T}$
- $\mathbf{O}_\mathrm{r} = -Q_\mathrm{r}^{-1}\mathbf{q}_\mathrm{r} = [64000, -3000, -2]^\mathsf{T}$
- $\tilde{\mathbf{e}}_\mathrm{l} = -[-64000, 3000, 2]^\mathsf{T} + [0, 0, 0]^\mathsf{T} = [64000, -3000, -2]^\mathsf{T}$
  $\rightarrow \mathbf{e}_\mathrm{l} = [-32000, 1500]$
- $\tilde{\mathbf{e}}_\mathrm{r} = -[0, 0, 0]^\mathsf{T} + [-64000, 3000, 2]^\mathsf{T} = [-64000, 3000, 2]^\mathsf{T}$
  $\rightarrow \mathbf{e}_\mathrm{r} = [-32000, 1500]$

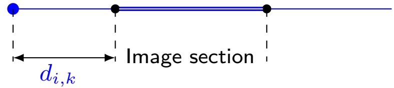
Epipolar Relations for Rectifying a Stereo Pair



Original stereo pair $(\mathbf{g}_1, \mathbf{g}_2)$

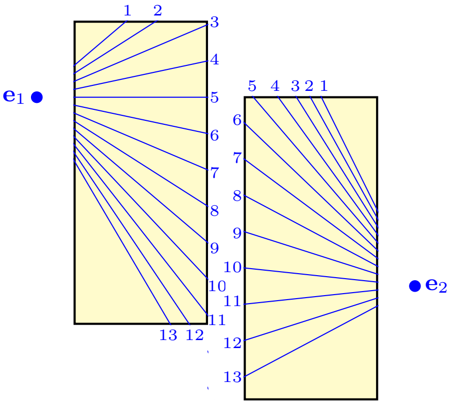


Rectified epipolar pair $(\widehat{\mathbf{g}}_1, \widehat{\mathbf{g}}_2)$

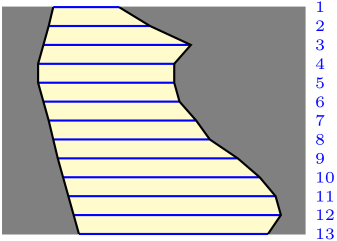


$d_{i:k}$ – the distance to the closest intersection point $\mathbf{s}_{c:i:k}$ of an epipolar ray k with the frame of the image $\mathbf{g}_i$

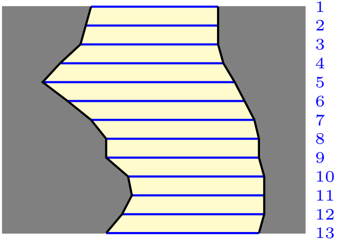
Epipolar Relations for Rectifying a Stereo Pair



Original stereo pair ($\mathbf{g}_1$, $\mathbf{g}_2$)



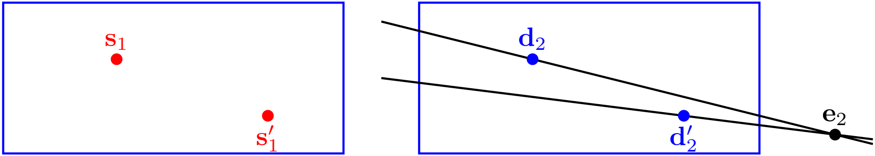
Rectified epipolar pair ($\widehat{\mathbf{g}}_1$, $\widehat{\mathbf{g}}_2$)



Epipolar Relations: Calibrated Cameras ($\widetilde{\ldots}$ – homogeneous coordinates)

As shown in Slide 10, a point $\mathbf{s}_i$ on one image, i, of a stereo pair and the projection matrices, $P_i = [Q_i \;\; \mathbf{q}_i]$; $i = 1, 2$, determines on the other image j; $j \neq i$, an epipolar line containing the point corresponding to $\mathbf{s}_i$

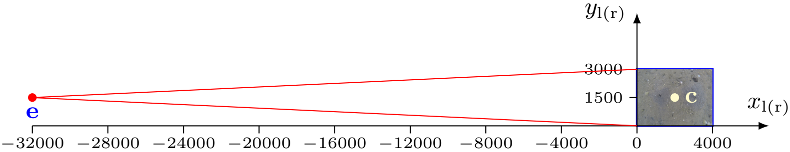
- This epipolar line is the projection of the optical ray producing $\mathbf{s}_i$
- This epipolar line is drawn through the epipole $\mathbf{e}_j$ and the projection $\mathbf{d}_j$ of the point $\mathbf{D}_i$ at infinity of the inverse optical ray $\overline{\mathbf{s}_i\mathbf{O}_i}$
  - The epipole $\widetilde{\mathbf{e}}_j = P_j \begin{bmatrix} \mathbf{O}_i \\ 1 \end{bmatrix} = [Q_j \;\; \mathbf{q}_j] \begin{bmatrix} -Q_i^{-1}\mathbf{q}_i \\ 1 \end{bmatrix}$
  - The projection $\widetilde{\mathbf{d}}_j = Q_j Q_i^{-1} \widetilde{\mathbf{s}}_i$
- The 2D epipolar line: $\mathbf{e}_j + \lambda (\mathbf{d}_j - \mathbf{e}_j)$; $\lambda \in (-\infty, \infty)$



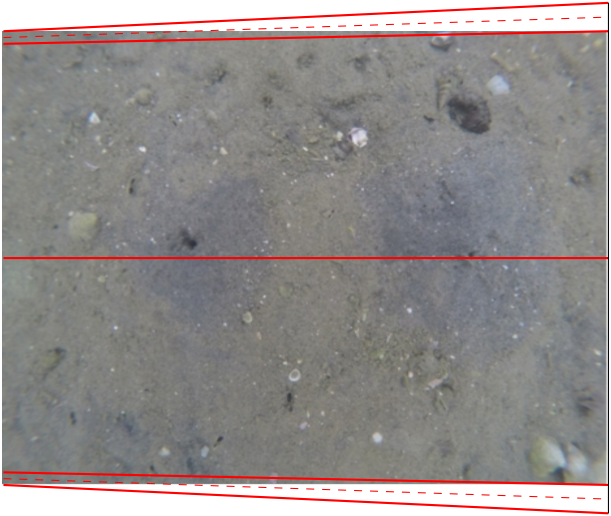
Calibration Parameters: An Example

$Q_\mathrm{l} = Q_\mathrm{r} = I_{3\times 3}$; $\mathbf{q}_\mathrm{l} = [0, 0, 0]^\mathsf{T}$, and $\mathbf{q}_\mathrm{r} = [-64000, 3000, 2]^\mathsf{T}$, giving:

- $\mathbf{e}_\mathrm{l} = \mathbf{e}_\mathrm{r} = [-32000, 1500]^\mathsf{T}$
- The projection ray to a left image point $\mathbf{s}_\mathrm{l} = [s_{\mathrm{l:x}}, s_{\mathrm{l:y}}]$ maps into the line through the right image point $\mathbf{d}_\mathrm{r} = \mathbf{s}_\mathrm{l}$.
- The projection ray to a right image point $\mathbf{s}_\mathrm{r} = [s_{\mathrm{r:x}}, s_{\mathrm{r:y}}]$ maps into the line through the left image point $\mathbf{d}_\mathrm{l} = \mathbf{s}_\mathrm{r}$.



Calibration Parameters: An Example



Calibration Parameters: An Example