DUE: Wed, Oct 6, 4:30pm, Andre's office.

1. (a) Show that if $A \in$ PSPACE then there is $B \in$ SPACE(n) such that $A \leq_P B$. (2 marks)

   (b) Show that SPACE(n^2) $\neq P$. (3 marks)

   You can use the fact proved in Chapter 9 that SPACE(n) is a proper subclass of SPACE(n^2).

2. (5 marks) Write a 1-tape Turing program for the alphabet $\{0, 1\}$ that decrements a binary counter. Numbers are written in binary with the least significant digit on the *left* (contrary to everday usage). For instance, 11001 represents the number 1+2+16=19. When 11001 is fed to the machine, it halts with 01001 on the tape and the head on the leftmost position. When 00001 if fed to the machine, it ends up with 1111, and input 1 yields the empty tape.

   Draw a TM diagram similar to the ones on pg. 144 (which you can give to me in hand-drawn form) and write your program in the language for the TM simulator at www.ironphoenix.org/tril/tm, and try it out. Attach a printout of the program to the assignment.

3. (a) State the Hennie-Stearns theorem. How is it used in the proof of the time hierarchy theorem? Why is no such theorem needed for the proof of the space hierarchy theorem? (3 marks)

   (b) Use the time hierarchy theorem to show that TIME(n^2) is a proper subclass of TIME($n^{5/2}$).

   (2 marks)

4. (5 marks) Explain why the following inclusions are proper (cite literature if appropriate):

   (a) Regular languages $\subset$ $\text{TIME}_{1-tape}(n \lg n)$

   (b) TIME(n) $\subset$ SPACE(n)

   (c) NTIME(n) $\subset$ NTIME($n \lg n$)