CompSci 373 Tutorial

Test Preparation

Dot Product 1

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$a \cdot b = a^T b = \begin{pmatrix} a_1 & a_2 & a_3 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$= a_1 * b_1 + a_2 * b_2 + a_3 * b_3$$

- The result is a single scalar, not a vector.
- Properties:
  - Give us information about the angle formed by the two vectors
  - Dot product of 2 perpendicular vectors is 0

Dot Product 2

$$a = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$$

- Compute their dot product
- What the angle between them?

Dot Product 2

$$a = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$$

- Compute their dot product

$$a \cdot b = 1 * 0 + 2 * 4 + 3 * 1 = 11$$

- What the angle between them?

$$a \cdot b = |a||b| \cos\phi$$

$$\cos\phi = \frac{|a||b|}{a \cdot b} = \frac{\sqrt{14} + \sqrt{9}}{11}$$

$$\phi = \arccos(\frac{\sqrt{14} + \sqrt{9}}{11})$$

Cross Product 1

$$a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$a \times b = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - b_2 a_3 \\ b_1 a_3 - a_1 b_3 \\ a_1 b_2 - b_1 a_2 \end{pmatrix}$$

- The result is a vector which has the same dimension and **perpendicular** the two input vectors.
- Properties:
  - The length of cross product vector is the area of the rectangle formed by the two vectors

Cross Product 2

$$a = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \qquad b = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

- What is the cross product?
- What is the area of the rectangle formed by these two vectors?

Cross Product 2

$$a = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \qquad b = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$$

- What is the cross product?

$$a \times b = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -4 \end{pmatrix}$$

- What is the area of the rectangle formed by these two vectors?

$$|a \times b| = \left|\begin{pmatrix} 5 \\ 2 \\ -4 \end{pmatrix}\right| = \sqrt{5^2 + 2^2 + (-4)^2} = \sqrt{45}$$

Plane Equation

- What is equation of the plane, which contains 3 points

$$A = \begin{pmatrix}1 & -6 & 0\end{pmatrix} \qquad B = \begin{pmatrix}-4 & 2 & -5\end{pmatrix} \qquad C = \begin{pmatrix}-2 & 4 & 1\end{pmatrix}$$

- Find the normal
  - Find the 2 vectors on the plane
  - Get cross product of them to get the normal

$$vec_1 = B - A = \begin{pmatrix}-5 & 8 & -5\end{pmatrix}$$

$$vec_2 = C - A = \begin{pmatrix}-3 & 10 & 1\end{pmatrix}$$

$$n = a \times b = \begin{pmatrix}58 \\ 20 \\ -26\end{pmatrix}$$

Plane Equation

- Plane equation has the form

$$n_x x + n_y y + n_z z + d = 0$$

- We have **n**, now find **d**

$$d = -(n \cdot A) = -(58*1 + 20*(-6) + (-26)*0) = -62$$

- **Plane equation**

$$58x + 20y - 26z - 62 = 0$$

Distance of a point to a plane

- **Plane equation**

$$2x + y - 2z = 3$$

- How far is it from the origin?
  - To vector form, and normalise it

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \Leftrightarrow \frac{1}{3} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{3}{3} = 1$$

Distance from the origin

Distance of a point to a plane

- **Plane equation**

$$2x + y - 2z = 3$$

- How far is the point (3, 6, 3) from the plane?

$$\frac{1}{3}\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} = 1$$

$$\Rightarrow \frac{6+6-6}{3} - \frac{3}{3} = 1$$

World-to-Viewport mapping.

A world window with left bottom corner at (0, 0) and the top right corner (100, 200). What is the homogeneous matrix mapping this world window to a window on the screen which is aligned with the top-left corner of the screen and has a width of 100 pixels and height of 200 pixels?

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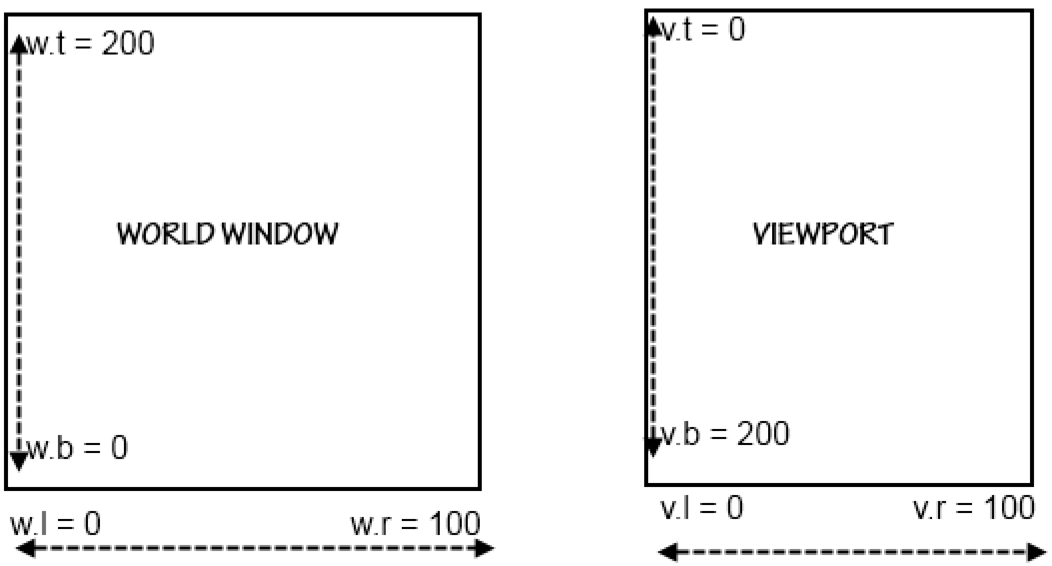


World-to-Viewport mapping.

Mapping a pixel **x** in the world coordinate to pixel **u** in the viewport coordinate.

$$\begin{pmatrix} u_x \\ u_y \\ 1 \end{pmatrix} = \begin{bmatrix} A & 0 & C \\ 0 & B & D \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_x \\ x_y \\ 1 \end{pmatrix}$$

$$A = \frac{v.r - v.l}{w.r - w.l} \qquad B = \frac{v.t - v.b}{w.t - w.b}$$

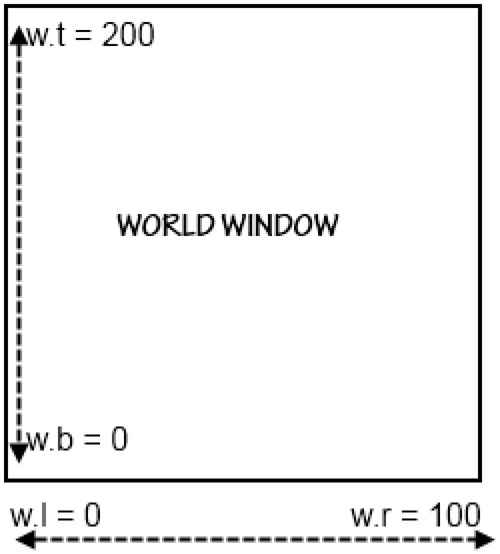
$$C = v.l - A * w.l \qquad D = v.b - B * w.b$$

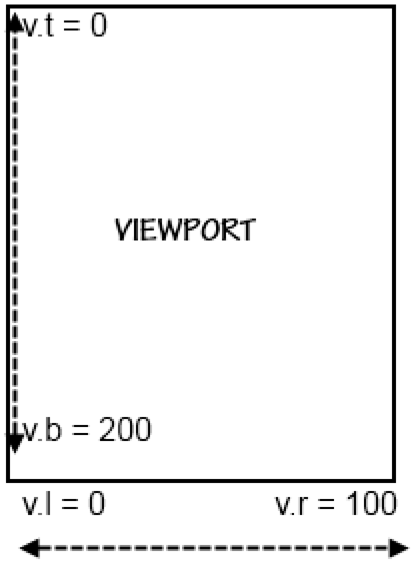
World-to-Viewport mapping.

Mapping a pixel **x** in the world coordinate to pixel **u** in the viewport coordinate.

$$A = \frac{v.r - v.l}{w.r - w.l}$$

$$A = \frac{100 - 0}{100 - 0} = 1$$



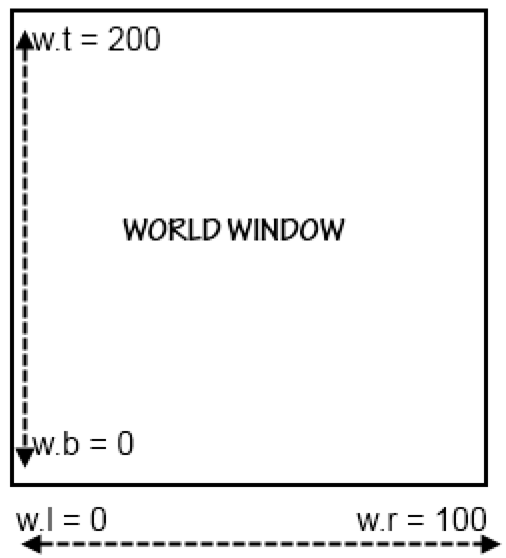


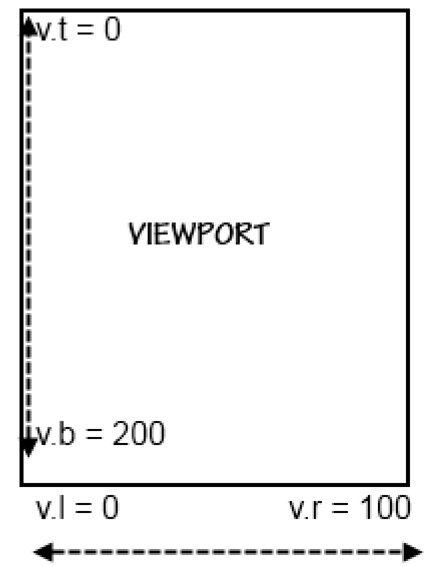
World-to-Viewport mapping.

Mapping a pixel **x** in the world coordinate to pixel **u** in the viewport coordinate.

$$B = \frac{v.t - v.b}{w.t - w.b}$$

$$B = \frac{0 - 200}{200 - 0} = -1$$



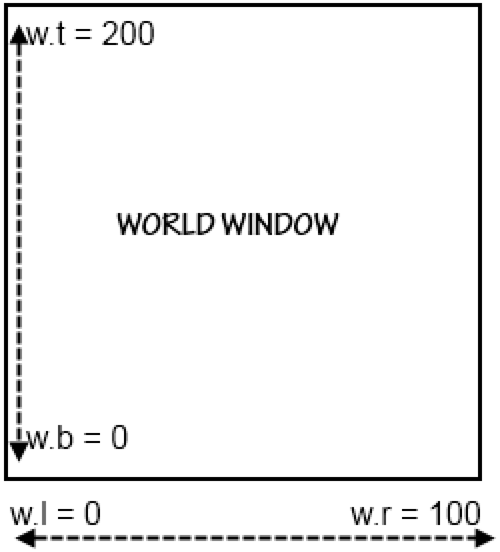


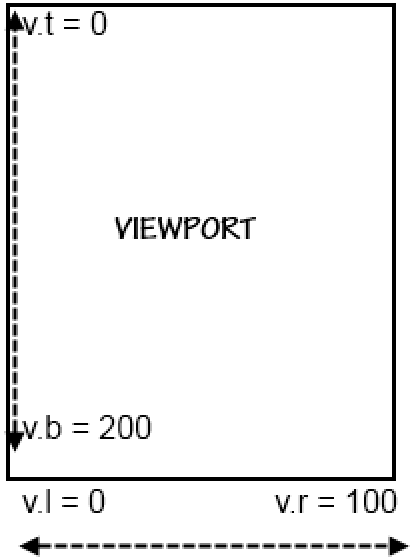
World-to-Viewport mapping.

Mapping a pixel **x** in the world coordinate to pixel **u** in the viewport coordinate.

$$C = v.l - A * w.l$$
$$= 0 - 1 * 0 = 0$$

$$D = v.b - B * w.b$$
$$= 200 - (-1) * 0 = 200$$





World-to-Viewport mapping.

The homogeneous matrix mapping the world window to the view port window is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 200 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformations

Order to apply:

- From RIGHT-TO-LEFT
- Or from BOTTOM-TO-TOP

For example:

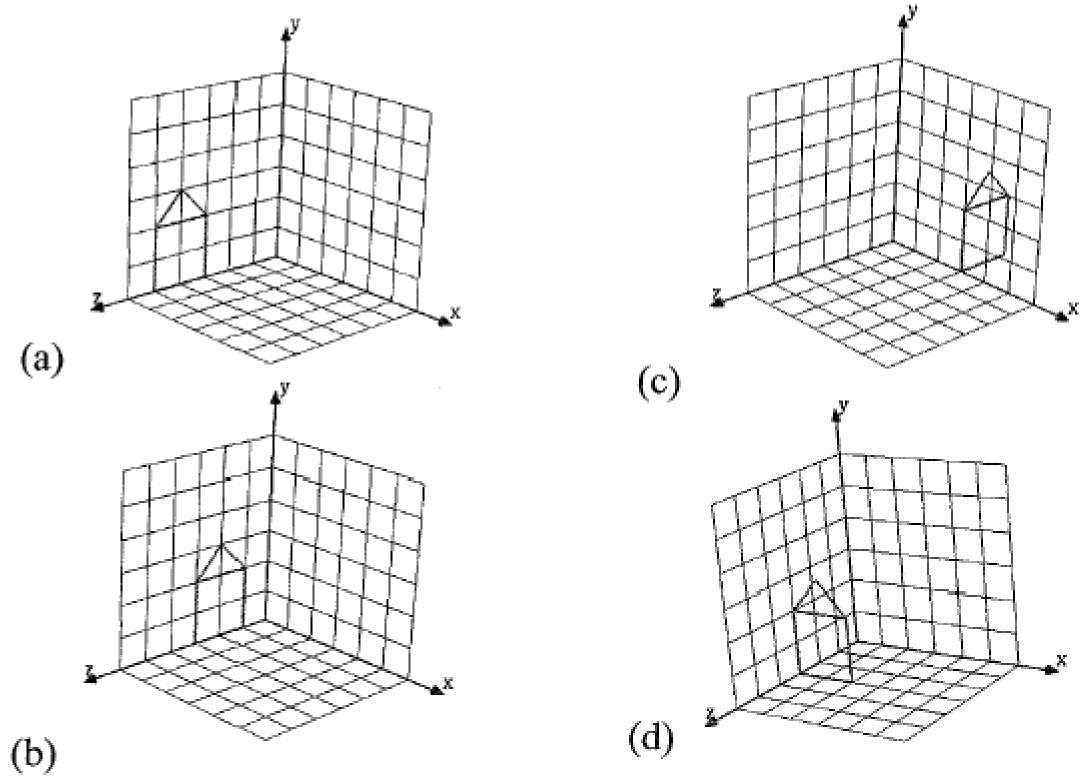
```
glScalef(1.0, 1.0, 2.0);
glRotatef(45, 1, 0, 0);
glTranslatef(0, 0, 5)
```

$$\mathbf{S}(1, 1, 2)\ \mathbf{R}(45, 1, 0, 0)\ \mathbf{T}(0, 0, 5)$$

Transformations

Ex: Suppose that a method *drawHouse()* draws a house at the origin, What is the scene generated by the following code segment?

```
glPushMatrix();
glTranslatef(0, 0, 3);
glRotatef(90, 0, 1, 0);
glPopMatrix();
```



(a) (b) (c) (d)

(e) None of the above

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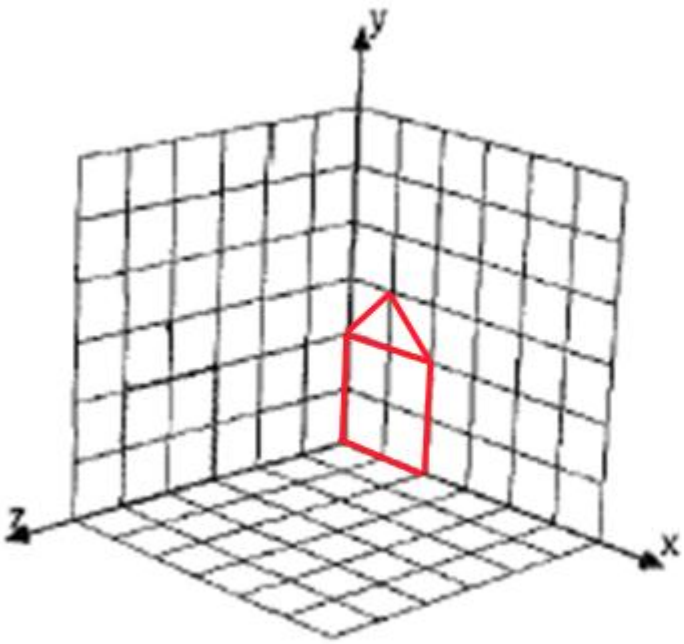

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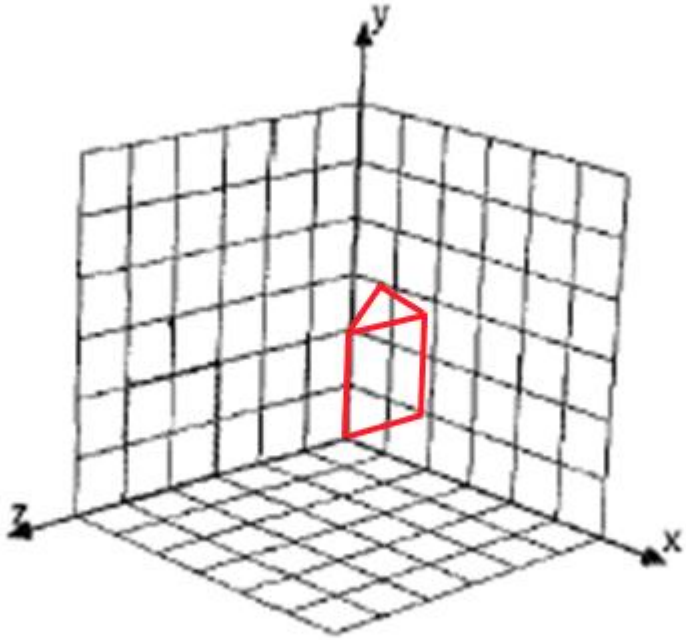

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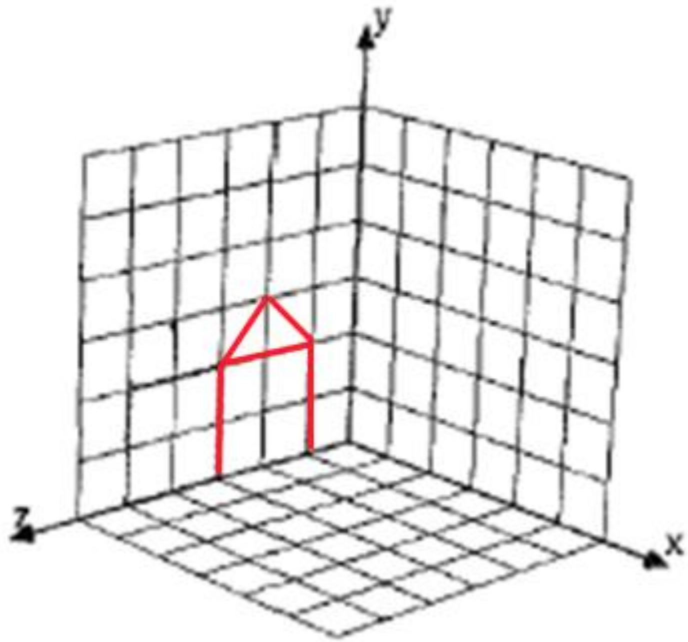
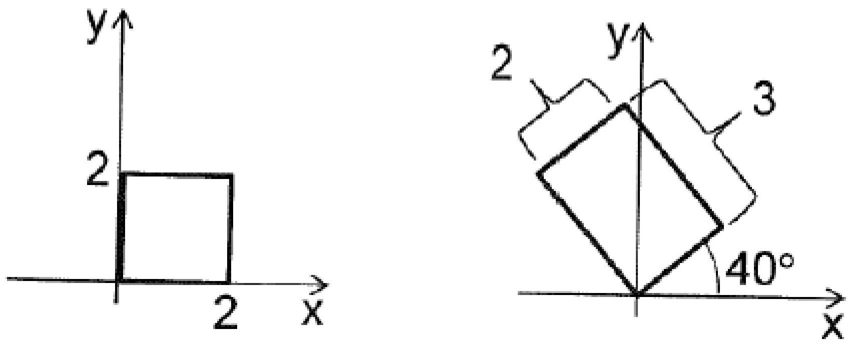


Transformations

Ex: Which homogeneous 2D matrix transforms the figure on the left to the one on the right?



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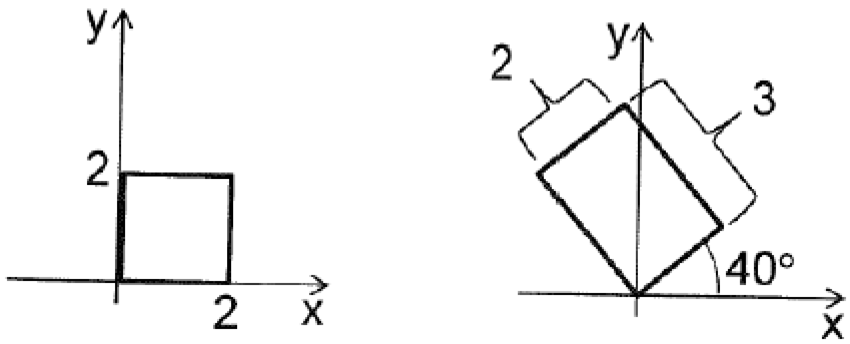

Two transformation involved: Scaling and Rotation

- Scale(1, 1.5)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate(40, 0, 1)

$$\begin{bmatrix} \cos(40) & -\sin(40) & 0 \\ \sin(40) & \cos(40) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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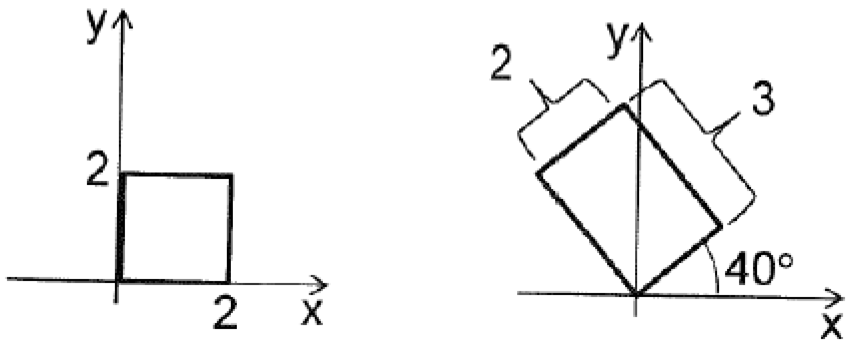

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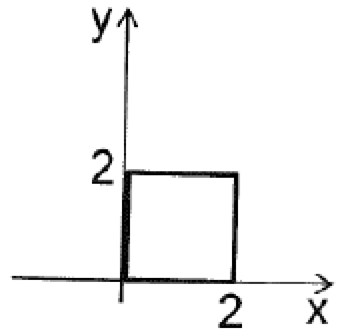

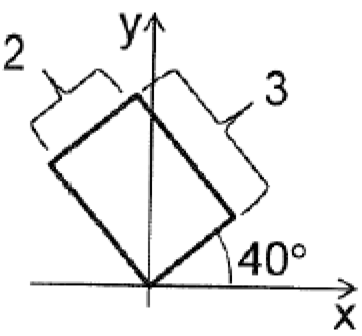

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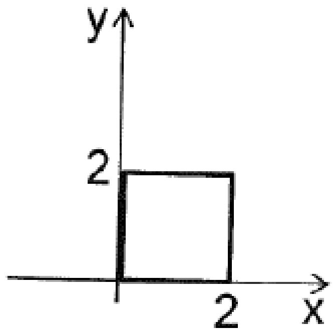
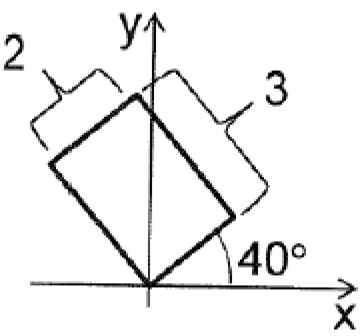
- Scale(1, 1.5) Rotate(40, 0, 1)

$$T = \begin{bmatrix} \cos(40) & -\sin(40) & 0 \\ \sin(40) & \cos(40) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(40) & -1.5\sin(40) & 0 \\ \sin(40) & 1.5\cos(40) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformations





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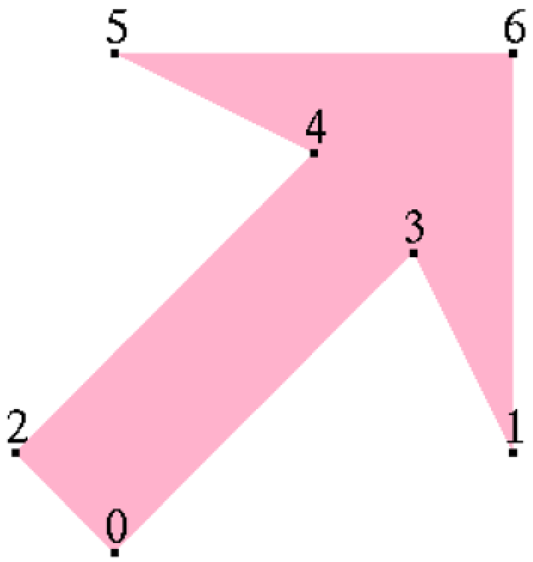
$$= \begin{bmatrix} \cos(40) & -1.5\sin(40) & 0 \\ \sin(40) & 1.5\cos(40) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Drawing

Given are the vertices

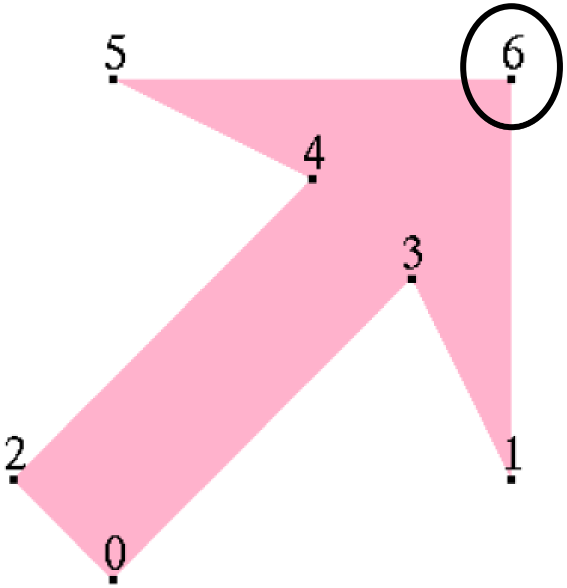
```
const int numVertices=7;
const float vertices[numVertices][2] =
   {{100,50},{300,100},{50,100},{250,200},{
    200,250},{100,300},{300,300}};
```

Which calling sequence of these vertices (using `glVertex2fv`) results in the shape on the right if we use the OpenGL commands `glBegin(GL_TRIANGLE_FAN)` and `glEnd()`?



Drawing

GL_TRIANGLE_FAN uses the FIRST specified vertex and the last vertex of the previous triangle for the next triangle



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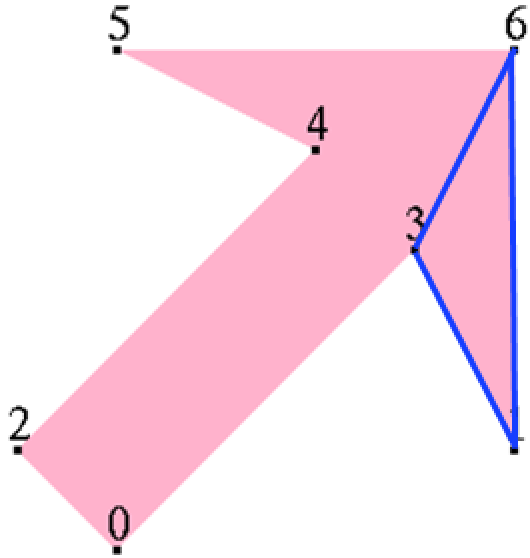

6, 1, 3

Drawing

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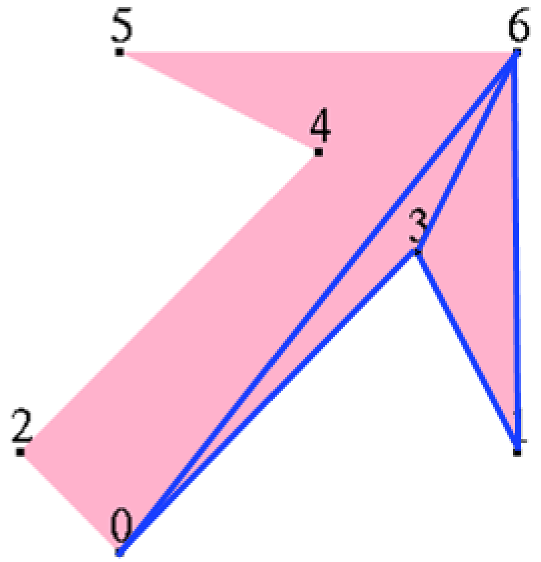
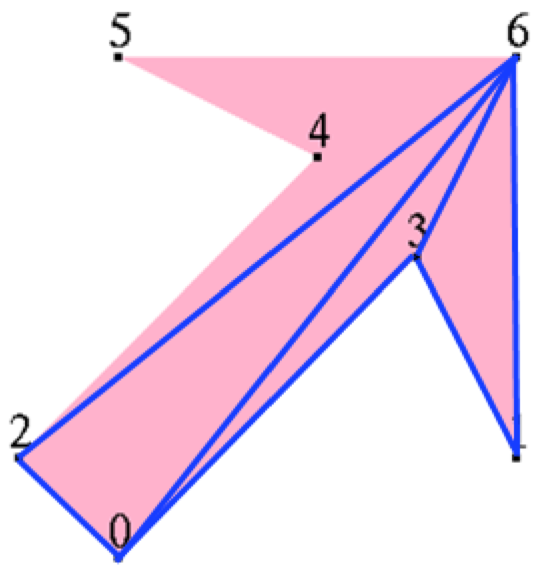


6, 1, 3, 0

Drawing

GL_TRIANGLE_FAN uses the FIRST specified vertex and the last vertex of the previous triangle for the next triangle

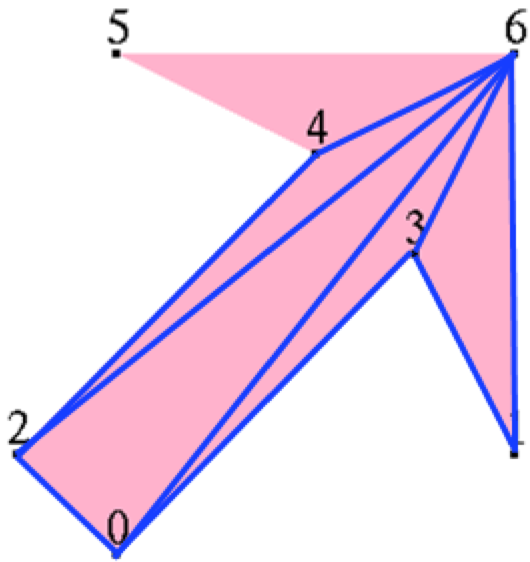


6, 1, 3, 0, 2

↑ ↑

Drawing

GL_TRIANGLE_FAN uses the FIRST specified vertex and the last vertex of the previous triangle for the next triangle



6, 1, 3, 0, 2, 4

Drawing

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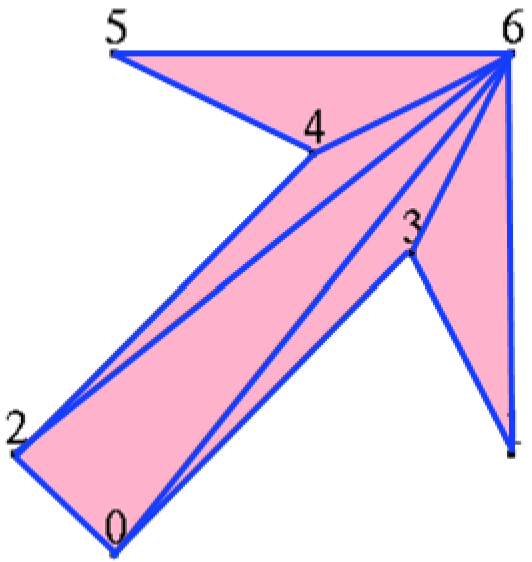
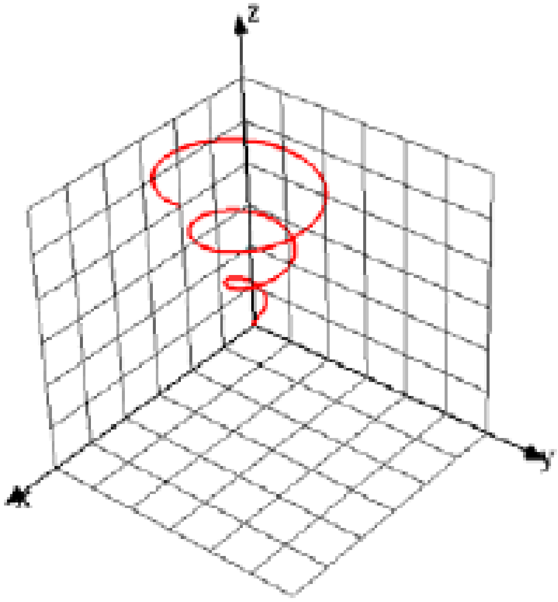


6, 1, 3, 0, 2, 4, 5

Parametric Curves

The picture on the right shows a parametric curve which is a spiral with 3 revolutions, a height of 4 units, and a radius rising from 0 to 2 units. The spiral starts at the origin and its center axis is the z-axis.

What is the parametric equation of this curve?



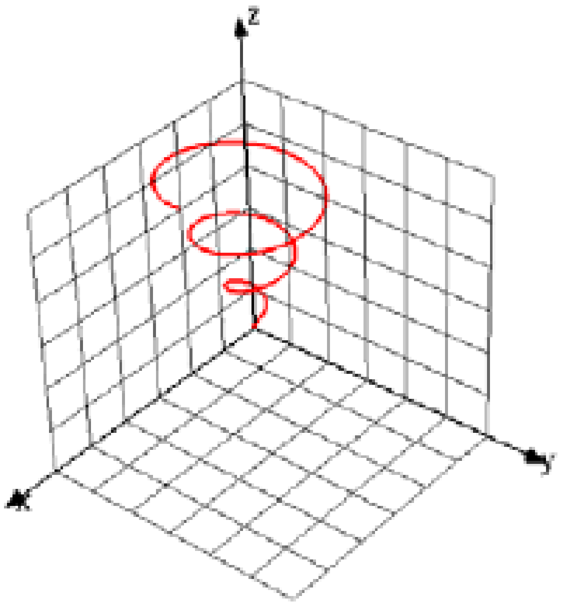
Parametric Curves

Parametric curve rotating around the z axis has the form

$$p(t) = \begin{bmatrix} x(t)\cos(t) \\ x(t)\sin(t) \\ z(t) \end{bmatrix} \qquad t \in [0,1]$$

If the angle is 2 * PI, it rotates 1 full circle, we want it to rotate 3 full circles -> the angle is 3 * (2 * PI) = 6 * PI

$$p(t) = \begin{bmatrix} x(t)\cos(6\pi t) \\ x(t)\sin(6\pi t) \\ z(t) \end{bmatrix}$$



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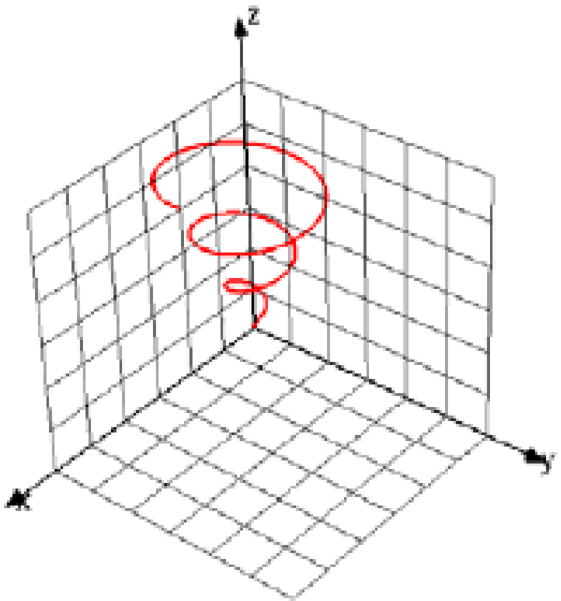
The height is 4 units, => z(t) = 4t

$$p(t) = \begin{bmatrix} x(t)\cos(6\pi t) \\ x(t)\sin(6\pi t) \\ 4t \end{bmatrix}$$



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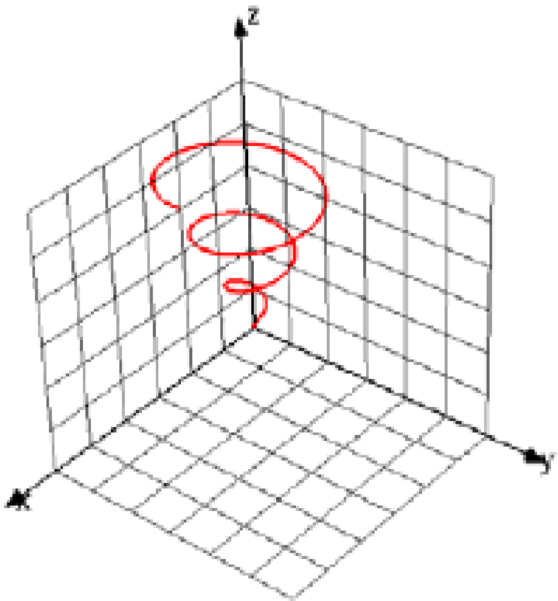
The radius is going from 0 to 2, ➔

$$p(t) = \begin{bmatrix} 2t\cos(6\pi t) \\ 2t\sin(6\pi t) \\ 4t \end{bmatrix}$$



GOOD LUCK