CompSci 372 S2 C 2008 Computer Graphics

Mid Term Test – Wednesday, 24th September 2008, 7 pm – 8 pm

VERSION CODE 0000001

Instructions:

- 1. Attempt **ALL** questions.
- 2. Fill in the appropriate boxes in the answer sheet using a **pencil**. There is only one correct answer for each question.
- 3. If you want to change your answer erase the previously filled in box completely using an eraser.
- 4. The test is for one (1) hour.
- 5. This is a **closed book** test.
- 6. Calculators and electronic devices are **NOT** permitted.
- 7. Questions total 50 Marks. Each question is worth 2 marks.
- 8. This test is worth 10% of your final marks for CompSci372 S1 C

Question 1:

The dot product of
$$\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$
 and $\mathbf{v} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$ equals
(a) -7
(b) -6
(c) 1
(d) None of the above

Solution:
$$\mathbf{u} \bullet \mathbf{v} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix} = 2 * 3 + 0 * (-3) + 3 * (-4) = -6$$

Question 2:

The cross product (vector product) of $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix}$ equals

(a)
$$\begin{pmatrix} -6\\ -12\\ 4 \end{pmatrix}$$

(b)
$$\begin{pmatrix} 6\\ 0\\ -12 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 9\\ 17\\ -6 \end{pmatrix}$$

(d) None of the above

Solution:
$$\mathbf{u} \bullet \mathbf{v} = \begin{pmatrix} 2\\0\\3 \end{pmatrix} \bullet \begin{pmatrix} 3\\-3\\-4 \end{pmatrix} = \begin{pmatrix} 0*(-4)-3*(-3)\\3*3-2*(-4)\\2*(-3)-0*3 \end{pmatrix} = \begin{pmatrix} 9\\17\\-6 \end{pmatrix}$$

Question 3:

Which of the following statements is NOT correct?

- (a) Scientific Visualization helps with analysing, understanding and communicating large scientific data by representing them as images.
- (b) Medical Imaging has revolutionized Computer Graphics through new algorithms with higher precision and correctness and the emergence of Direct Volume Rendering.
- (c) Animatronics is always cheaper than using computer generated models for movie production.
- (d) "Second Life" is a popular "Virtual World", which is increasingly used for ecommerce.

Question 4:

The area of a triangle in 3D can be computed from two of its edge vectors as

(a) half the length of their cross product

- (b) half the absolute value of their dot product
- (c) half the product of their lengths
- (d) None of the above

Solution: See slide 20 of chapter 6 of the lecture handout

Question 5:

A 3D object is deformed by the transformation matrix $\mathbf{M} = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} \cos \varphi & 0 & \sin \varphi \\ 0 & 1 & 0 \\ -\sin \varphi & 0 & \cos \varphi \end{pmatrix}$

Then its surface normals must be transformed using the matrix

(a)
$$\mathbf{M} = \begin{pmatrix} \cos -\varphi & 0 & \sin -\varphi \\ 0 & 1 & 0 \\ -\sin -\varphi & 0 & \cos -\varphi \end{pmatrix} \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$$

(b) $\mathbf{M} = \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \begin{pmatrix} \cos -\varphi & 0 & -\sin -\varphi \\ 0 & 1 & 0 \\ \sin -\varphi & 0 & \cos -\varphi \end{pmatrix} \begin{pmatrix} \cos -\varphi & 0 & -\sin -\varphi \\ 0 & 1 & 0 \\ \sin -\varphi & 0 & \cos -\varphi \end{pmatrix} \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$
(c) $\mathbf{M} = \begin{pmatrix} \cos -\varphi & 0 & -\sin -\varphi \\ 0 & 1 & 0 \\ \sin -\varphi & 0 & \cos -\varphi \end{pmatrix} \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix}$
(d) None of the above

Solution: According to slide 32 of chapter 5 of the lecture handouts the normal is transformed by $(\mathbf{M}^{-1})^T$.

$$\begin{aligned} \left(\mathbf{M}^{-1}\right) &= \left(\begin{pmatrix} -3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix} \right)^{-1} = \begin{pmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{pmatrix}^{-1} \begin{pmatrix} -3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}^{-1} \\ &= \left(\begin{pmatrix} \cos-\varphi & 0 & \sin-\varphi \\ 0 & 1 & 0 \\ -\sin-\varphi & 0 & \cos-\varphi \end{pmatrix} \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} \cos-\varphi & 0 & \sin-\varphi \\ 0 & 1 & 0 \\ -\sin-\varphi & 0 & \cos-\varphi \end{pmatrix} \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 \end{pmatrix} \right)^{T} = \left(\begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 0 \end{pmatrix} \right)^{T} =$$

Question 6:

Given are three vectors \mathbf{a} , \mathbf{b} and \mathbf{c} with components x, y, and z. Three of the following four expressions are equivalent. Which of the four expressions is different?

(a)
$$|\mathbf{a} \times \mathbf{b}||\mathbf{c}|$$

(b) $\begin{vmatrix} a_x & b_x & c_x \\ a_y & b_y & c_y \\ a_z & b_z & c_z \end{vmatrix}$

- (c) $(\mathbf{a} \times \mathbf{b}) \bullet \mathbf{c}$
- (d) $|\mathbf{a}||\mathbf{b}||\mathbf{c}|\sin\varphi\sin\theta$, where φ is the angle between **c** and the plane spanned by **a** and **b** and θ is the angle between **a** and **b**.
- Solution: The formulas in (b), (c) and (d) all compute the volume of the parallelepiped spanned by the vectors **a**, **b** and **c**. The equivalence of (b) and (c) can be seen by computing the determinant using co-factor expansions (e.g. http://algebra.math.ust.hk/determinant /02_computation/lecture2.shtml). The equivalence of (c) and (d) can be seen by using the definitions of the vector and scalar product.

An easier way to obtain the correct solution is by recognizing that (a) is the only expression which doesn't change if the angle between c and the plane spanned by a and b varies.

Question 7:

The crowd animation software used for the battle scenes in the "Lord of the Rings" movies is called:

- (a) Peoplesoft
- (b) Maya
- (c) Character Studio
- (d) None of the above

Solution: The name of the company is MASSIVE (see assignment 1).

Question 8:

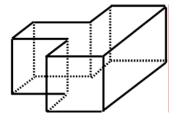
Given is a closed manifold polygon mesh with F=7 faces and V=7 vertices. Then the number of edges E is

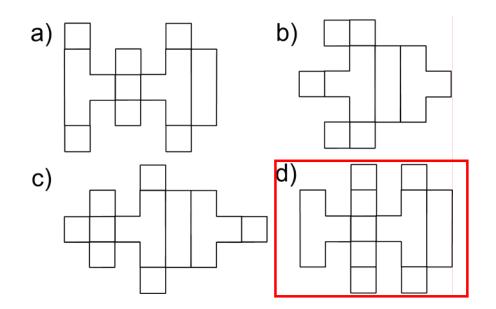
(a) 8
(b) 10
(c) 12
(d) None of the above

Solution: With slide 8 of chapter 2 of the lecture notes we get E=F+V-2=7+7-2=12. Note that even if you don't remember the exact formula you can derive it easily by using simple polyhedra as example such as the tetrahedron and the cube.

Question 9:

Which of the four 2D shapes below results, when folded appropriately, into the 3D object on the right?





Question 10:

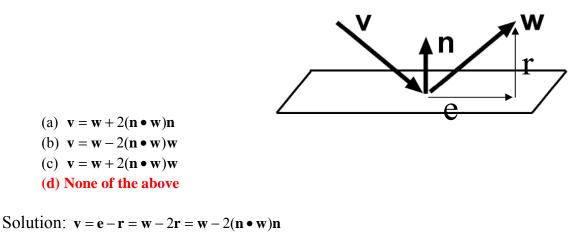
Given are two parallel vectors \mathbf{a} and \mathbf{b} with length 3. Then $\mathbf{a} \cdot \mathbf{b}$ equals

(a) 0
(b) 3
(c) 6
(d) None of the above

Solution: If **a** and **b** are parallel the angle φ between them is zero, i.e. $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \varphi = |\mathbf{a}| |\mathbf{b}| = 9$.

Question 11:

A 3D vector \mathbf{v} is reflected on a surface with a unit normal \mathbf{n} . Given the reflected vector \mathbf{w} which formula computes the original vector **v**?



Question 12:

What transformation is performed by the homogeneous matrix $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$?

(a) It mirrors a 3D point on the plane z=2.

- (b) It mirrors a 3D point on the plane z=4.
- (c) It mirrors a 3D point on the xy-plane

(d) None of the above

Solution: Let
$$\mathbf{p} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$
 then $\mathbf{Mp} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 4-z \\ 1 \end{pmatrix}$. Since the transformation keeps

the x- and y-coordinates unchanged it must be a mirroring on a plane parallel to the xy-plane. By testing different z-values it can be easily seen that the mirroring is on the plane z=2.

Question 13:

What is the distance of the point (5,2,2) to the plane given by the point (4,0,0) and the normal (1,2,2)?

(a) 3

(b) 11

(c) 12

(d) None of the above

Solution: The plane equation is
$$\mathbf{n} \bullet \mathbf{p} = d$$
 where $d = \mathbf{n} \bullet \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = 4$. $dist = \frac{\mathbf{n} \bullet \mathbf{p} - d}{|\mathbf{n}|} = \frac{(5+4+4)-4}{3} = 3$.

Question 14:

Given is the parametric surface $\mathbf{p}(s,t) = \begin{pmatrix} 3e^s \\ s \\ t^2 + \sin t \end{pmatrix}$. The normal of this surface at the point

 $\mathbf{p}(s,t)$ is given by

(a)
$$\begin{pmatrix} 3e^{s} \\ 1 \\ 2t + \cos t \end{pmatrix}$$

(b)
$$\begin{pmatrix} 3e^{s} \\ 1 \\ t^{2} + \sin t \end{pmatrix} \times \begin{pmatrix} 3e^{s} \\ s \\ 2t + \cos t \end{pmatrix}$$

Solution: $\mathbf{n} = \frac{\partial \mathbf{p}}{\partial s} \times \frac{\partial \mathbf{p}}{\partial t} = \begin{pmatrix} 3e^s \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 2t + \cos t \end{pmatrix}$

(c)
$$\begin{pmatrix} 3e^s \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 2t + \cos t \end{pmatrix}$$

(d) None of the above

Question 15:

What is the intersection point of the <u>line</u> $\mathbf{p}(t) = \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} + t \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$ and the plane $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \bullet \mathbf{p} = 5$?

(a)
$$\begin{pmatrix} 3\\3\\1 \end{pmatrix}$$

(b) $\begin{pmatrix} 3\\5\\0 \end{pmatrix}$
(c) $\begin{pmatrix} 3\\-1\\3 \end{pmatrix}$
(c) $\begin{pmatrix} 3\\-1\\3 \end{pmatrix}$
(c) $\begin{pmatrix} 3\\-1\\3 \end{pmatrix}$
(c) $\begin{pmatrix} 3\\-1\\3 \end{pmatrix}$

Solution: Let $\mathbf{p}(\hat{t})$ be the intersection point. The intersection point lies on the plane, i.e

$$\begin{pmatrix} 0\\1\\2 \end{pmatrix} \bullet \mathbf{p}(\hat{t}) = 5 \Leftrightarrow \begin{pmatrix} 0\\1\\2 \end{pmatrix} \bullet \begin{pmatrix} 3\\2\\6 \end{pmatrix} + \hat{t} \begin{pmatrix} 0\\-1\\2 \end{pmatrix} = 5 \Leftrightarrow 14 + 3\hat{t} = 5 \Leftrightarrow \hat{t} = -3 \quad \Rightarrow \mathbf{p}(\hat{t}) = \begin{pmatrix} 3\\5\\0 \end{pmatrix}.$$

Question 16:

Consider the code segment below:

```
CVec3df v1,v2,v3;
CVec3df n=cross(v2-v1,v3-v1);
n.normaliseDestructive();
x=dot(v1,n);
```

Which of the following statements is correct:

- (a) x is the distance of v1 to the line going through v2 and v3.
- (b) The points v1, v2 and v3 lie on the plane n. p = x.
- (c) x is the area of the triangle defined by the points v1, v2 and v3 .
- (d) None of the above

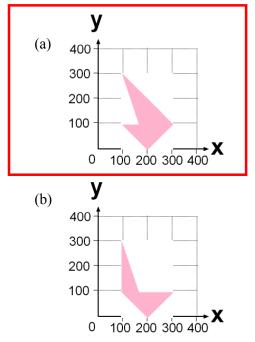
Solution: Let $\mathbf{n} \cdot \mathbf{p} = d$ be the plane defined by the three points $\mathbf{v1}$, $\mathbf{v2}$ and $\mathbf{v3}$. Then $\mathbf{v2} \cdot \mathbf{v1}$ and $\mathbf{v3} \cdot \mathbf{v1}$ lie on the plane and the plane normal is given by the cross product of these vectors. The value *d* is obtained by inserting any point on the plane, e.g. $\mathbf{v1}$, into the plane equation.

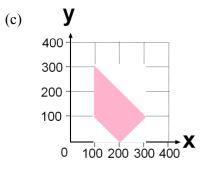
Question 17:

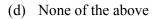
Given is an OpenGL program with the display method

```
void display(void)
{
    // clear all pixels in frame buffer
    glClear(GL_COLOR_BUFFER_BIT);
    glColor3f(0.0, 0.0, 0.0);
    glBegin(GL_TRIANGLE_STRIP);
    glVertex2f(100, 100);
    glVertex2f(200, 0);
    glVertex2f(300, 100);
    glVertex2f(100, 300);
    glEnd();
    glFlush();
}
```

What shape does this display method draw?





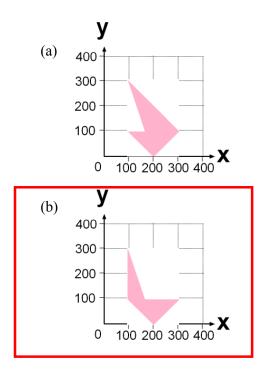


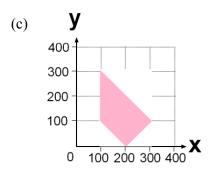
Question 18:

Given is an OpenGL program with the display method

```
void display(void)
{
    // clear all pixels in frame buffer
    glClear(GL_COLOR_BUFFER_BIT);
    glColor3f(0.0, 0.0, 0.0);
    glBegin(GL_TRIANGLE_FAN);
    glVertex2f(100, 100);
    glVertex2f(100, 300);
    glVertex2f(200, 0);
    glVertex2f(300, 100);
    glEnd();
    glFlush();
}
```

What shape does this display method draw?





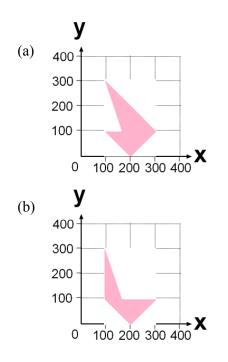
(d) None of the above

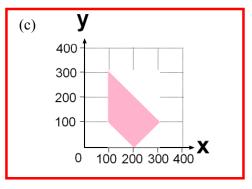
Question 19:

Given is an OpenGL program with the display method

```
void display(void)
{
    // clear all pixels in frame buffer
    glClear(GL_COLOR_BUFFER_BIT);
    glColor3f(0.0, 0.0, 0.0);
    glBegin(GL_QUADS);
    glVertex2f(300, 100);
    glVertex2f(100, 300);
    glVertex2f(100, 100);
    glVertex2f(200, 0);
    glEnd();
    glFlush();
}
```

What shape does this display method draw?





(d) None of the above

Question 20:

What is the model view matrix after executing the following code?

glMatrixMode(GL MODELVIEW); glLoadIdentity(); glRotatef(ϕ , 1, 0, 0); glTranslatef(0, 2, 0); $(\cos \varphi - \sin \varphi \ 0 \ 0)$ 2 $\sin \varphi$ $\cos \varphi$ 0 (a) 0 0 1 0 0 0 0 1 0 0 0) 1 $0 \cos \varphi - \sin \varphi$ 2 (b) 0 $0 \sin \varphi$ $\cos \varphi$ 1 0 0 0

(c)	(1	0	0	0)
	0	$\cos \varphi$	$-\sin \varphi$	$2\cos\varphi$
	0	$\sin \varphi$	$\cos \varphi$	$2\sin\varphi$
	0	0	0	$\begin{array}{c} 0\\ 2\cos\varphi\\ 2\sin\varphi\\ 1 \end{array}$

⁽d) None of the above

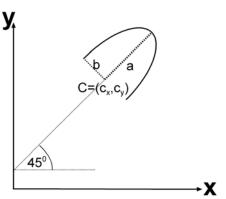
Solution: The code initializes the model view matrix with an identity matrix, multiplies it on the right with a matrix for a rotation around the x-axis, and then multiplies the result with a translation matrix. Hence

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi & 0 \\ 0 & \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi & 2\cos\varphi \\ 0 & \sin\varphi & \cos\varphi & 2\sin\varphi \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Question 21:

What is the parametric equation $\mathbf{p}(t)$ for the halfellipsoid in the image on the right? Note that the ellipsoid has the centre (c_x, c_y) and it's major axis forms an angle of 45 degree with the x-axis.

Hint: In order to get the correct shape the curve equation, the transformation and the parameter range must be correct.



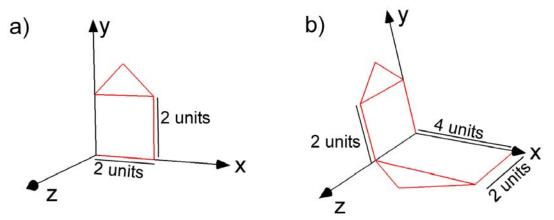
(a)
$$\mathbf{p}(t) = \begin{pmatrix} c_x \\ c_y \end{pmatrix} + \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} a \cos 2\pi t \\ b \sin 2\pi t \end{pmatrix}$$

 $t \in [0,0.5]$
(c) $\mathbf{p}(t) = \begin{pmatrix} c_x + a \cos 2\pi t \\ c_y + b \sin 2\pi t \end{pmatrix}$
 $t \in [-0.25, 0.25]$
(d) None of the above
 $t \in [0.5, 1.0]$

Solution: The shape in the resulting drawing is a half ellipsoid consisting of the fourth and first quadrant, i.e. we can use the parametric equation for an ellipsoid with parameter interval [-0.25, 0.25]. This ellipsoid is then rotated by 45 degree counter clockwise and then translated by \mathbf{c} . Hence:

$$\mathbf{p}(t) = \begin{pmatrix} c_x \\ c_y \end{pmatrix} + \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} a\cos 2\pi t \\ b\sin 2\pi t \end{pmatrix}$$
$$t \in [0.25, 0.25]$$

Given is a function *drawHouse()* which draws a wire frame house in the xy-plane as shown in part (a) of the image below.



The OpenGL transformations for converting this scene into the one shown in part (b) of the image are:

glPushMatrix(); // MISSING CODE SEGMENT 1 drawHouse(); glPopMatrix(); // MISSING CODE SEGMENT 2 drawHouse();

Question 22:

The missing code segment 1 in the above program (drawing the scaled house) is

(a) glRotatef(-90, 1, 0, 0); glScalef(2, 1, 1);
(b) glScalef(2, 1, 1); glRotatef(90, 1, 0, 0);

(c) glScalef(2, 1, 1); glRotatef(90, 0, 1, 0);

(d) None of the above

Solution: The shape is first rotated by 90 degree clockwise around the x-axis, such that it lies in the xz-plane. It is then scaled by 2 in x-direction.

Question 23:

The missing code segment 2 in the above program (drawing the house in the yz-plane) is

(a) glTranslatef(0, 0, 2); glRotatef(90, 0, 1, 0);

(b) glTranslatef(0, 0, 2); glRotatef(-90, 0, 1, 0);

- (c) glRotatef(-90, 0, 1, 0); glTranslatef(0, 0, -2);
- (d) None of the above

Solution: The shape is first rotated by 90 degree clockwise around the y-axis, such that it lies in the yz-plane (in the half with negative z-values). Translating it shifts it into the half with positive z-values.

Question 24:

Given is a profile curve $\mathbf{p}(t) = (x(t), y(t), 0)$. A parametric surface $\mathbf{p}(s,t)$ is created by rotating the profile curve around the x-axis. The resulting surface of revolution $\mathbf{p}(s,t)$ is:

(a)
$$\begin{pmatrix} x(t)\cos(2\pi s) \\ x(t)\sin(2\pi s) \\ y(t) \end{pmatrix}$$

(b)
$$\begin{pmatrix} x(t)\sin(2\pi s) \\ y(t) \\ x(t)\sin(2\pi s) \end{pmatrix}$$

(c)
$$\begin{pmatrix} x(t) \\ y(t)\cos(2\pi s) \\ y(t)\sin(2\pi s) \end{pmatrix}$$

(d) None of the above

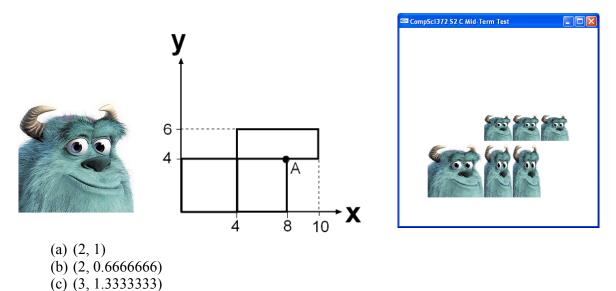
Solution: The parametric surface is obtained by rotating the profile curve around the x-

axis, i.e:
$$p(s,t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(2\pi s) & -\sin(2\pi s) \\ 0 & \sin(2\pi s) & \cos(2\pi s) \end{pmatrix} \begin{pmatrix} x(t) \\ y(t) \\ 0 \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t)\cos(2\pi s) \\ y(t)\sin(2\pi s) \end{pmatrix}$$

Question 25:

(d) None of the above

Given is the texture shown in the image below on the left and three polygons shown in the image below in the middle. Assume the texture coordinate of the bottom left vertex is (0,0) and the texture parameters GL_REPEAT are set. Which texture coordinates do we have to define for the vertex A in order to get the image shown below on the right?



Solution: At the point A the image has been three times repeated in x-direction and one time in y-direction, i.e. the texture coordinates are (3,1).