COMPSCI 373 S1C: Computer Graphics and Image Processing

Part 3 – Image Processing

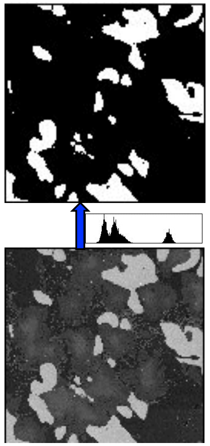
Mathematical Morphology: Applications to Digital Images

Outline:

1. Morphological processing of binary images
   1. Main concepts
   2. Basic operations: Erosion / dilation
   3. Compound operations: Closing / opening
   4. Boundary detection and other operations
2. Generalising to grayscale images

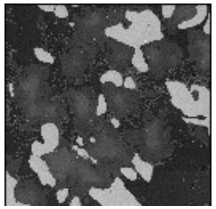
Basic Concepts

- **Morphology**: *a study of structure or form* (http://www.merriam-webster.com/dictionary/morphology)
- **Morphological image processing** may remove imperfections of an image
  - Regions in binary images produced by simple thresholding are typically distorted by noise
- Morphological operations are non-linear and account for structure and forms of regions (objects) to improve an image
- Morphological operations can be extended to greyscale images

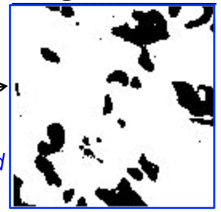


Binary Images: Object/Background

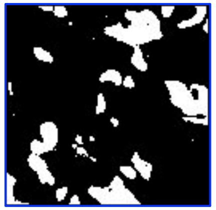
- White background – black foreground regions (objects)



For morphological processing, the foreground (i.e. object) and background pixels are treated as "1"s and "0"s, respectively, independently of their visual black-white coding

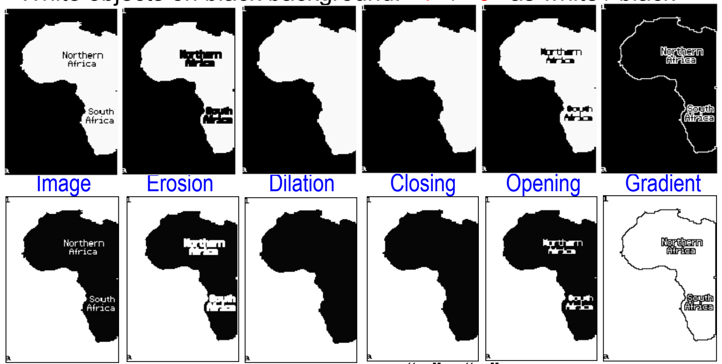


- Black background – white foreground regions (objects)



Morphological Operations: Examples

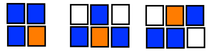
- White objects on black background: "1" / "0" as white / black



- Black objects on white background: "1" / "0" as black / white

Basic Concepts

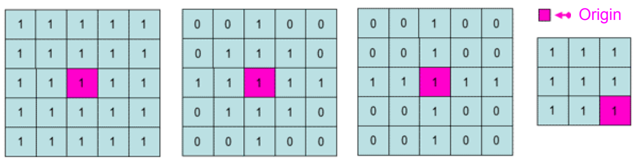
- Morphological operations rely on relative ordering of pixel values, not on their numerical values
  - Thus the operations are especially suited to binary image processing
  - These operations can be applied also to greyscale images such that their absolute pixel values are of no or minor interest
    - E.g. images with unknown light transfer functions
- Morphological operations probe an image with a small shape or template called a **structuring**, or structure **element** (SE)
  - SE resembles a convolution kernel in linear filtering



Structuring Element

A small binary image, i.e. a small matrix of pixels, each with a value of zero ("0") or one ("1")

- Zero-valued pixels of the SE are ignored
- ***Size*** of the SE: the matrix dimensions
- ***Shape*** of the SE: the pattern of ones and zeros
- ***Origin*** of the SE: usually, one of its pixels
  - Generally, the origin can be also outside the matrix

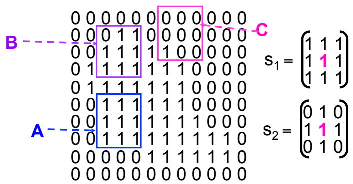


Square 5×5 SE Diamond 5×5 SE Cross 5×5 SE Square 3×3 SE

Structuring Element

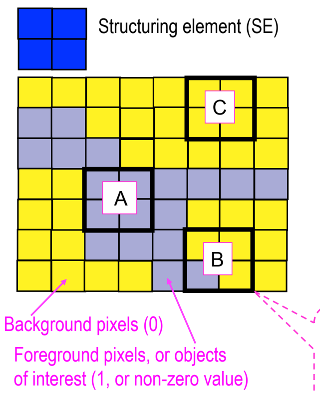
When an SE is placed in a binary image, each its pixel is associated with the pixel of the area under the SE

- The SE **fits** the image if **for each** of its pixels set to 1 the corresponding image pixel is also 1
- The SE **hits** (intersects) the image if **at least for one** of its pixels set to 1 the corresponding image pixel is also 1



		A	B	C
fit	s_1	yes	no	no
	s_2	yes	yes	no
hit	s_1	yes	yes	yes
	s_2	yes	yes	no

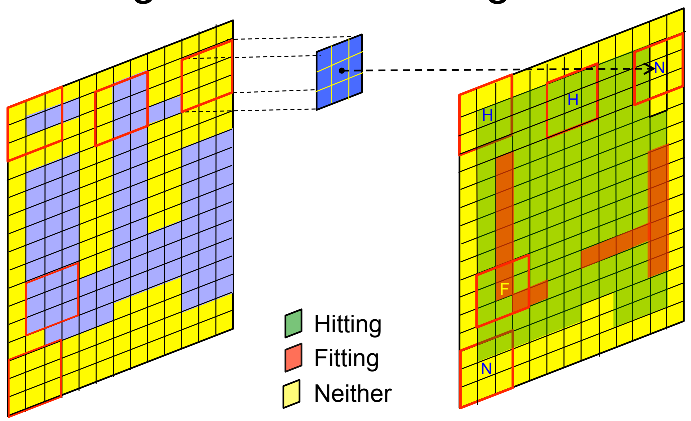
Probing with a Structuring Element



- SE is positioned at all possible locations in an image and compared with the pixel neighbourhood at each location.
- Operations test whether the SE "fits" within the neighbourhood or "hits" the neighbourhood.
- An operation on a binary image creates a new binary image in which the pixel has a foreground value only if the test at that place in the input image is successful.

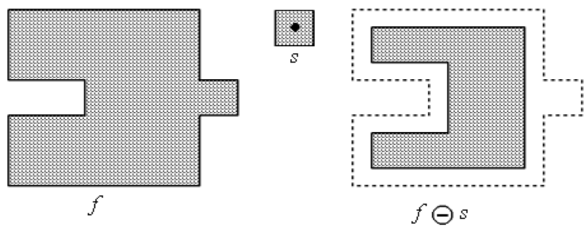
A – the SE **fits** the foreground object
B – the SE **hits** (intersects with) the object
C – the SE neither fits, nor hits the object

Probing with a Structuring Element



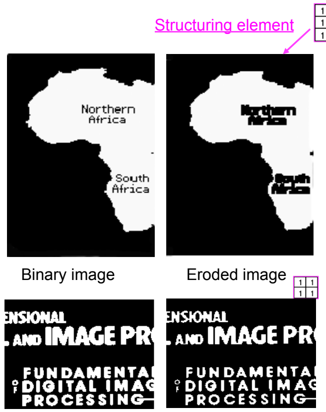
Fundamental Operation: Erosion

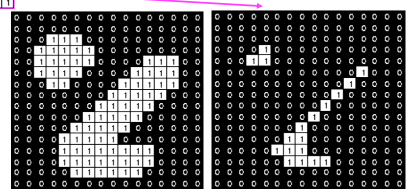
- **Erosion** $f \ominus s$ of a binary image f by an SE s produces a new binary image $g = f \ominus s$
  - Eroded image g has ones in all locations (x,y) of an origin of the structuring element s at which s fits the input image f
    - For all pixel coordinates (x,y), $g(x,y) = 1$ if s fits f and 0 otherwise



From: http://www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

Erosion of a Binary Image





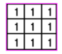
- Erosion with small (e.g. 2×2 - 5×5) square SEs shrinks an image by stripping away a layer of pixels from both inner and outer region boundaries
- Holes in and gaps between foreground objects become larger, and small details are eliminated

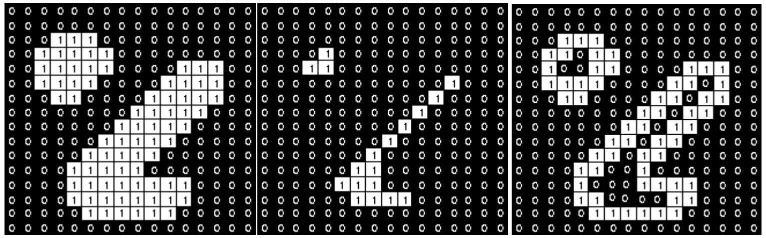
www.cs.princeton.edu/~pshilane/mosaic/
www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html
http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Erosion of a Binary Image

- Larger SEs have a more pronounced effect
  - Erosion with a large SE is similar to an iterated erosion with a smaller SE of the same shape
  - If a pair of SEs s_1 and s_2 are identical in shape, with s_2 twice the size of s_1, then $f \ominus s_2 \approx (f \ominus s_1) \ominus s_1$
- Erosion removes small-scale details and noise from a binary image
  - Simultaneously erosion reduces the size of regions of interest (objects), too
  - Background areas are growing, i.e. an image with the black or white background becomes “blacker” or “whiter”, respectively

Boundary Detection by Erosion

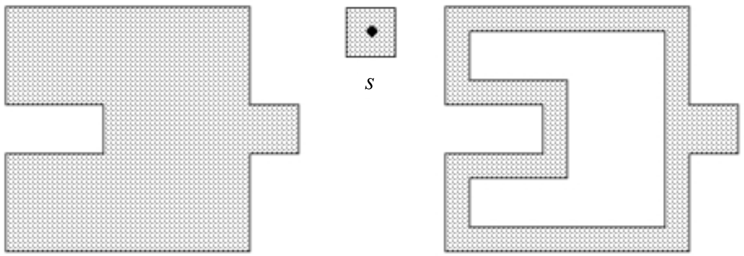
- **Internal gradient** of each region: by subtracting the eroded image from an original image: $b = f - (f \ominus s)$
  - Gradient is computed with the 3×3 square SE: 



Binary image f | Eroded image $f \ominus s$ | Boundary image $b = f - (f \ominus s)$

Boundary Detection by Erosion

- **Internal gradient** of each region

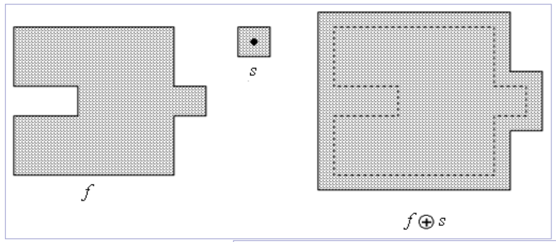


f $\qquad$ $f - (f \ominus s)$

http://www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

Fundamental Operation: Dilation

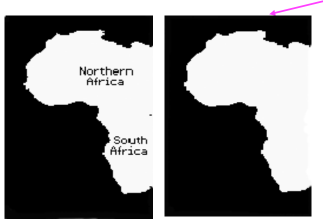
Dilation $f \oplus s$ of a binary image f by a SE s produces a new binary image $g = f \oplus s$
- Dilated image has ones in all locations (x,y) of an origin of the SE s at which s hits the input image f
  - For all pixel coordinates (x,y), $g(x,y) = 1$ if s hits f and 0 otherwise



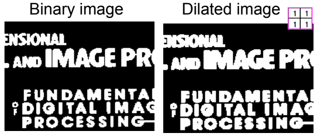
From: http://www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

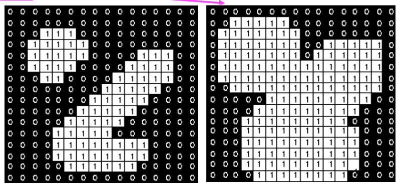
Dilation of a Binary Image

Structuring element



Binary image Dilated image





Dilation is opposite to erosion: it adds a layer of pixels to the inner and outer boundaries of regions

- Holes enclosed by a single region and gaps between different regions become smaller; small intrusions into boundaries of a region are filled in

www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html
www.cs.princeton.edu/~pshilane/class/mosaic/
http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

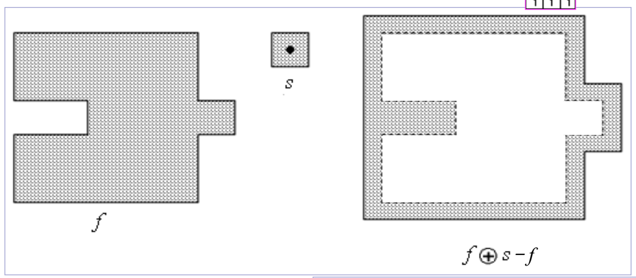
Dilation and Erosion: Duality

- Dilation and erosion are *dual* operations in that they have opposite effects: $f \oplus s = f^c \ominus s_{\text{rot}}$
  - f^c - the complement of f produced by replacing 1 / 0 with 0 / 1
  - s_{rot} is the SE s rotated by 180°
  - If a SE is symmetric with respect to rotation, then s_{rot} does not differ from s
- If a binary image is considered as a collection of connected regions of "1"s on background of "0"s:
  - **Erosion** is the fitting of a SE to these regions
  - **Dilation** is the fitting of a SE, rotated if necessary, into the background, followed by inversion of the result

Boundary Detection by Dilation

- **External gradient** of each region: by subtracting an original image from the dilated image: $b = (f \oplus s) - f$
  - Gradient is computed with the 3×3 square SE:

1	1	1
1	1	1
1	1	1

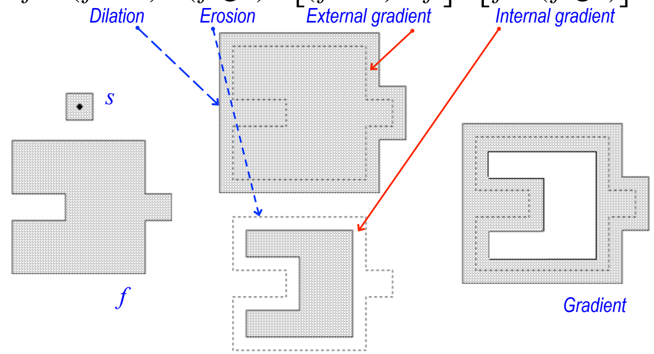


From: http://www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

Morphological Gradient

- Difference between the dilation and erosion:

$$\nabla f = (f \oplus s) - (f \ominus s) \equiv [(f \oplus s) - f] + [f - (f \ominus s)]$$

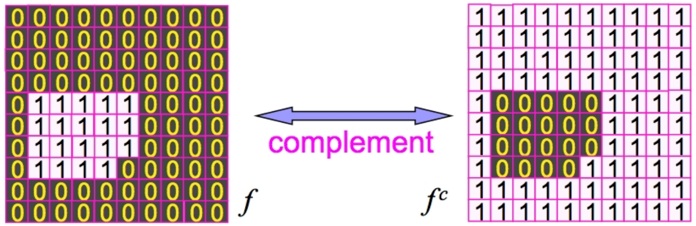


Set-theoretic Binary Operations

- Many morphological operations are combinations of erosion, dilation, and simple set-theoretic operations
- Set-theoretic **complement** of a binary image:

  $f^c(x,y) = 1$ if $f(x,y) = 0$, and

  $f^c(x,y) = 0$ if $f(x,y) = 1$

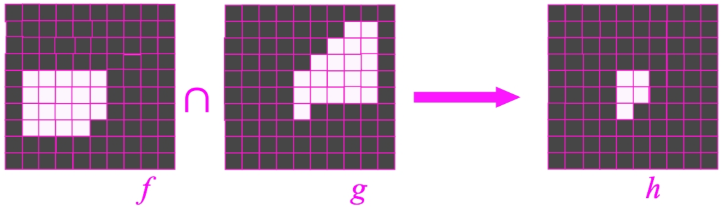


Set-theoretic Binary Operations

- **Intersection** $h = f \cap g$ of two binary images f and g:

  $h(x,y) = 1$ if $f(x,y) = 1$ **AND** $g(x,y) = 1$;

  $h(x,y) = 0$ otherwise

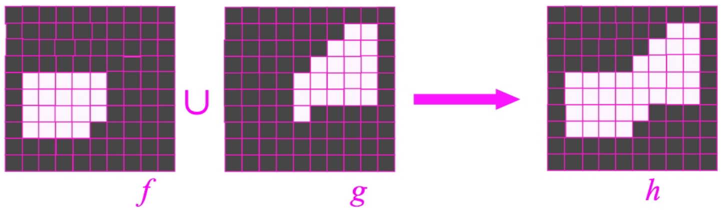


Set-theoretic Binary Operations

- **Union** $h = f \cup g$ of two binary images f and g:

  $h(x,y) = 1$ if $f(x,y) = 1$ **OR** $g(x,y) = 1$;

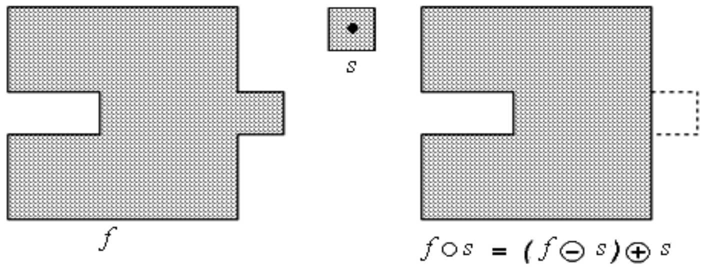
  $h(x,y) = 0$ otherwise



Opening

- **Opening** $f \circ s$ of an image f by a structuring element s is an *erosion* followed by a *dilation*:

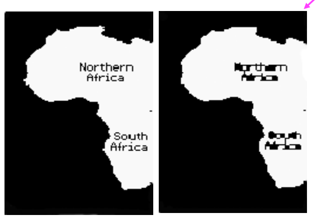
  $$f \circ s = (f \ominus s) \oplus s$$

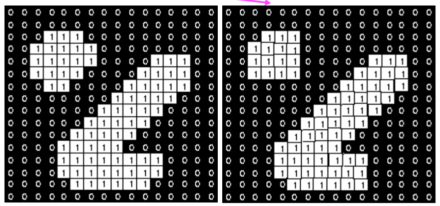


http://www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

Opening of a Binary Image

Structuring element



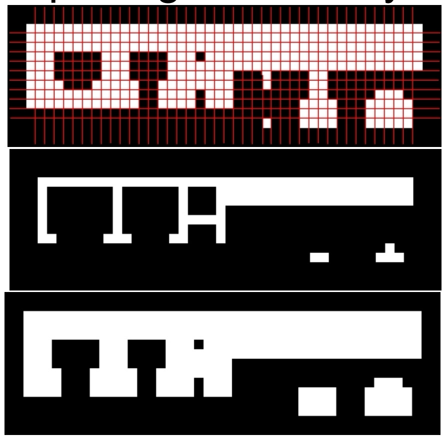


Binary image Opened image

Opening is so called because it can open up a gap between objects connected by a thin bridge of pixels

www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html
www.cs.princeton.edu/~pshilane/class/mosaic/
http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Opening of a Binary Image

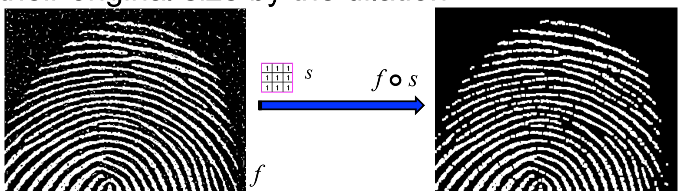


- Initial image f: white "1"s and black "0"s
- SE s
- Erosion $f \ominus s$
- Opening $f \circ s = (f \ominus s) \oplus s$

www.cs.ru.nl/~ths/rt2/col/h11/11morphENG.html

Opening of a Binary Image

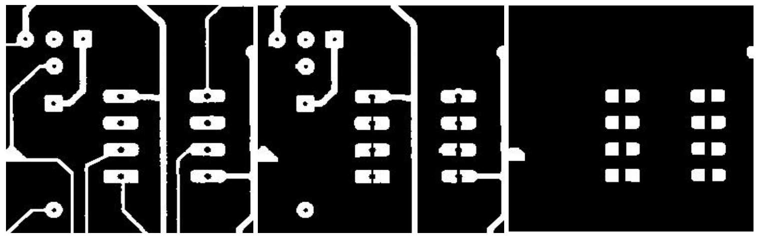
- Any regions survived the erosion are restored to their original size by the dilation



http://www.cs.ru.nl/~ths/rt2/col/h11/11morphENG.html

- **Idempotent operation**: $(f \circ s) \circ s = f \circ s$
  - Once an image is opened, next openings with the same structuring element have no further effect

Opening of a Binary Image



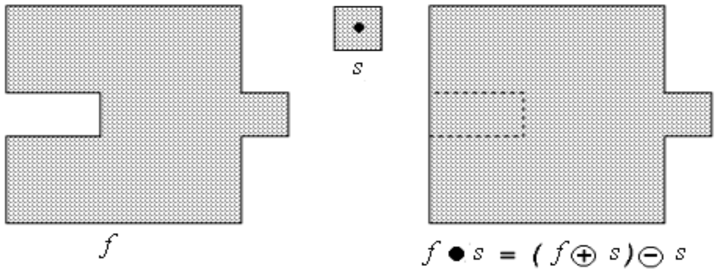
Binary image | Opening with a 5 × 5 square structuring element | Opening with 9 × 9 square structuring element

www.mmorph.com/html/morph/mmopen.html/

Closing

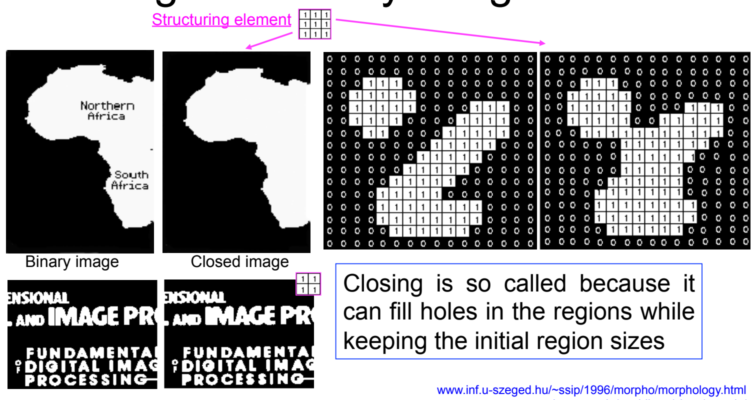
- **Closing** $f \bullet s$ of an image f by a structuring element s is a *dilation* followed by an *erosion*:

$$f \bullet s = (f \oplus s) \ominus s$$



http://www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

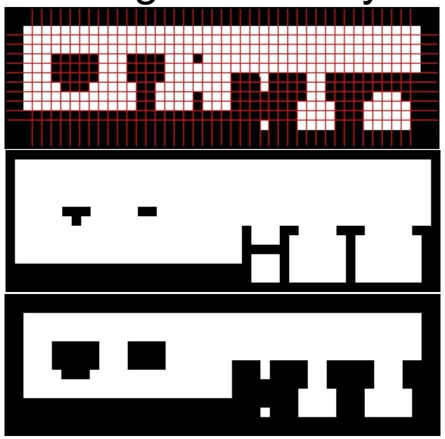
Closing of a Binary Image



Closing is so called because it can fill holes in the regions while keeping the initial region sizes

www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html
www.cs.princeton.edu/~pshilane/class/mosaic/
http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Closing of a Binary Image

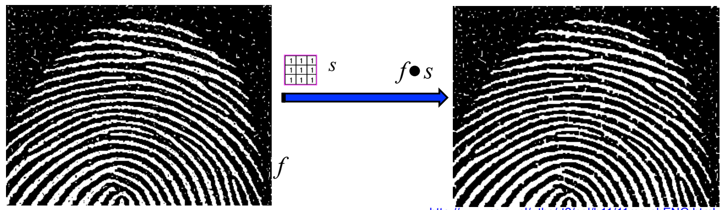


- Initial image f: white "1"s and black "0"s
- SE s
- Dilation $f \oplus s$
- Closing
  $f \bullet s = (f \oplus s) \ominus s$

www.cs.ru.nl/~ths/rt2/col/h11/11morphENG.html

Closing of a Binary Image

- Dilation and erosion with a rotated by 180° SE
  - Typical symmetric SE: the rotated and initial SE do not differ



http://www.cs.ru.nl/~ths/rt2/col/h11/11morphENG.html

- **Idempotent operation** $(f \bullet s) \bullet s = f \bullet s$
- Once an image is closed, next closings with the same structuring element have no further effect

Closing Vs. Opening

- Closing is the **dual operation** of opening
  - Just as opening is the dual operation of closing
- **Closing** of a binary image (*dual implementation*):
  - Take the **complement** of that image ("1 / 0" ➢ "0 / 1")
  - Perform **opening** with the structuring element, and
  - Take the **complement** of the result
- **Opening** of a binary image (*dual implementation*):
  - Take the **complement** of that image
  - Perform **closing** with the structuring element, and
  - Take the **complement** of the result

Closing Vs. Opening

- Closing with a square or disk SE:
  - Fills thin connections within an object
  - Eliminates small holes and fills dents in contours
  - Fills small gaps in parts of an object
- Opening with a square or disk SE:
  - Breaks thin connections within an object
  - Eliminates small islands and sharp protrusions

Binary Image: Closing + Opening

Initial image f

Closed, then opened image f: $(f \bullet s) \circ s$

http://www.cs.ru.nl/~ths/rt2/col/h11/11morphENG.html

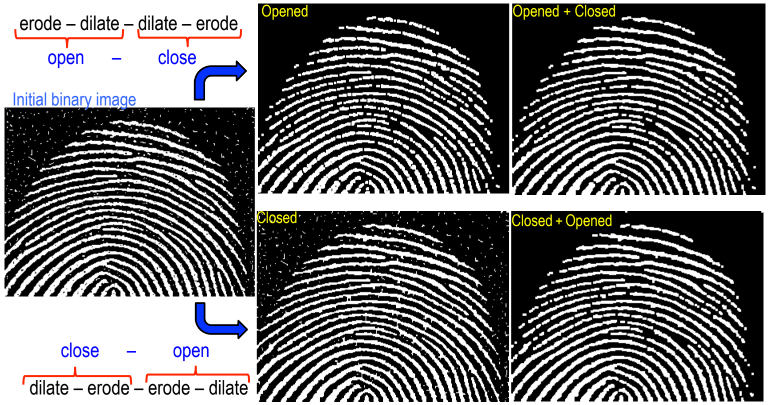
Binary Image: Opening + Closing

Initial image f

Opened, then closed image f: $(f \circ s) \bullet s$

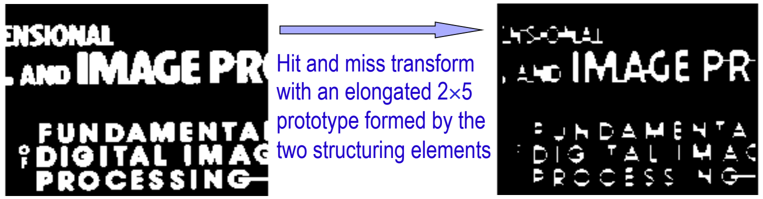
http://www.cs.ru.nl/~ths/rt2/col/h11/11morphENG.html

Opening+Closing vs. Closing+Opening



Hit and Miss Transform

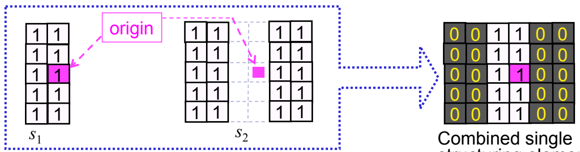
- Hit and miss transform tests how objects in a binary image are related to their surroundings
- Matched pair of structuring elements, $\{s_1, s_2\}$, probes the inside and outside, respectively, of objects in the image: $f \ominus \{s_1, s_2\} = (f \ominus s_1) \cap (f^c \ominus s_2)$



http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

Hit and Miss Transform

- The transform preserves a pixel of a region if and only if s_1 for that pixel fits inside the region AND s_2 for that pixel fits outside the region
  - Structuring elements s_1 and s_2 do not intersect, otherwise it would be impossible for both fits to occur simultaneously



Combined single structuring element

Hit and Miss Transform

- Easier description: by considering s_1 and s_2 as a single element with "1"s for pixels of s_1 and "0"s for pixels of s_2
- The transform assigns 1 to an output pixel only if:
  - The "1"s and "0"s in the structuring element exactly match "1"s and "0"s, respectively, in the input image
  - Otherwise that pixel is set to 0
- If the two SEs together present a specific shape (spatial arrangement of foreground "1"s and background "0"s), the transform can detects the desired shapes
- The transform can be used for thinning or thickening of linear elements of objects

Morphological Filtering

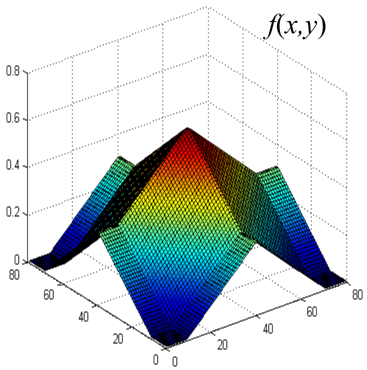
- Compound operations (e.g. opening and closing) act as non-linear filters of shape in a binary image
  - Opening and closing with a disc SE smooth corners from the inside and the outside, respectively
  - Details smaller in size than the disc are also filtered out
    - Opening is filtering at a scale of the size of the SE
    - Only those portions of the image that fit the SE are passed by the filter
    - Smaller structures are blocked and excluded
  - The size of the SE is most important in order to eliminate noisy details but not to damage objects
    - If the SE is too large, the object could be degraded by the operation

Greyscale Morphology

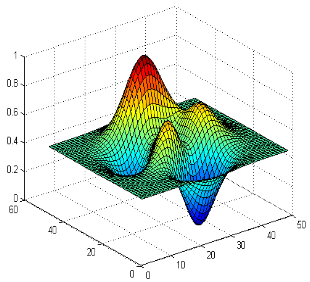
- Definitions of operations: similar to the binary case but with the additional dimension of **pixel grey level**
  - An image is considered as a 3D surface $f(x,y)$ with the height at any point representing the integer grey level f at that point
  - The structuring element $s(\xi,\eta)$ is also 3D surface, but its pixels take any integer value s, including zero and negative values

Greyscale Morphology

$f(x,y)$



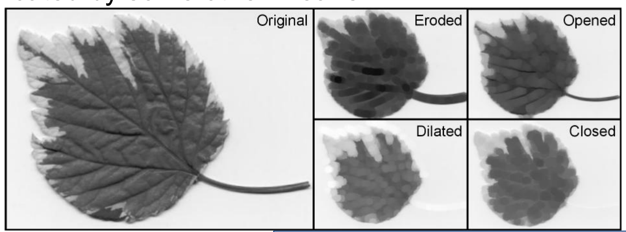
$s(\xi,\eta)$



http://www.cs.wits.ac.za/~michael/Lab5.html

Greyscale Morphology

- The SE is sometimes referred in the greyscale morphology to as a ***structuring function***
- Zero value is now significant: pixels that do not participate in morphological operations have to be indicated by some other means



http://rsb.info.nih.gov/ij/plugins/images/gray-morphology.jpg

Greyscale Erosion

- Greyscale erosion $f \ominus s$ of an image f by the SE s

220	210	120	45	50
225	200	130	67	53
202	199	100	73	45
189	190	110	68	49
190	200	134	71	57

−1	−9	−1
11	11	11
−1	−9	−1

⇒

	101	56	42	
	89	62	34	
	101	57	38	

  - Replacing the pixel grey level corresponding to the origin of s with the minimum difference between each pixel grey level in f and the corresponding value in s over the domain of the SE

Greyscale Erosion

- Greyscale erosion $f \ominus s$ of an image f by the SE s

$$(f \ominus s)(x,y) = \min_{\xi,\eta}[\, f(x+\xi, y+\eta) - s(\xi,\eta)\,]$$

  - (ξ, η) - 2D indices of a pixel in the SE s , i.e. the pixel coordinates with respect to the origin of the SE s
  - Indices $\xi,\eta \in [-k,k]$ for a $(2k+1) \times (2k+1)$ square SE s

220	210	120	45	50
225	200	130	67	53
202	199	100	73	45
189	190	110	68	49
190	200	134	71	57

−1	−9	−1
11	11	11
−1	−9	−1

$$\min\{220+1, 210+9, 120+1, 225-11, 200-11, 130-11, 202+1, 199+9, 100+1\} = \min\{221,210,121,214,189,119, 203,208,101\} = \mathbf{101}$$

Greyscale Erosion

- Flat structuring element: $s(\xi,\eta) = 0$
- Erosion with the flat SE:

$$(f \ominus s)(x,y) = \min_{\xi,\eta}[\, f(x+\xi, y+\eta)\,]$$

  - Same effect as the **minimum rank**, or **minimum** filter

Greyscale image

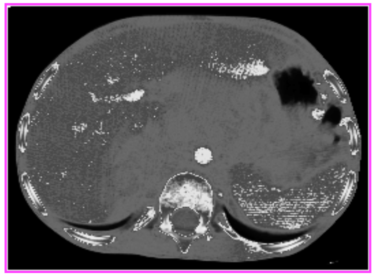
Eroded image: a 3 x 3 minimum filter

http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

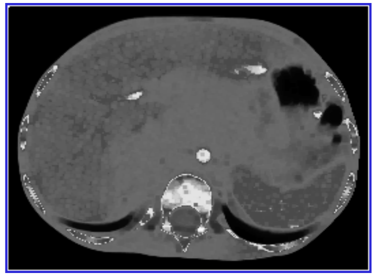
The Flat SE: The Minimum Filter

- Selects the minimum (bottom-ranked) grey level from the neighbourhood as the output value

Greyscale image



Eroded image: a 3 x 3 minimum filter



http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

Greyscale Dilation

- Greyscale erosion $f \oplus s$ of an image f by the SE s

220	210	120	45	50
225	200	130	67	53
202	199	100	73	45
189	190	110	68	49
190	200	134	71	57

−1	−9	−1
11	11	11
−1	−9	−1

⇨

	236	211	141	
	224	210	111	
	201	201	133	

  - Replacing the pixel grey level corresponding to the origin of s with the maximum sum of each pair of the pixel grey level in f and the corresponding value in s over the domain of the SE

Greyscale Dilation

- Greyscale dilation $f \oplus s$ of an image f by the SE s

$$(f \oplus s)(x,y) = \max_{\xi,\eta}[f(x-\xi, y-\eta) + s(\xi,\eta)]$$

  - (ξ, η) - 2D indices of the pixel in the SE s
  - Greyscale dilation is a dual operation with respect to erosion

220	210	120	45	50
225	200	130	67	53
202	199	100	73	45
189	190	110	68	49
190	200	134	71	57

−1	−9	−1
11	11	11
−1	−9	−1

$$\max\{220-1, 210-9, 120-1, 225+11, 200+11, 130+11, 202-1, 199-9, 100-1\}$$
$$= \max\{219,201,119,236,211,143, 201,190, 99\} = \mathbf{236}$$

Greyscale Dilation

- Flat structuring element: $s(\xi,\eta) = 0$
- Dilation with the flat SE:

$$(f \oplus s)(x,y) = \max_{\xi,\eta}[f(x-\xi, y-\eta)]$$

  - Same effect as the **maximum rank**, or **maximum** filter

Greyscale image

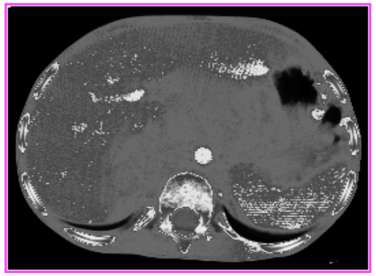
Dilated image: a 3 x 3 maximum filter

http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

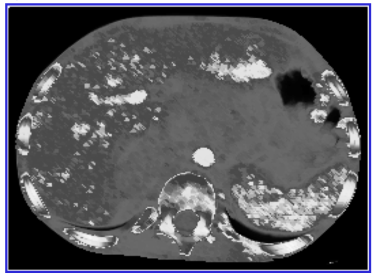
The Flat SE: The Maximum Filter

- Selects the maximum (top-ranked) grey level from the neighbourhood as the output value

Greyscale image



Dilated image: a 3 x 3 maximum filter



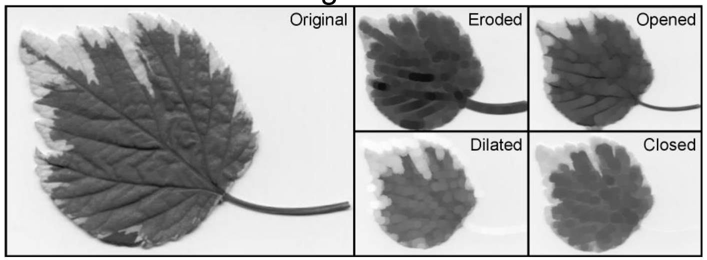
http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html
www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

Greyscale Opening / Closing

- **Opening** and **closing** for greyscale images are defined just as those for binary images:
  - Opening $f \circ s = (f \ominus s) \oplus s$
  - Closing $f \bullet s = (f \oplus s) \ominus s$
- With an appropriate structuring element, these operations can smooth the image
  - However, it is the **surface of grey levels** that is smoothed, rather than the contours of shapes in a binary image

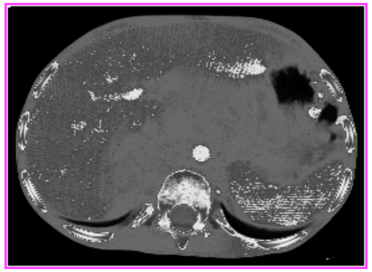
Greyscale Opening / Closing

- Opening tends to smooth away small-scale **bright** details in an image
- Closing tends to smooth away the small-scale **dark** details in an image

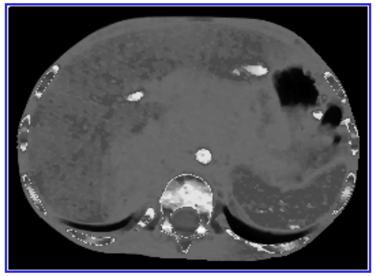


http://rsb.info.nih.gov/ij/plugins/images/gray-morphology.jpg

Greyscale Opening



Greyscale image



Opened image: a 3 x 3 square flat SE

Smoothing out bright small-scale “noisy” details of the initial image

http://www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

Erosion - Opening / Dilation - Closing

The pairs differ like the corresponding binary operations

- **Erosion** shrinks bright features and enlarges dark features
- **Opening** removes small bright features but does not enlarge dark ones
- Similar considerations apply to **dilation** and **closing**

Greyscale image

3 x 3 square flat SE

Erosion

Dilation

Opening

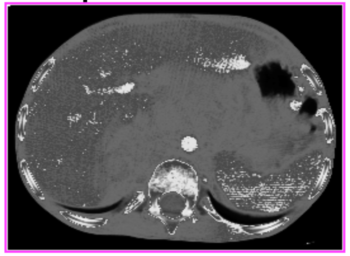
Closing

http://documents.wolfram.com/applications/digitalimage/UsersGuide/Morphology/ImageProcessing6.3.html

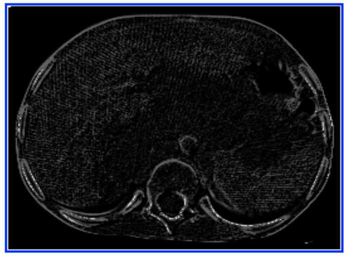
Smoothing / Top-hat Transform

- **Morphological smoothing**: an opening-closing iteration
  - Removes small-scale bright and dark details
  - Resembles extreme forms of the median filter
- **Top-hat transform**: $g = f - (f \circ s)$
  - Opening $f \circ s$ removes small-scale bright details
  - Top-hat transform selects only these details
  - Therefore the top-hat transform acts as a detector of peaks and ridges of the grey level surface

Top-hat Transform



Greyscale image



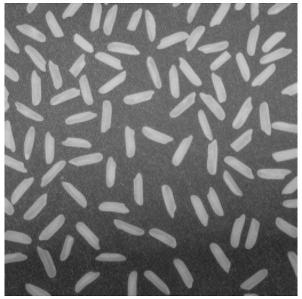
Top-hat transform: a 3 x 3 square flat structuring element

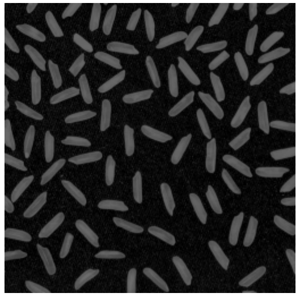
- The dual of the top-hat transform, $(f \bullet s) - f$, detects pits and valleys in the grey level surface

http://www.inf.u-szeged.hu/~ssip/1996/morpho/morphology.html

Top-hat Transform

- Correcting uneven illumination if the background is dark with a 12 pixel wide disk-shaped SE





http://www.mathworks.fr/help/toolbox/images/ref/imtophat.html