Computer Graphics and Image Processing Introduction

Part 3 – Image Processing

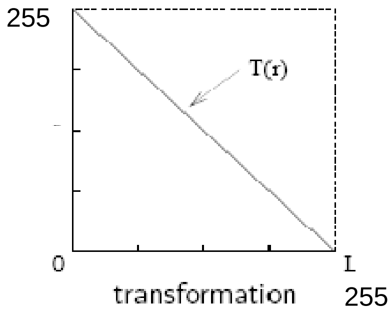
Lecture 3 Histogram equalization

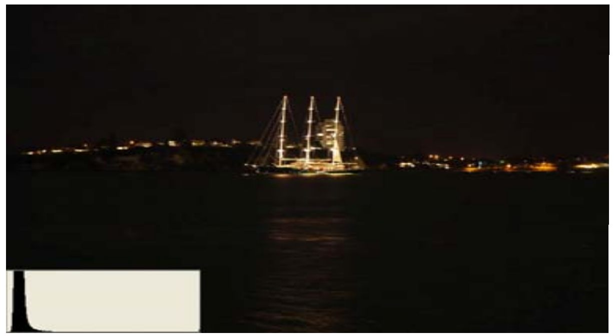
Negation

The negative of an image with grey levels f in the range [0, 255] is obtained by the negative transformation shown in the figure to the right; given by the expression, $g_{out} = 255 - f$

This expression results in reversing of the grey level intensities of the image produces a negative like image.

The image to the right shows the result of applying a negation to the dark image given earlier.





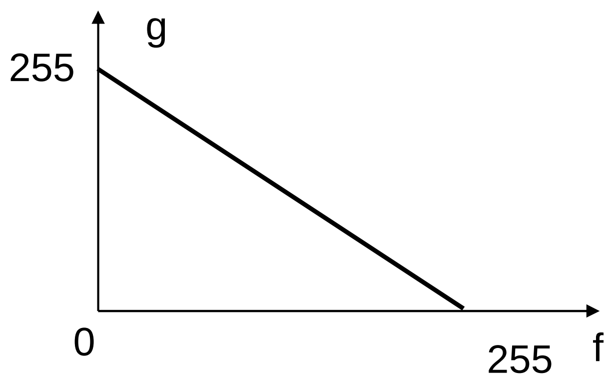
Negation example

f

0	50	75	75
175	30	105	75
150	205	30	25
150	150	175	0

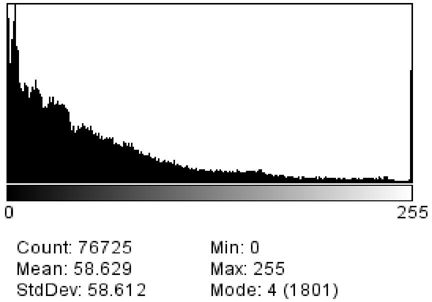
g

255	205	180	180
80	225	150	180
105	50	225	230
105	105	80	255

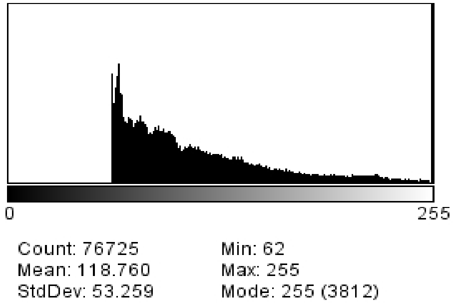


negation mapping

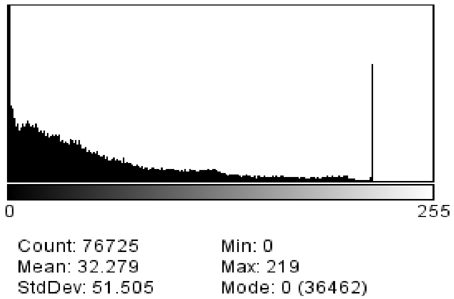
Linear mapping – brightness: histogram



b>0



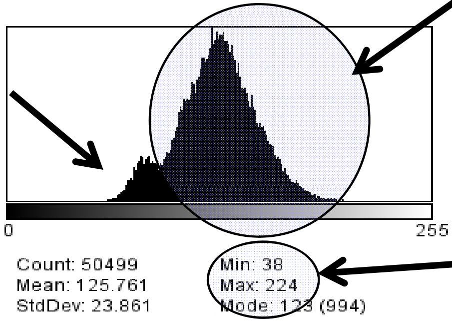
b<0



Linear mapping – scaling pixel values

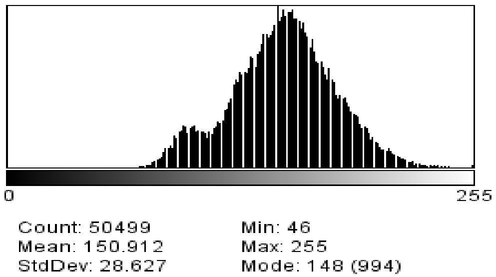
foliage

urban

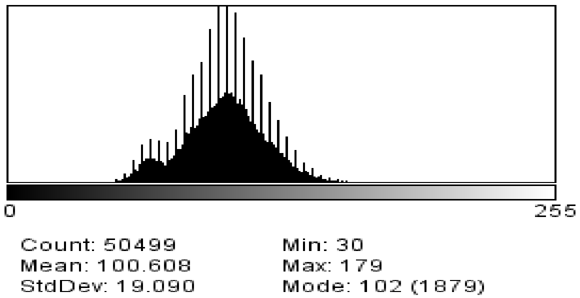


Pixel range

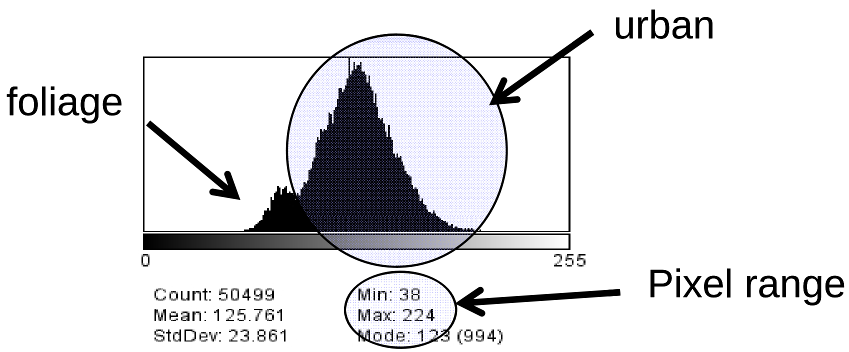
a>1



a<1



Histogram stretching

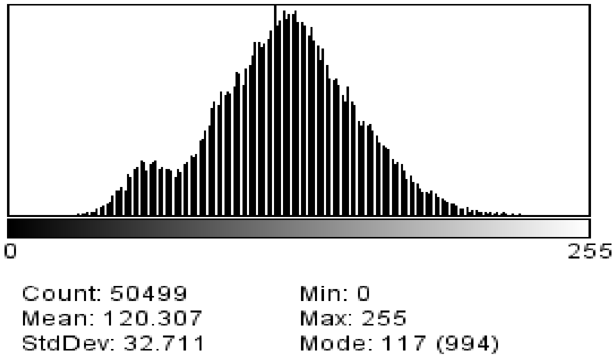


From Image min-max to max range (0-255)

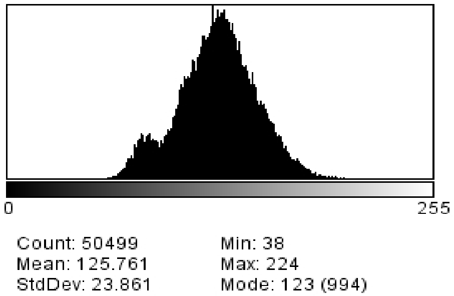
C=38, d=224, g_{max}=255, g_{min}=0

$$g_{out} = (f - c)\left(\frac{g_{max} - g_{min}}{d - c}\right) + g_{min}$$

$$g_{out} = (f - 38)\left(\frac{255}{186}\right)$$



Histogram stretching (5-95 percentile)

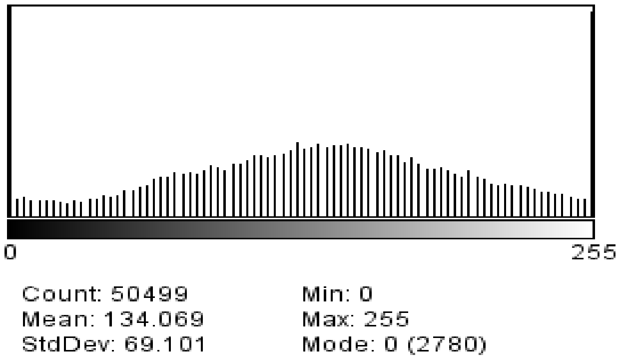


Here we suppose we want 5-95 percentile of histogram distribution to max range (0-255):

- **Compute cumulative histogram H_cumulative**
- **Find c the smallest value such that H_cumulative(c) is larger than 5% of image overall pixel count**
- **Find d the largest value such that H_cumulative(c) is smaller than 95% of image overall pixel count**
- **Do the stretching (linear mapping)**
- **Keep values below 0 at 0 and above 255 at 255**

$$g_{out} = (f - c)\left(\frac{g_{max} - g_{min}}{d - c}\right) + g_{min}$$

C=83, d=164, g_{max}=255, g_{min}=0



$$g_{out} = (f - 83)\left(\frac{255}{81}\right)$$

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Histogram Equalisation

Histogram equalisation redistributes pixel intensity values in an attempt to flatten (evenly distribute) the Image histogram, thus increasing the dynamic range and as a result increasing the image contrast.

The method is useful in images with backgrounds and foregrounds that are both bright or both dark.

It tends to reveal details that would be otherwise hidden.

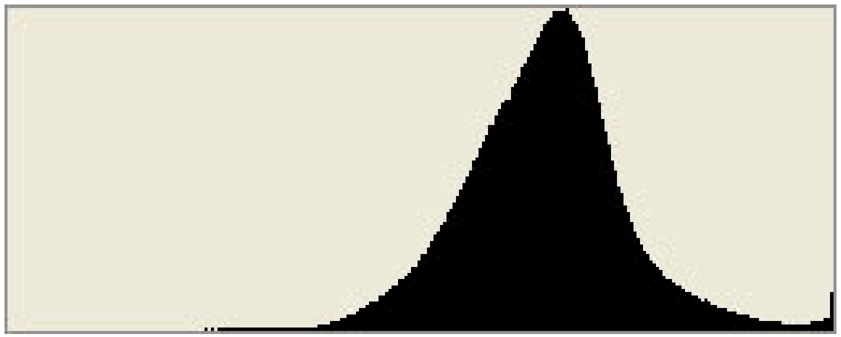
It often produces unrealistic effects in photographs, but is very useful in scientific images such as x-ray, satellite or thermal images.

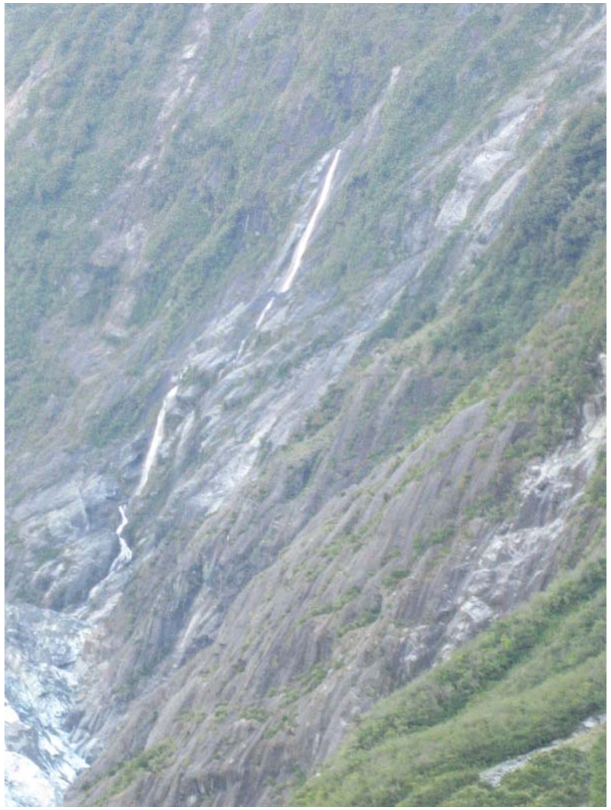
Histogram equalisation differs from Contrast stretching in that it uses non-linear transfer functions to map between pixel intensity values in the input and output images.

- **Once completed, the operation cannot be reserved to recover the original image**

Histogram Equalisation

An example of an image with poor contrast is shown on the left.
The histogram confirms what we can see by visual inspection: this image has poor dynamic range.





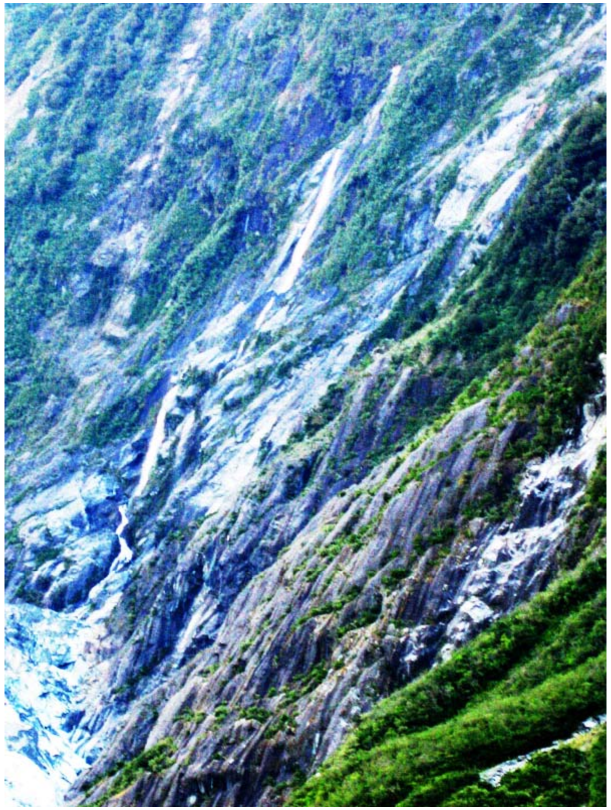
Under/Over exposure

Under-exposed

Over-exposed

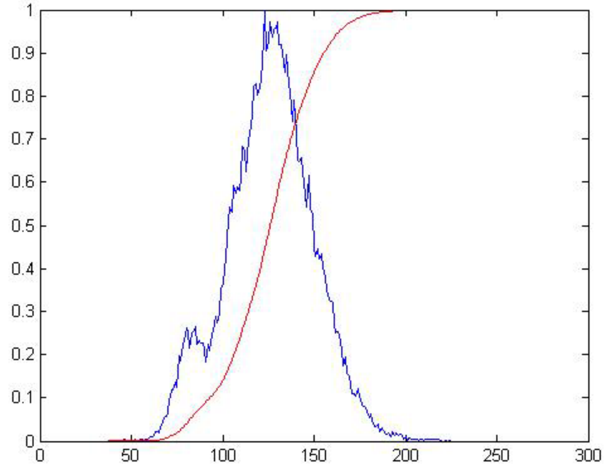
Histogram Equalisation

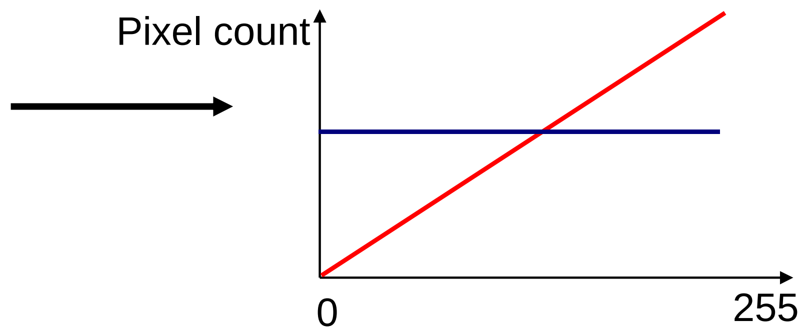
The same image after equalisation is shown to the left.
Now the Image histogram shows a much more even distribution of values.
What will the cumulative histogram for this image look like?



How to Adjust the Image?

- Histogram equalization
  - Basic idea: find a map T(x) such that the histogram of the modified (equalized) image is flat (uniform).





Cumulative histogram

Histogram Equalisation

Assuming an image I where n_i is the number of pixels with intensity level *i*. Then the probability of a pixel of intensity *i* occurring in the image is:

$$p_I(i) = \frac{n_i}{n}, 0 \le i \le 255$$

Where *n* is the total number of pixels in the image, and $p_I(i)$ is the image histogram for pixel value I, normalised to [0,1].

We define the cumulative distribution function corresponding to p_I as:

$$cdf_I(i) = \sum_{j=0}^{i} \frac{n_i}{n} = \sum_{j=0}^{i} p_I(i)$$

Histogram Equalisation

We want to produce a transformation of the form *J = T(I)* to generate a new image J such that its cumulative distribution function will be **uniform** across the pixel values range:

$$cdf_J(i) = iK$$

for some constant *K*. The transform is defined as:

$$J = T(I) = cdf_I(I)$$

Note that *T* maps the intensities into the range [1,0] so the following transformation needs to be applied to the result to map the values back to their original range i.e. the minimal and maximal values taken by I

$$J' = J(\max(I) - \min(I)) + \min(I)$$

Alternatively assuming range [0,255] $$J' = 255 * J$$

Histogram Equalisation

The image histogram equalisation algorithm is as follows:

Assuming we already have an Image histogram of intensity values. First, build a cumulative histogram as a Look up table (LUT)

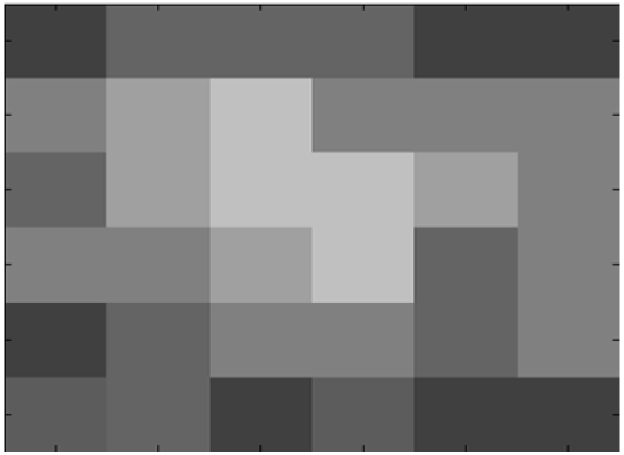
```
Sum=0
for number of intensity values, i, do
    sum = sum + histogram[i]
    CumulativeHistogram[i] = sum
end for
```

Then transform the image using the LUT

```
for number of pixels, j, do
    EqualisedImage[j] = MaxIntensity*(CumulativeHistogram[OriginalImage[j]] –
    min(CumulativeHistogram))/(number of pixels- min(CumulativeHistogram))
end for
```

Histogram Equalisation example

The following example uses the 6 pixel by 6 pixel image shown at the bottom right.
Its corresponding matrix of greyscale values are given in the table at bottom left.

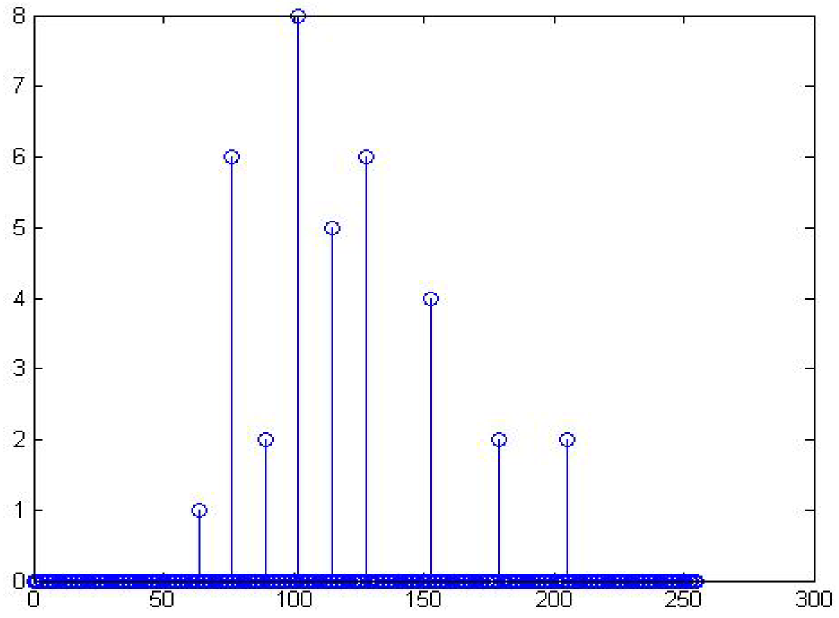


64	102	102	102	76	76
115	153	205	128	115	128
102	153	205	179	153	128
115	128	153	179	102	115
76	102	128	115	102	128
89	102	76	89	76	76

Histogram Equalisation example

First we create the Image histogram:

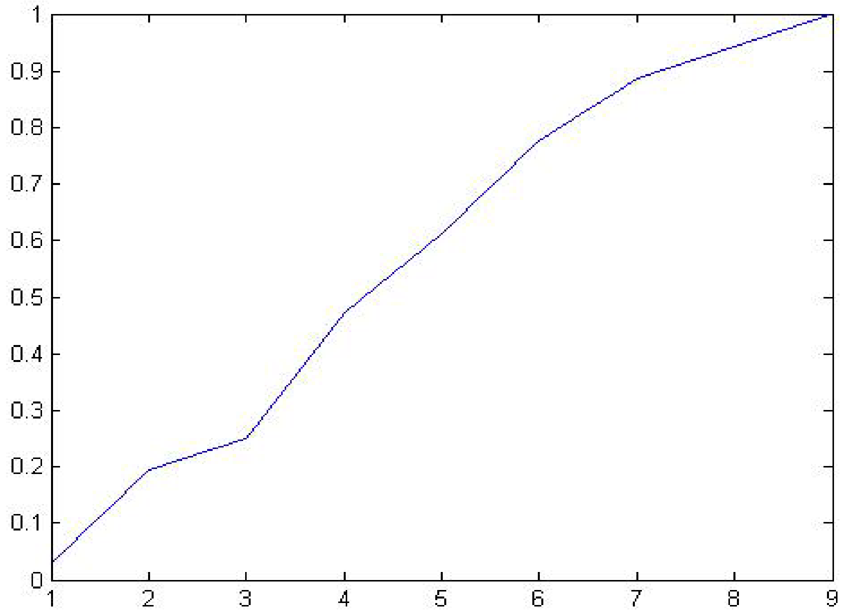
Value	Count
64	1
76	6
89	2
102	8
115	5
128	6
153	4
179	2
205	2



Histogram Equalisation example

Next, we create the cumulative distribution function:

Value	Count
64	1/36
76	7/36
89	9/36
102	17/36
115	22/36
128	28/36
153	32/36
179	34/36
205	36/36



Histogram Equalisation example

The cumulative histogram shows that the minimum intensity value in the image is 64 and the maximum is 205.

The cumulative distribution function needs to be normalised to [0,255].

We use the general histogram equalisation formula:

$$T(i) = round\left(\frac{H(i) - H_{\min}}{M \times N - H_{\min}} * 255 \right) = round\left(\frac{cdf(i) - cdf_{\min}}{1 - cdf_{\min}} * 255 \Big/ M \times N \right)$$

Where cdf_{min} is the minimum value of the cumulative distribution function, in this case 1/36.

M * *N* (*M* = width, *N* = height) is the number of pixels in the image, in this case 36.

Histogram Equalisation example

The equalisation formula for this specific example is:

$$T(i) = round\left(\frac{cumulative\_histo_I(i) - 1}{35} * 255\right)$$

$$= round\left(\frac{cdf_I(i) - 1/36}{1 - 1/36} * 255\right)$$

We can now calculate each value for image J

Value	Cdf	J	J'
64	1/36	0	0
76	7/36	6/35	44
89	9/36	8/35	58
102	17/36	16/35	117
115	22/36	21/35	146
128	28/36	27/35	197
153	32/36	31/35	219
179	34/36	33/35	233
205	36/36	1	255

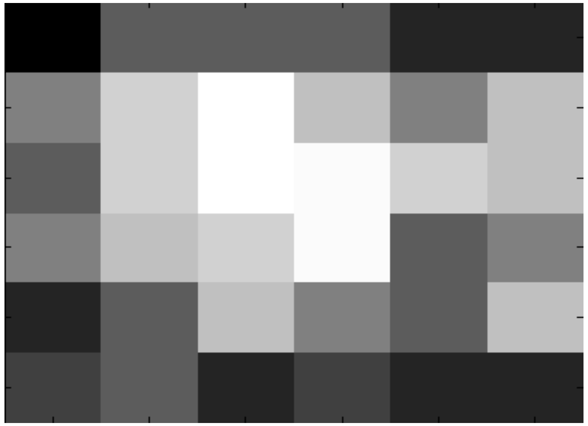
Histogram Equalisation example

The values of the equalised image are then taken directly from the normalised cumulative distribution function.

The equalised values and corresponding image are given below.

Note that the minimum value (64) is now 0, and the maximum (205) is now 255.

0	117	117	117	44	44
153	226	248	197	153	197
117	226	255	248	226	197
153	197	226	248	117	153
44	117	197	153	117	197
58	117	44	58	44	44



Histogram equalization vs stretching

Stretching

Stretching 5-95