COMPSCI 320SC 2006 Midterm Exam

Attempt *all* questions. Put the answers in the space below the questions. Write clearly! You may continue your answers onto the "overflow" page provided at the end of the test, if necessary.

Marks for each question are shown below and just before each answer box.

Use of calculators is NOT permitted.

This one hour (60 minutes) test is worth 10% of your final grade for the course.

Question #:	1	2	3	4	5	Total
Possible marks:	10	10	10	10	10	50
Awarded marks:						

University ID: _____

Student Name: _____

Student Signature: _____

Time Finished: _____

1. Suppose that I have a new algorithm for multiplication of matrices that works by dividing each of the $n \times n$ matrices x and y into 4 parts of as equal size as possible, and computing the product xy by means of 7 multiplications of the parts, plus a fixed number of matrix additions and subtractions.

   Let $g(n)$ be the total worst-case time taken to perform the additions, subtractions, and overhead.

   (a) Write down a recurrence describing the worst-case running time of this algorithm on an instance of size n. (***5 marks***)

   (b) Is this algorithm likely to be better than the standard matrix multiplication algorithm when used on large inputs? Give full explanation. (***5 marks***)

2. Recall that *selection sort* sorts an array in place by iteratively finding the minimum of the set of unsorted elements (by scanning the array) and swapping it with the leftmost of the set of unsorted elements. We claim that selection sort is a greedy algorithm. Answer the following questions to show how selection sort fits into the greedy framework.

   (a) What is the *objective function*? (***2 marks***)

   (b) What is a *feasible solution*? (***2 marks***)

   (c) What is an *optimal solution*? (***2 marks***)

   (d) What is the *selection criterion*? (***2 marks***)

   (e) What is the *rejection criterion*? (***2 marks***)

3. (a) Explain the difference between *average-case* analysis and *amortized* analysis. (***4 marks***)

   (b) Show how to multiply 31 by 17 using "Russian peasant" multiplication. (***3 marks***)

   (c) What is the maximum value of objects that can be placed in a bag of capacity 100 if fractions of objects are allowed and the objects must be chosen from a set of objects whose weights are $10, 20, 5, 80, 50, 25$ and values are $30, 17, 5, 100, 24, 13$ respectively? (***3 marks***)

4. Recall the scheduling problem discussed in lectures. There is a single processor that can work on only one job at a time. are n jobs and the ith must start at time $s(i)$ and finish at time $f(i)$. We want to schedule as many (nonoverlapping) jobs as possible.

   (a) Give an example to show that the greedy rule "schedule in order of increasing $s(i)$" does not give an optimal solution in general. (***2 marks***)

   (b) Give an example to show that the greedy rule "schedule in order of increasing $f(i) - s(i)$" does not give an optimal solution in general. (***3 marks***)

   (c) Prove that the rule "schedule in order of increasing $f(i)$" always gives an optimal solution. (***5 marks***)

5. (a) Suppose that the sequence $a(n)$ satisfies $a(0) = 2, a(1) = 1$ and $a(n) = a(n-1) + a(n-2)$ for $n \geq 2$. Prove by constructive induction that $a(n)$ grows exponentially in n and give the correct exponential growth rate. (***5 marks***)

   (b) Explain in as much detail as you can how the Union-Find ADT can be implemented using a forest. (***5 marks***)

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