COMPSCI.320SC Test
19 September 2002

Attempt *all* questions. Put the answers in the boxes below the questions. You may continue your answers onto the "overflow" pages provided at the back of the book if necessary. Marks for each question are shown below and just before each answer box. Use of calculators is NOT permitted. This test is worth 15% of your final grade for the course.

NAME:

ID #

Question #:	1	2	3	4	5	6	Total
Possible marks:	10	10	5	10	10	5	50
Awarded marks:							

1. (a) Complete the table by answering YES or NO to each question. You score +0.5 for each correct answer and -0.5 for each incorrect one.

$f(n)$	$g(n)$	$f(n) \in O(g(n))$?	$f(n) \in \Omega(g(n))$?	$f(n) \in \Theta(g(n))$?
$\log(n)$	$\lg(n^{100})$			
$n^2 + 0.0001n^5$	$10^4 n^3$			
n^{10000}	$(1.00001)^n$			
$n^{1.001}$	$(\lg n)^{100}$			

[6 marks]

(b) Prove formally from the definition that if f is a positive function defined on the natural numbers, $a > 1$ is a constant and $f \in \Theta(a^n)$, then $\log f \in \Theta(n)$. [4 marks]

2. Solve the following recurrence exactly *assuming that n is a power of* 3 (here c, d are positive real constants):

$$t(n) = \begin{cases} c & \text{if } n = 1; \\ 5t(n/3) + dn & \text{if } n > 1. \end{cases}$$

[10 marks]

3. Recall that two sorted arrays of sizes k and l can be merged into a sorted array using $k + l - 1$ comparisons.

   (a) Write down a recurrence for $T(n)$, the number of comparisons done by mergesort on an input of size n. [3 marks]

   (b) Give an asymptotic solution to the recurrence above. You may assume that T is increasing. Justify your answer.

[2 marks]

4. Recall that *selection sort* sorts an array in place by iteratively finding the minimum of the set of unsorted elements (by scanning the array) and swapping it with the leftmost of the set of unsorted elements.

   We claim that selection sort is a greedy algorithm. Answer the following questions to show how selection sort fits into the greedy framework.

   (a) What is the *objective function*? [2 marks]

   (b) What is a *partial solution*? [2 marks]

   (c) What is a *solution*? [2 marks]

   (d) What is an *optimal solution*? [2 marks]

   (e) What is the *selection criterion*? [2 marks]

5. (a) Consider the polynomials $p(x) = a_0 + a_1 x, q(x) = b_0 + b_1 x, r(x) = p(x)q(x)$. Show how to compute the coefficients of $r(x)$ using only 3 multiplications. [2 marks]

(b) Multiply 23 by 19 using multiplication *à la russe* ("Russian style"). [4 marks]

(c) Explain the difference between *average-case analysis* and *amortized analysis*. [4 marks]

6. (a) Explain the difference between Monte Carlo and Las Vegas randomized algorithms. [1 mark]

(b) Explain the difference between biased and unbiased Monte Carlo algorithms for a decision problem (a problem where the only answers are YES or NO). [2 marks]

(c) Explain how randomization can be used to ensure that quicksort runs in $\Theta(n \log n)$ expected time regardless of the input. [2 marks]

Additional work pages

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