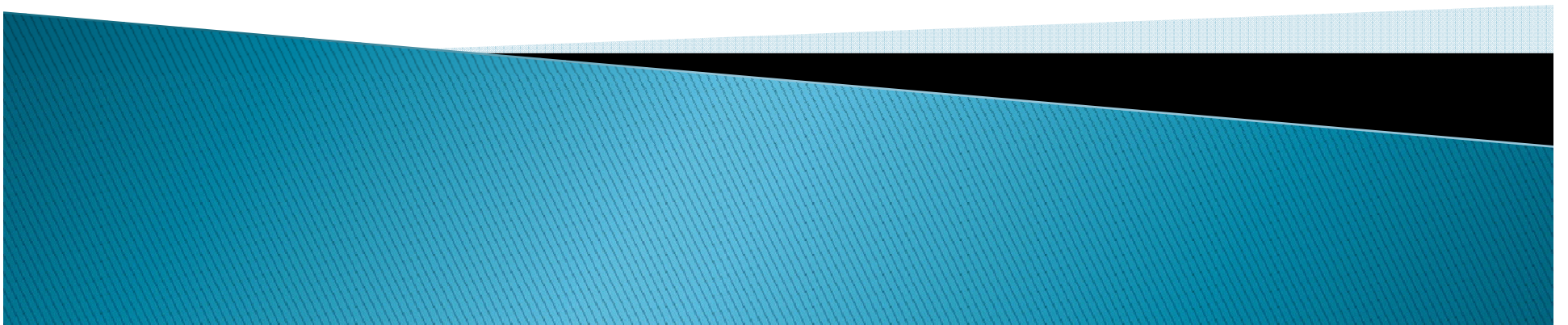


Computer Science 210

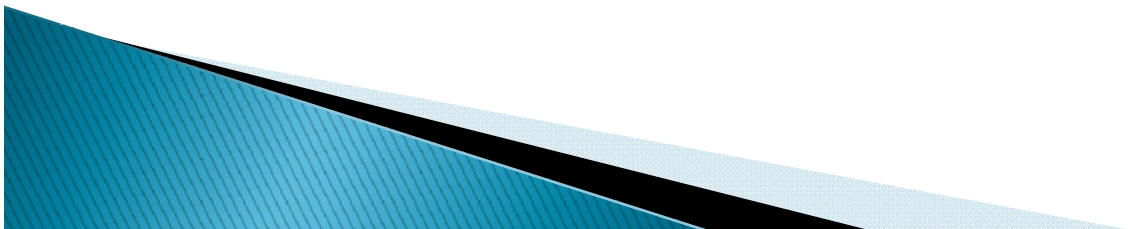
Revision

Revision



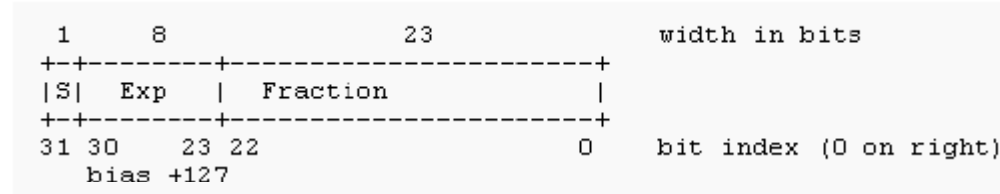
Exam overview

- ▶ Will be 3 parts:
 - Data representations
 - Assembly languages
 - C++ programming languages
- ▶ Probably will be all multiple choice questions.
- ▶ This slides will concentrate on the first part of Data representations.
- ▶ Assembly and C++ programming parts should be revised by looking at pass year exams and test (download link provided).



The IEEE formats 754:

1. Single precision(32bits)



An example:

Let's encode the decimal number -118.625 using the IEEE 754 system.

We need to get the sign, the exponent and the fraction.

Because it is a negative number, the sign is "1". Let's find the others.

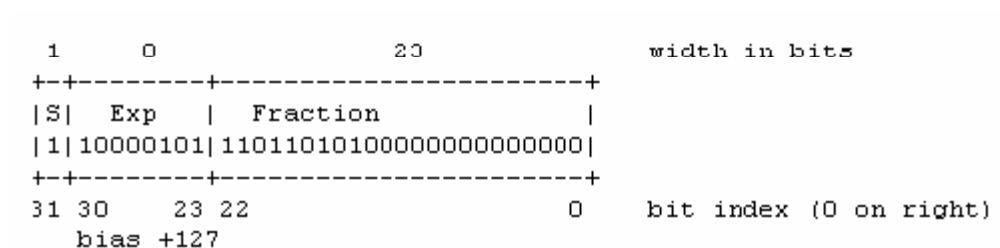
First, we write the number (without the sign) using binary notation. Look at binary numeral system to see how to do it. The result is 1110110.101

Now, let's move the radix point left, leaving only a 1 at its left: $1110110.101 = 1.110110101 \cdot 2^6$

The fraction is the part at the right of the radix point, filled with 0 on the right until we get all 23 bits. That is 11011010100000000000000 .

The exponent is 6, but we need to convert it to binary and bias it (so the most negative exponent is 0, and all exponents are non-negative binary numbers). For the 32-bit IEEE 754 format, the bias is 127 and so $6 + 127 = 133$. In binary, this is written as 10000101 .

Putting them all together:



Data Representation pass exams

1 COMPSCI210 2006 Exam – Data Representation

Information:

Question 8a: What is the output of the following program? [4 marks]

Question 8b: Convert the following ... to ... [4 marks]

Question 9a: Give the floating point representations in hexadecimal of ... and [8 marks]

Question 9b: Evaluate ... [5 marks]

Question 9c: Evaluate ... [5 marks]

2 Mock Exam Questions

2.1 Floating Point Representation

1. Evaluate $3FA00000 + 40600000$

```
1) Adjust exponents and align mantissa
   Start by adjusting the smaller exponent to be equal to the larger exponent
   Take 3FA00000 (0 01111111 01000...0)
   Original Value: E: 01111111 M: 010...0
   Shifted 1 place: E: 10000000 M: 1010...0 Note: "1" is the hidden bit)
2) Add mantissa bits
   1.11000...000
   +0.10100...00
   10.01100...000
3) Normalize result:
   Right shift-> Increase the exponent by 1
   Exponent = 10000000 + 1
   Mantissa = 1.00110...0
4) Check result: OK.
Answer = 0 10000001 00110...0
```

2. Evaluate 40600000 - 3FA00000

```
1) Adjust exponents and align mantissa
   Start by adjusting the smaller exponent to be equal to the larger exponent
   Take (3FA00000) 0 01111111 01000...0
       Original Value: E: 01111111 M: 010...0
       Shifted 1 place: E: 10000000 M: 1010...0 Note: "1" is the hidden bit)
2) Add mantissa bits
   1.11000
  -0.101...00
   1.001
3) Normalize result: OK
4) Check result: OK.
Answer = 0 10000000 0010...0
```

3. Evaluate 40600000 * 3FA00000

```
1) Sign = 0 XOR 0 = 0
2) Multiply mantissa (don't forget the hidden bit)
   1.11
  * 1.01
  -----
   1 11
   00 0
  -----
  111
  1000 11 = 10.0011
3) Add exponents
   10000000 + 01111111 - 01111111 = 10000000
4) Normalize the result
   Right shift by 1, increase the exponent by 1
   Exponent = 10000000 + 1
   Mantissa = 1.00011
5) Check result: OK.
Answer = 0 10000001 0001100...0
```

Pass year exams

3 Past Years Exam Questions on Data Representation

3.1 Binary Fraction

4. Convert the following decimal fraction into a binary fraction. (105SC 2003 Test)

0.8125

Answer = 0.1101_2

5. Convert the following hexadecimal fraction into binary fraction, and then into decimal fraction.

1A.C0 (105FT 2003 Exam)

Answer = 26.75

6. Convert the following decimal fraction into a binary fraction. (105SC 2002 Test)

0.59375

Answer = 0.10011_2

7. Convert the decimal number 33.3 to binary, giving a result with fraction accurate to 4 binary digits (105SC 2002 Exam)

Answer: $0.30 = 0.01001 = 0.0101$ to four bits.

result = 100001.0101

8. Convert the following ratio first into a decimal fraction, then into a 4-bit binary fraction. [2 marks] 13/16 (105SC 2000 Exam)

Answer = 0.1101

9. Convert the following decimal fractions into 4 digit octal fractions, and then into 12 bit binary fractions.

a) 0.75

b) 0.333333 (105SC 1996 Test)

a) 0.110 000 000 000

b) 0.010 101 010 101

Pass year exams

3.2 Floating Point Representation

10. A 32-bit IEEE floating point number consists of 1 sign bit, 8 exponent bits and 23 mantissa bits. Given that +7.5 is represented (in hexadecimal) as 40F00000, give the binary digits (in hexadecimal) of the same representations of -30. (105FT 2003 Exam)

Answer = C1F00000

11. A 32-bit IEEE-754 floating point number is represented in hexadecimal as C0400000.

What is this value in decimal notation? (105SC 2003 Test)

Answer = C0400000 in binary is 1100 0000 0100 0000 0000 etc Regrouping: 1 10000000 100000000... Sign 1 so negative Exponent is $128-127 = 1$ In binary, number is $-1.1 \times 2^1 =$
In decimal $-1.5 \times 2 = -3.0$

12. A 32-bit IEEE-754 floating point number consists of 1 sign bit, 8 exponent bits and 23 mantissa/significand bits. What decimal number is represented by 3FA00000? Show your working. [15 marks] (105FT 2003 Test)

Answer = 1.25

13. What is the representation of the decimal number 16.25 as a 32-bit IEEE-754 floating point number?

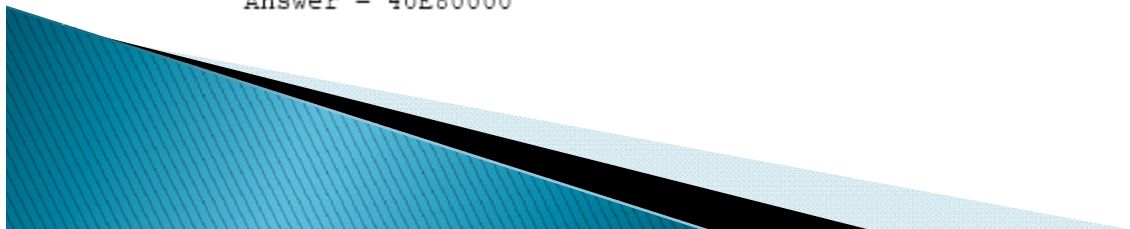
Express your result in hexadecimal. (105SC 2002 Test)

Answer = 41820000

14. A 32-bit IEEE-754 floating point number consists of 1 sign bit, 8 exponent bits and 23 significant bits.

What is the representation of the decimal number 7.25? Express your result in hexadecimal. (105FT 2002 Exam)

Answer = 40E80000



Pass year exams

15. (a) What is the value in decimal of the Java float variable that contains the hexadecimal pattern 0xC0F00000? (105SC 2002 Exam)

(b) The IEEE754 floating point standard format reserves the exponent pattern 0xFF for a value that is not a number. What is the bit pattern of the largest positive value of a float variable? (Give your answer in hexadecimal.) What value does this represent in decimal? (You may give an approximate value for the decimal representation, using a power of 2, e.g. 2^{10} .)

(c) The following calculations are performed using values stored in Java float variables. What values will result and what is the name for what has happened? (Be sure to give the full name.) (SC 2002 Exam)

a) $2^{100} * 2^{100}$

result value:

name:

b) $2^{-100} * 2^{-100}$

result value:

name:

Answer:

a) 0xC0F00000 = 1100 0000 1111 0000 0000 0000 0000 0000 = 1 10000001 1110000....

sign = 1, negative

exponent = (128+1) - 127 = 2

fraction = 1.111

binary value = $-1.111 \times 2^2 = -111.1 =$ decimal value = -7.5

b) 0 11111110 11111111... = 0111 1111 0111 1111 = Bit pattern in hex: 0x7F7FFFFF

Value in decimal: $1.1111111111..._{10} \times 2^{126} \sim 2 \times 2^{126} = 2^{127}$

c) (i) $2^{100} * 2^{100}$

result value: NaN or positive infinity, name: exponent overflow

(i) $2^{-100} * 2^{-100}$

result value: 0 (zero), name: (exponent) underflow

16. A 32-bit IEEE-754 floating-point number consists of 1 sign bit, 8 exponent bits and 23 significant bits.

Given that +3.75 is represented, in hexadecimal, as 40700000, give the floating-point representations in hexadecimal of 30 and 1.875. [10 marks] (105FT 2001 Test)

Answer = 3FF00000

Pass year exams

17. A 32-bit IEEE-754 floating-point number consists of 1 sign bit, 8 exponent bits and 23 significand bits. Given that +1.75 is represented, in hexadecimal, as 3FE00000, give the floating-point representations in hexadecimal of 14 and 0.4375. [8 marks] (105FT 2000 Test)

Answer = 41600000

Answer = 3EE00000

18. A 32-bit IEEE-754 floating point number consists of 1 sign bit, 8 exponent bits and 23 significand bits. What decimal number is represented by 40B00000? Show your working. (105SC 2000 Test)

Answer = 5.5

19. A 32-bit IEEE-754 floating point number consists of 1 sign bit, 8 exponent bits and 23 significand bits. What is the representation of the decimal number 8.75? Express your result in hexadecimal. [3 marks] (105SC 2000 Exam)

20. A 32-bit IEEE-754 floating point number consists of 1 sign bit, 8 exponent bits and 23 significand bits. Given that +5.6 is represented, in hexadecimal, as 40B33333, give the first 12 binary digits of the representations of +0.7 and -2.8. [4 marks]

Explain why the answer asks for only the first few bits. [2 marks] (105SC 1999 Test)

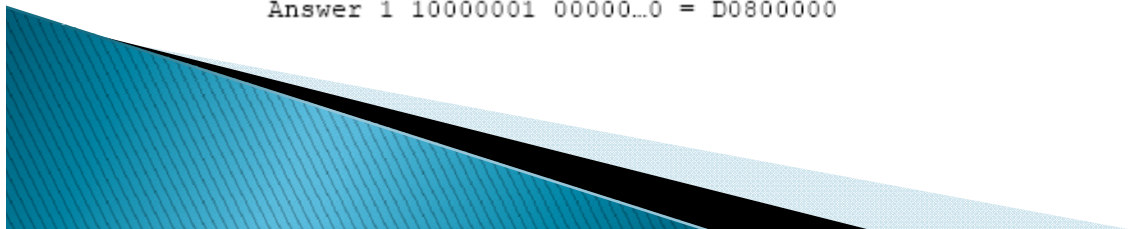
Answer = C0333333

21. A 32-bit IEEE format floating point number has the bit pattern 40000000 (base 16) or in binary and showing its component fields 0 10000000 000000000000000000000000.

a) (i) Explain why these bits represent the value 2.0.

b) (ii) Hence, or otherwise, obtain the representation of -4.00. (105SC 1999 Exam)

Answer 1 10000001 00000...0 = D0800000



Pass year exams

22. A 32-bit IEEE format floating point number consists of 1 sign bit, 8 exponent bits and 23 significant bits.

(i) Given that +1.35 is represented, in hexadecimal, as 3FACCCCD, explain why +1.85 is represented by 3FECCCCD. [6 marks]

(ii) Hence, or otherwise, obtain the representation of -1.60. (105SC 1998 Exam)

Answer

$$1.35 = (1.0 + \text{significant}) * 2^{(127-127)} = (1.0 + \text{significant}) * 1$$

$$\text{Then, } 1.85 = 1.35 + 0.5 = (1.0 + \text{significant} + 0.5) * 1$$

Therefore, we need to change the value of significant, but keep the sign and exponent bits unchanged.

$$\text{Significant} = 010\ 1100\ 1100\ 1100\ 1100\ 1100\ 1101 + 100000\dots0000$$

$$= 110\ 1100\ 1100\ 1100\ 1100\ 1100\ 1101$$

$$\text{Therefore, } 1.85 = 0011\ 1111\ 1110\ 1100\ 1100\ 1100\ 1100\ 1100\ 1101 = 3FECCCCD$$

(ii) -1.60

$$1.6 = 1.35 + 0.25$$

Therefore, we need to change the value of significant, but keep the sign and exponent bits unchanged.

$$\text{Significant} = 010\ 1100\ 1100\ 1100\ 1100\ 1100\ 1101 + 010\ 0000\dots0000$$

$$= 100\ 1100\ 1100\ 1100\ 1100\ 1100\ 1101$$

$$\text{Therefore, } -1.60 = 1011\ 1111\ 1100\ 1100\ 1100\ 1100\ 1100\ 1100\ 1101 = BFCCCCD$$

23. A 32-bit IEEE floating point number consists of 1 sign bit, 8 exponent bits and 23 significant bits.

a) Given that +0.45 is represented, in hexadecimal, as 3EE66666, give the first 12 binary digits of the representations of +0.225 and -1.8. [4 marks]

b) Explain why the preceding question asks for only the first few bits. [2 marks] (105SC 1998 Test)

Answer

0.225 is half of 0.45, so the exponent is reduced by 1, the answer is 3E666666.

-1.8 is $4 * 0.45$, with a sign change, the exponent must have 2 added, the answer is BFE66666.

Pass year exams

24. A 32-bit IEEE floating point number consists of 1 sign bit, 8 exponent bits and 23 significand bits. Given that +0.65 is represented, in hexadecimal, as 3F266666, give the first 12 binary digits of the representations of +0.325 and -1.3. Explain why the answer asks for only the first few bits. (105SC 1997 Test)

Answer:

0.325 is half of 0.65, and -1.3 is twice 0.65, with sign change, so the only differences are in the sign and exponent. The exponent in 0.65 is 011 1111 0 and the first few bits of significand 010 0110.

For 0.325 the exponent will be 1 less, or 011 1110 1, the sign still 0, to give the first 12 bits as 0 00111110 1 010.

For -1.30 the exponent will increase by 1 to 011 1111 1 and the sign will change, giving the first 12 bits as 1 011 1111 1010

25. A 32-bit IEEE floating point number consists of 1 sign bit, 8 exponent bits and 23 significand bits. Given that 0.4 is represented, in hexadecimal, as 3ECCCCCD, give the first 12 binary digits of the representations of +0.8 and -0.2. Explain why the answer asks for only the first few bits. (105SC 1996 Test)

Answer:

Last part first - a sign change changes only the sign bit, and a multiplication or division by a power of 2 just changes the exponent, so the other bits of the significand do not change. The 12 most significant bits of the representation of 0.4 are 0011 1110 1100, with the underlined ones being the exponent. 0.8 is twice 0.4, so the exponent is just increased by 1 giving 0011 1111 0100. 0.2 is half of 0.4, so the exponent decreases by 1. The negative sign means that the sign bit changes from 0 to 1, giving a final 1011 1110 0100.

