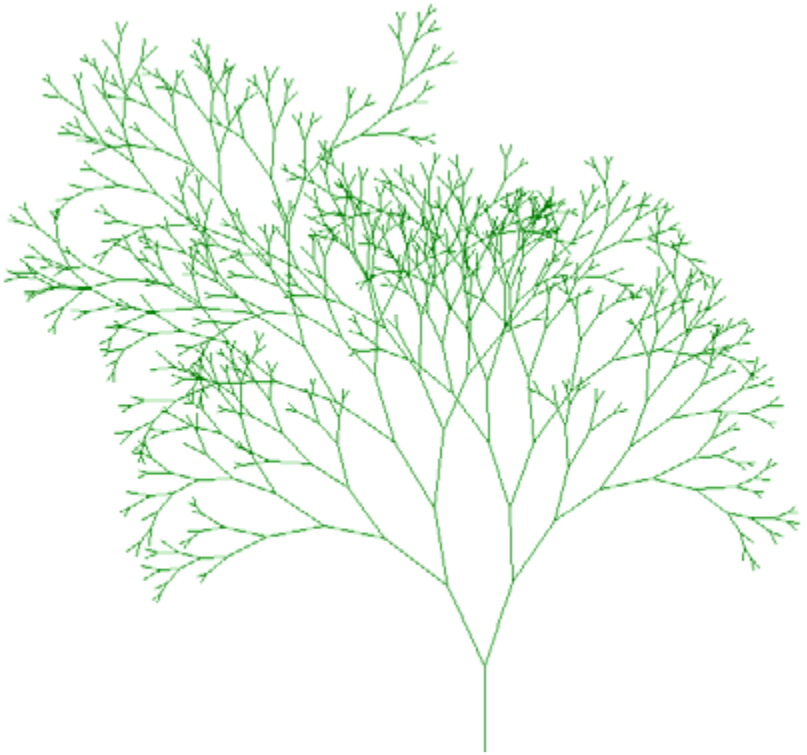
COMPSCI 105 S1 2017
Principles of Computer Science

21-Recursion(2)

Agenda & Readings

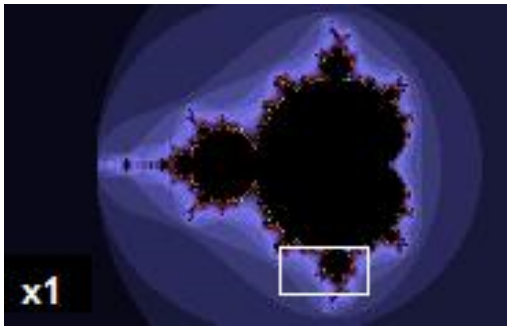
- Agenda
  - Introduction
  - Using the Python Turtle
    - Recursive Drawing
    - Drawing a Spiral
    - Drawing a KochUp
    - Drawing a C-curve
    - Call Tree

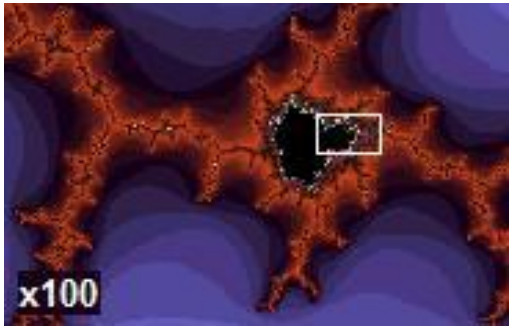


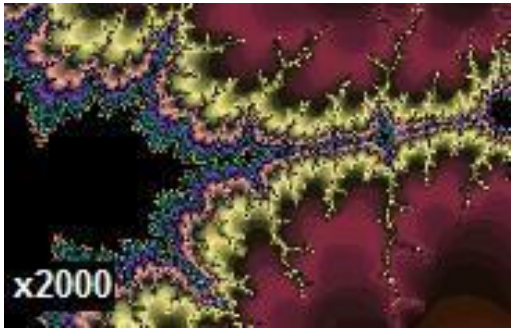
- Reference:
  - Textbook:
    - Problem Solving with Algorithms and Data Structures
      - Chapter 4 – Recursion

Self-similarity

- A fractal is a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole
  - This a property is called self-similarity

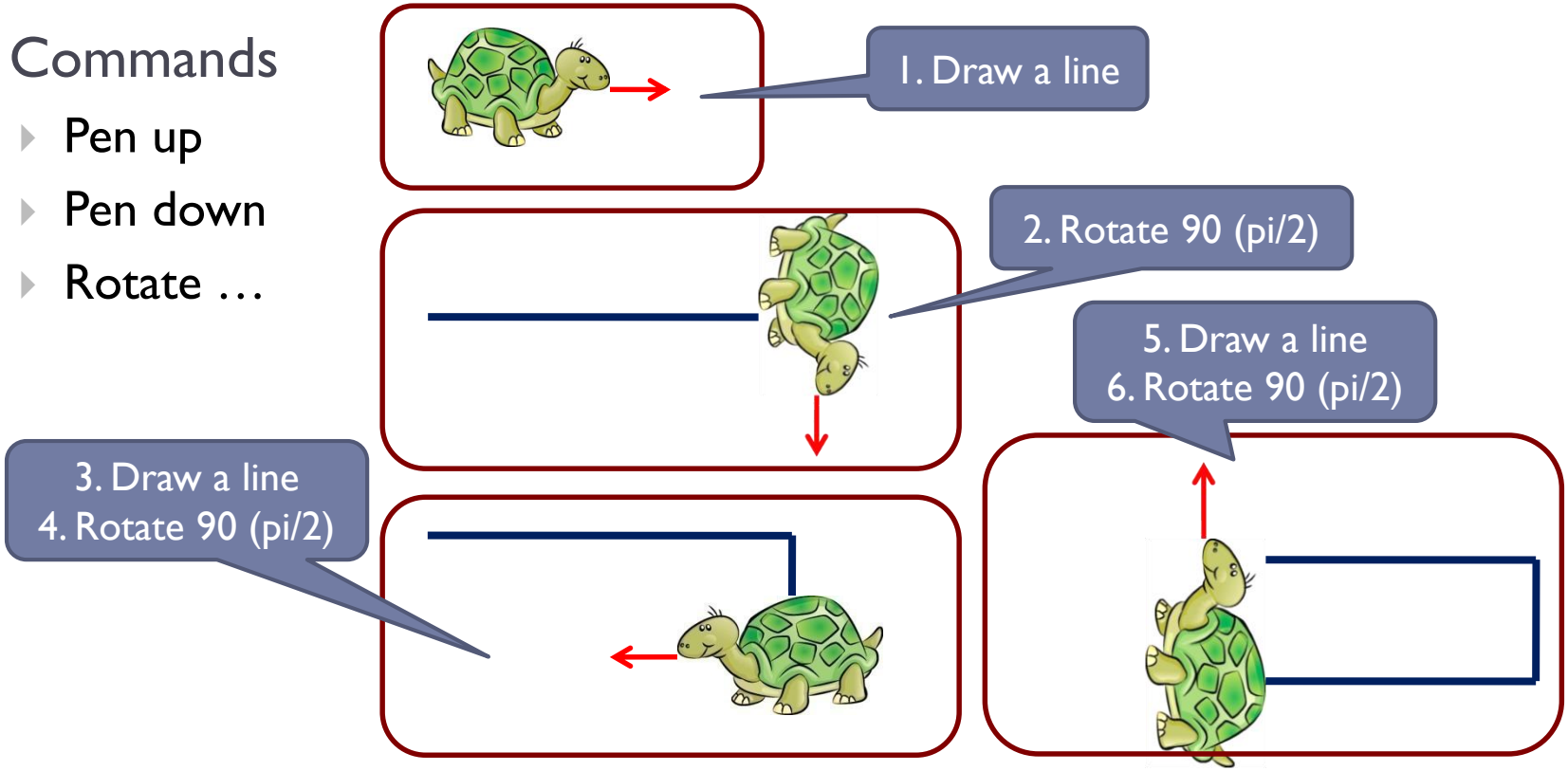






Turtle Class

- Can be drawn using a "Turtle"
  - Named after Logo programming language
  - Pen location used to draw on the screen
  - Commands
    - Pen up
    - Pen down
    - Rotate …



Turtle Class

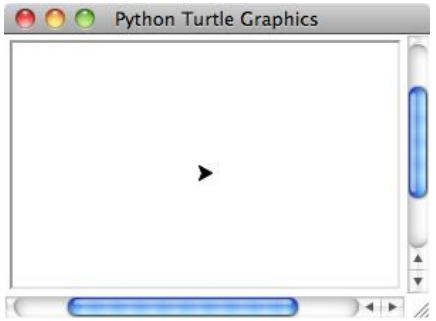
- Steps:
  - Import the turtle module which defines the Turtle and the Screen types `import turtle`
  - Create and open a window `my_win = turtle.Screen()`
    - The window contains a canvas, which is the area inside the window on which the turtle draws
  - Create a turtle object which can move forward, backwards, turn left, turn right, the turtle can have its tail up/down
  - If the tail is down, the turtle draws as it moves `tess = turtle.Turtle()`
    - The width and colour of the turtle tail can be changed
  - When the user clicks somewhere in the window, the turtle window closes and execution of the Python program stops

`turtle.exitonclick()`

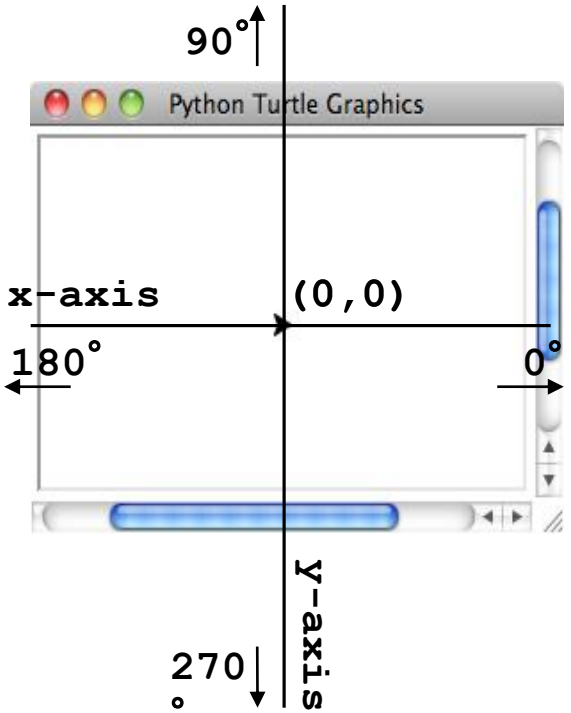
Turtle Class

- Instantiate a Turtle object:

```
tess = turtle.Turtle()
```



- The turtle appears as an icon
  - Initial position: (0, 0)
  - Initial direction: East (0°)
  - Colour: black
  - Line width: 1
  - pixel Pen: down (ready to draw)



Methods

- forward(distance) – move the turtle forward
- backward(distance) – move the turtle backwards
- right(angle) – turn the turtle clockwise
- left(angle) – turn the turtle anti-clockwise
- up() – puts the turtle tail/pen up, i.e., no drawing
- down() – puts the turtle tail/pen down, i.e., drawing

Methods

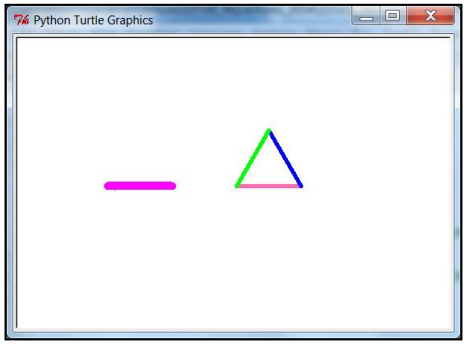
- pencolor(colour_name) – changes the colour of the turtle's tail
- heading() – returns the direction in which the turtle is pointing
- setheading(angle) – set the direction in which the turtle is pointing
- position() – returns the position of the turtle
- goto(x, y) – moves the turtle to position x, y
- speed(number) – set the speed of the turtle movement

Drawing Examples

- Examples:

```
my_win = turtle.Screen()
tess = turtle.Turtle()
tess.pencolor("hotpink")
tess.pensize(5)
tess.forward(80)
tess.pencolor("blue")
tess.left(120)
tess.forward(80)
tess.pencolor("green")
tess.left(120)
tess.forward(80)
```

```
tess.pensize(10)
tess.pencolor("magenta")
tess.left(120)
tess.right(180)
tess.up()
tess.forward(80)
tess.down()
tess.forward(80)
my_win.exitonclick()
```



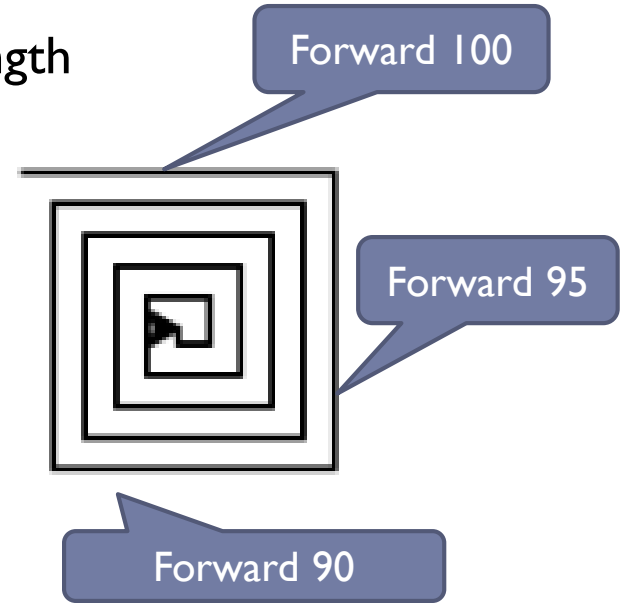
Recursive Drawing

- In the previous section, we looked at some problems that were easy to solve using recursion
- In this section, we will look at a couple of examples of using recursion to draw some interesting pictures
  - Drawing a spiral recursively
  - Drawing a Koch Up shape
  - Drawing a C Curve

Drawing a Spiral

- Define the draw_spiral function
  - The base case is when the length of the line is zero or less
  - The recursive step: (length of the line > 0; len > 0)
    - Instruct the turtle to go forward by len units, and
    - Turn right 90 degrees
    - Call draw_spiral again with a reduced length

```
draw_spiral(my_turtle,100)
draw_spiral(my_turtle,95)
draw_spiral(my_turtle,90)
 …
draw_spiral(my_turtle,0)
```



Drawing a Spiral

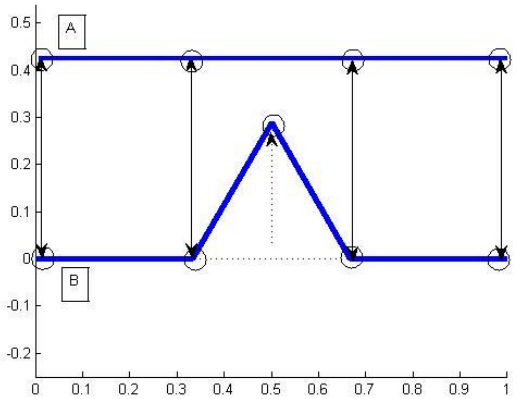
- The draw_spiral function
- Steps:
  - Define the draw_spiral function
  - Create a turtle
  - Call the recursive function

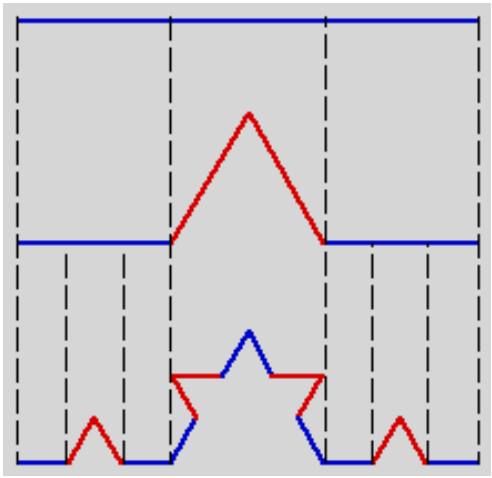
```
def draw_spiral(my_turtle, line_len):
    if line_len > 0:
        my_turtle.forward(line_len)
        my_turtle.right(90)
        draw_spiral(my_turtle,line_len-5)
```

```
import turtle
 ...
my_win = turtle.Screen()
my_turtle = turtle.Turtle()
draw_spiral(my_turtle,100)
turtle.exitonclick()
```

Drawing a KochUp

- Idea: recursively applying a simple rule to each of the triangles sides
- Examples:

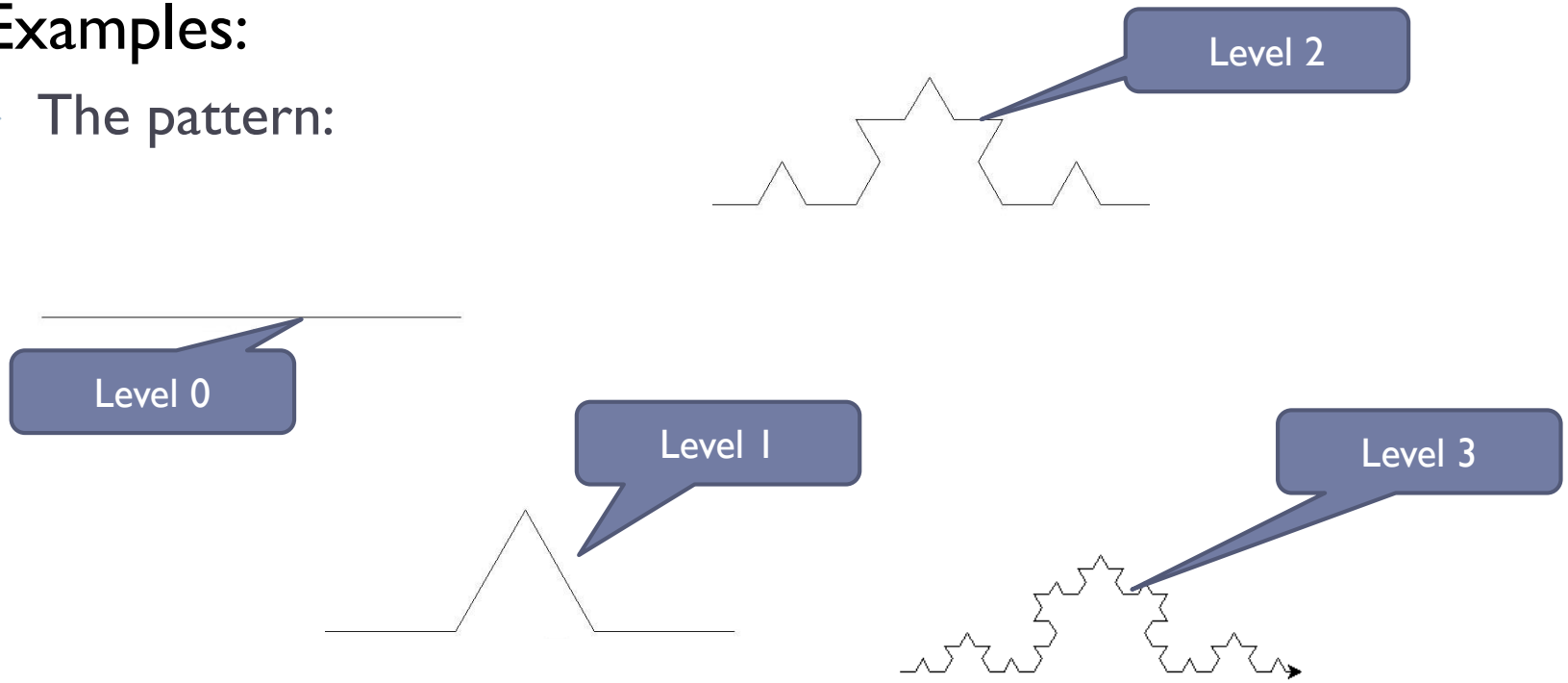




  - The pattern:
    - Cut the side (line_len) into 3 equal parts (line_len/3)
    - Replace the center part with 2 sides of length line_len/3, such that it forms a spike
    - Repeat the process for each of the 4 sides, until the length of each side is smaller than a given value.

Drawing a KochUp

- Idea: recursively applying a simple rule to each of the triangles sides
- Examples:
  - The pattern:



Drawing a KochUp

- Define the draw_kochup function
  - The base case is when the level is zero or less:
    - Instruct the turtle to go forward by line_len units
  - The recursive step: (level> 0)
    - Call draw_kochup again with a reduced length and a reduced level
    - Turn left 60 degrees (anti-clockwise)
    - Call draw_kochup again with a reduced length and a reduced level
    - Turn right 120 degrees
    - Call draw_kochup again with a reduced length and a reduced level
    - Turn left 60 degrees (anti-clockwise)
    - Call draw_kochup again with a reduced length and a reduced level

Drawing a KochUp

- The draw_kochup function

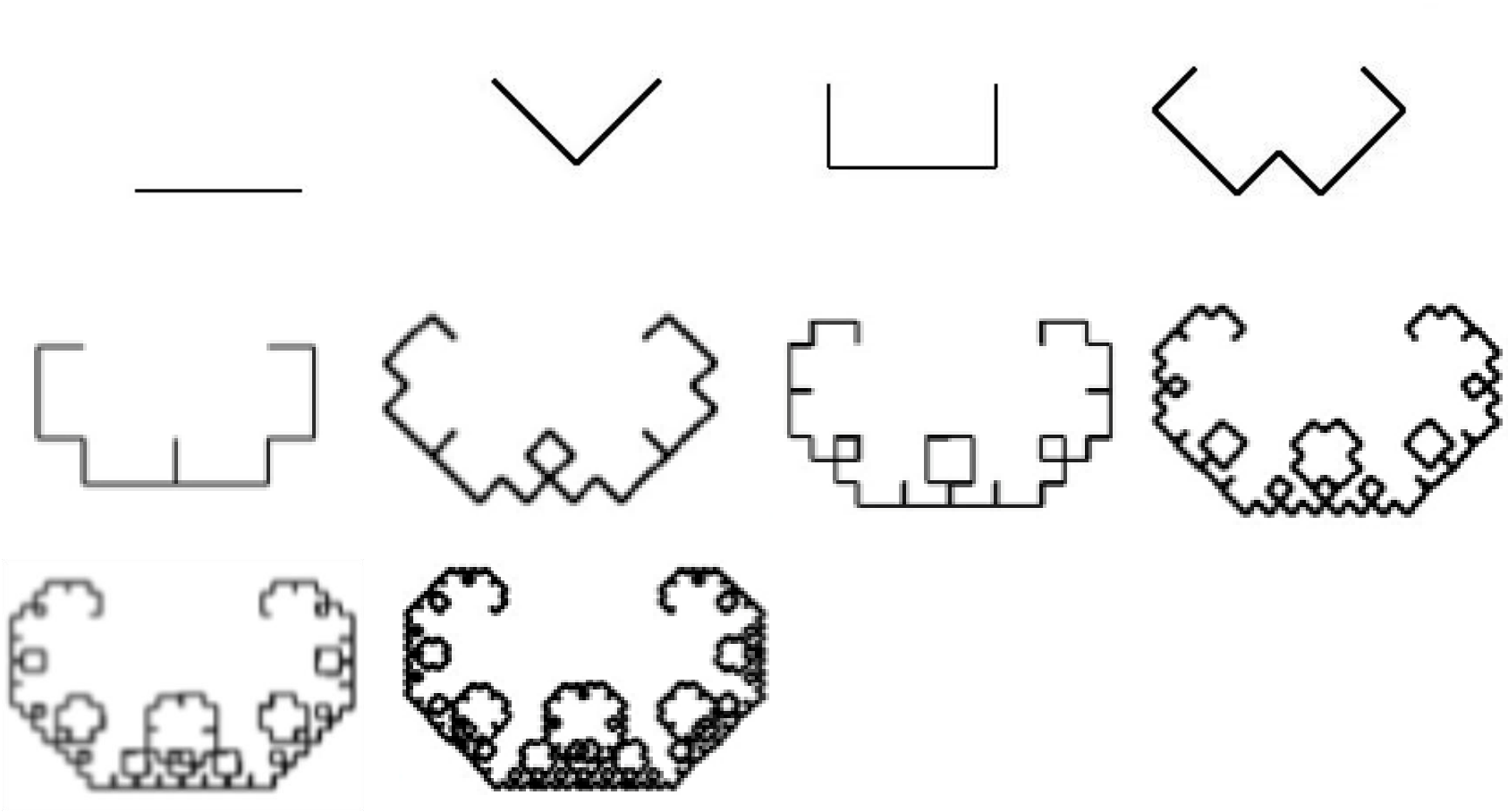
```
def draw_kockup(my_turtle, level, line_len):
    if level > 0:
        draw_kockup(my_turtle, level-1, line_len/3)
        my_turtle.left(60)
        draw_kockup(my_turtle, level-1, line_len/3)
        my_turtle.right(120)
        draw_kockup(my_turtle, level-1, line_len/3)
        my_turtle.left(60)
        draw_kockup(my_turtle, level-1, line_len/3)
    else:
        my_turtle.forward(line_len)
```

Drawing a C Curve

- A C-curve is a fractal pattern
  - A level 0 C-curve is a vertical line segment
  - A level 1 C-curve is obtained by bisecting a level 0 C-curve and joining the sections at right angles
  - …
  - A level N C-curve is obtained by joining two level N - 1 C-curves at right angles

Drawing a C Curve

- Examples:

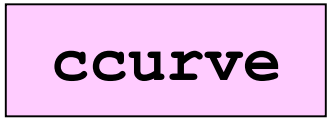


Drawing a C Curve

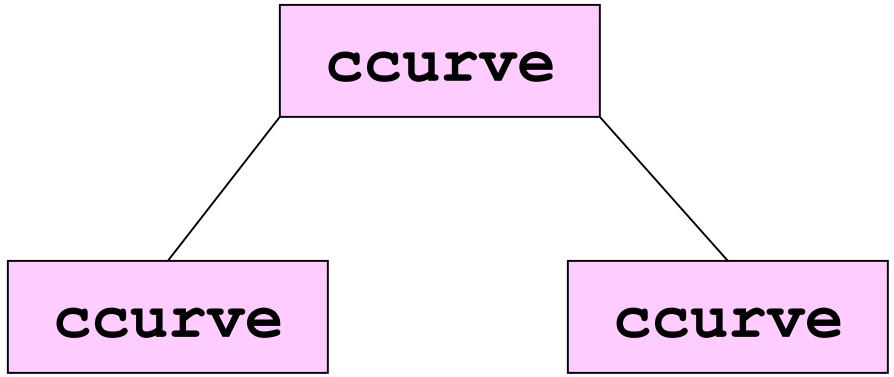
- Define the draw_c_curve function
  - The base case is when the level is zero or less:
    - Instruct the turtle to go forward by line_len units
  - The recursive step: (level> 0)
    - Turn right 45 degrees
    - Call draw_c_curve again with a reduced length and a reduced level
    - Turn left 90 degrees
    - Call draw_c_curve again with a reduced length and a reduced level
    - Turn right 45 degrees

Drawing a C Curve

- A call tree diagram shows the number of calls of a function for a given argument value
  - ccurve(0) uses one call, the top-level one

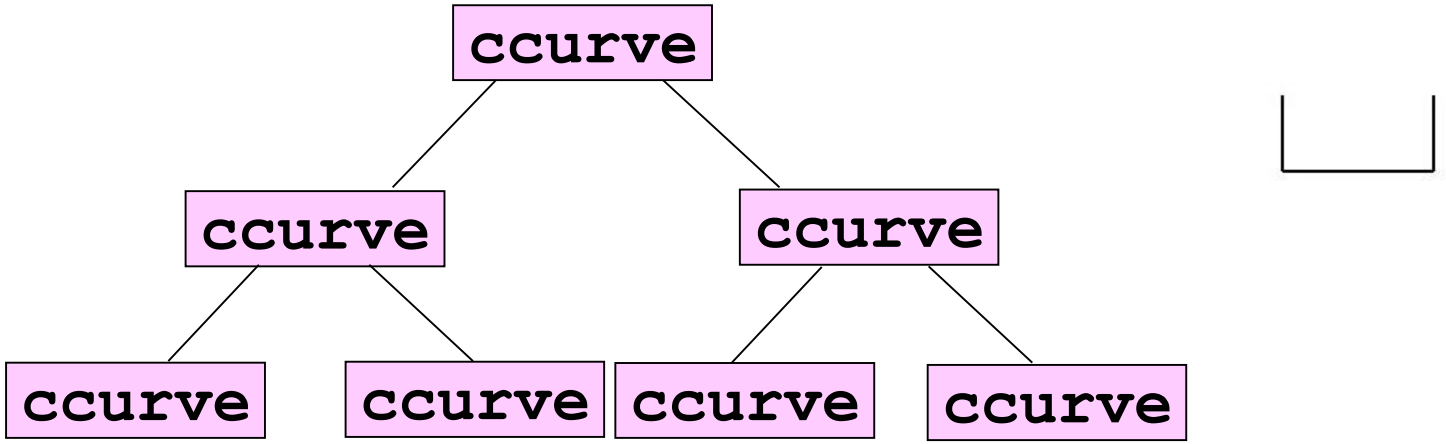


  - ccurve(1) uses three calls, a top-level one and two recursive calls



Drawing a C Curve

- A call tree diagram shows the number of calls of a function for a given argument value
  - ccurve(2) uses 7 calls, a top-level one and 6 recursive calls



  - ccurve(n) uses 2n+1 calls, a top-level one and 2n recursive calls

Drawing a C Curve

- The ccurve function

```
def ccurve(my_turtle, level, line_len):
    if level > 0:
        my_turtle.right(45)
        ccurve(my_turtle, level-1, line_len/2)
        my_turtle.left(90)
        ccurve(my_turtle, level-1, line_len/2)
        my_turtle.right(45)
    else:
        my_turtle.forward(line_len)
```

Summary

- The application of recursion is practiced by using Python Turtles