COMPSCI 105 S1 2017
Principles of Computer Science

20-Recursion(1)

Agenda & Readings

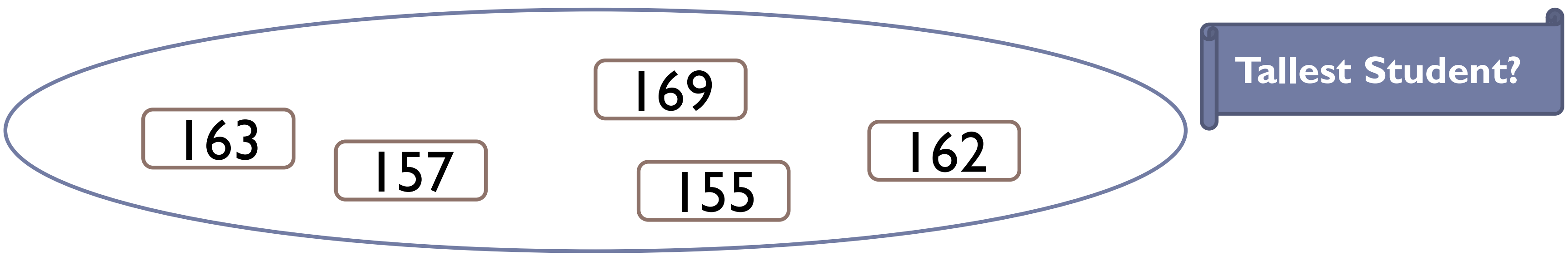
- Agenda
  - What is recursion?
  - Recursive solutions, examples:
    - The Factorial of N
    - Box Trace Example
    - Write a String Backward
    - Tail Recursion
- Reference:
  - Textbook:
    - Problem Solving with Algorithms and Data Structures
      - Chapter 4 – Recursion

Definitions

- Problem Domain:
  - The space consisting of all elements for which the problem is solved
  - Examples: An array of integers, all people in this room, the days of the month, all "All Blacks" rugby games
- Problem Size:
  - The number of elements of the problem domain
  - Examples: An array with N elements, the number of people in this room, a list of N cities, the number of games played by the "All Blacks"
- GOAL: Design algorithms to solve problems!

Iterative Algorithm

- Algorithm which solves a problem by applying a function to each element of the problem domain
  - Example: Find the tallest person in a group of N>0 students

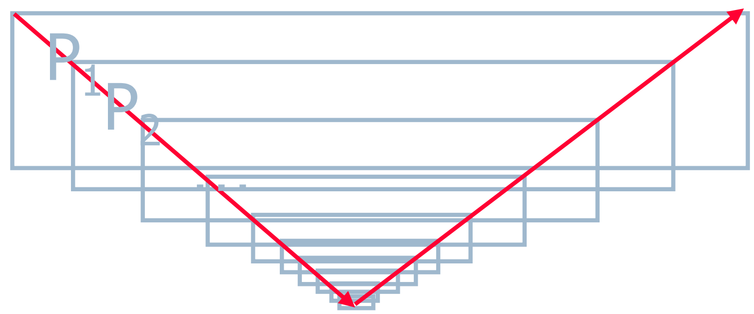



```
def Student FindTallestStudent(Group_of_students)
    TallestStudent = Take any student from group;
    Repeat until nobody left
        Take next student from group
        If student is taller than TallestStudent then
            TallestStudent = student
    Return TallestStudent
```

20.1 Introduction

Recursion

- Recursion is a powerful problem solving technique where a problem is broken into smaller and smaller identical versions of itself until a smaller version is small enough that it has an obvious solution



A **base case** is a special case whose solution is known

- Note:
  - Complex problems can have simple recursive solutions It is an alternative to iteration (involves loops)
  - BUT: Some recursion solutions are inefficient and impractical!

Recursion

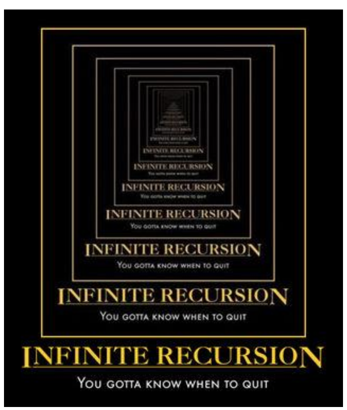
- Recursion involves a function calling itself
  - Example: Find the tallest person in a group of N>0 students

FindTallestStudent([162,155,169,157]) unknown	taller	169
FindTallestStudent([155,169,157]) unknown	taller	169
FindTallestStudent([169,157]) unknown	taller	169
FindTallestStudent([157])		157

```
def FindTallestStudent(Group of students)
    If only one student in group
        return this student
    else
        StudentA = Take any student from group
        StudentB = FindTallestStudent(Remaining Group)
        return the taller person of StudentA and studentB;
```

Recursive Solutions

- Properties of a recursive solution
  - A recursive method calls itself
  - Each recursive call solves an identical, but smaller, problem
  - A test for the base case enables the recursive calls to stop
    - Base case: a known case in a recursive definition
  - Eventually, one of the smaller problems must be the base case (problem not allowed to become smaller than base case)



Recursive Solutions

- Four questions for constructing recursive solutions
  - How can you define the problem in terms of a smaller problem of the same type?
  - How does each recursive call diminish the size of the problem?
  - What instance of the problem can serve as the base case?
  - As the problem size diminishes, will you reach this base case?

Example – Calculate the Sum

- Get the sum by:
  - Taking the first number + the sum of the rest of the list

recursive_sum([2, 1, 5, 6]) 2+ 14

recursive_sum([1, 5, 6]) 1+ 12

recursive_sum([5, 6]) 5+ 11

recursive_sum([6]) 6+ 6

0

```
def recursive_sum(num_list):
    if len(num_list) == 0:
        return 0
    return num_list[0] + recursive_sum(num_list[1:])
```

Example – – Bad Recursion 1

- Problem:
  - Compute the sum of all integers from 1 to n

```
def bad_sum(n):
    return n + bad_sum(n-1)
```

No base case!!!

Example – – Bad Recursion 2

- Problem:
  - If n is odd compute the sum of all odd integers from 1 to n, if it is even compute sum of all even integers

```
def bad_sum(n):
    if (n == 0):
        return 0
    return n + bad_sum(n-2)
```

Base case cannot be reached!!!

Definition

- Problem
  - Compute the factorial of an integer n >=0
- An iterative definition of factorial(n)
  - If n = 0, factorial(0) = 1
  - If n > 0, factorial(n) = n * (n-1) * (n-2) * … * 1
- Examples:
  - 4! = 4 * 3 * 2 * 1 = 24
  - 7! = 7 * 6 * 5 * 4 * 3 * 2 * 1 = 5040

```
def factorial(n):
    result = 1
    for i in range(n, 1, -1):
        result = result * i
    return result
```

Definition

- A recurrence relation
  - A mathematical formula that generates the terms in a sequence from previous terms
    - factorial(n) = n * [(n-1) * (n-2) * … * 1]
    - factorial(n) = n * factorial(n-1)
  - A recursive definition of factorial(n)
    - $\text{factorial}(n) = \begin{cases} 1, \text{ if } n = 0 \\ n * \text{factorial}(n-1), \text{ if } n > 0 \end{cases}$

```
def fact (n):
    if n <= 0:
        return 1
return n * fact(n-1)
```

Four Criteria

- fact(n) satisfies the four criteria of a recursive solution
  - fact(n) calls itself
  - At each recursive call, the integer whose factorial to be computed is diminished by 1
  - The methods handles the factorial 0 differently from all other factorials, where fact(0) is 1
    - Thus the base case occurs when n is 0
  - Given that n is non-negative, item 2 of this assures that the computation will always reach the base case

```
def fact (n):
    if n <= 0:
        return 1
return n * fact(n-1)
```

Box Trace

- A systematic way to trace the actions of a recursive method
  - Create a new box for each recursive method call
  - Describe how return value is computed
  - Provide link to box (or boxes) for recursive method calls within the current method call
  - Each box corresponds to an activation record
    - Contains a method's local environment at the time of and as a result of the call to the method

The local environment contains:
Value of argument, local variables, return value, address of calling method, …, etc.

Box Trace

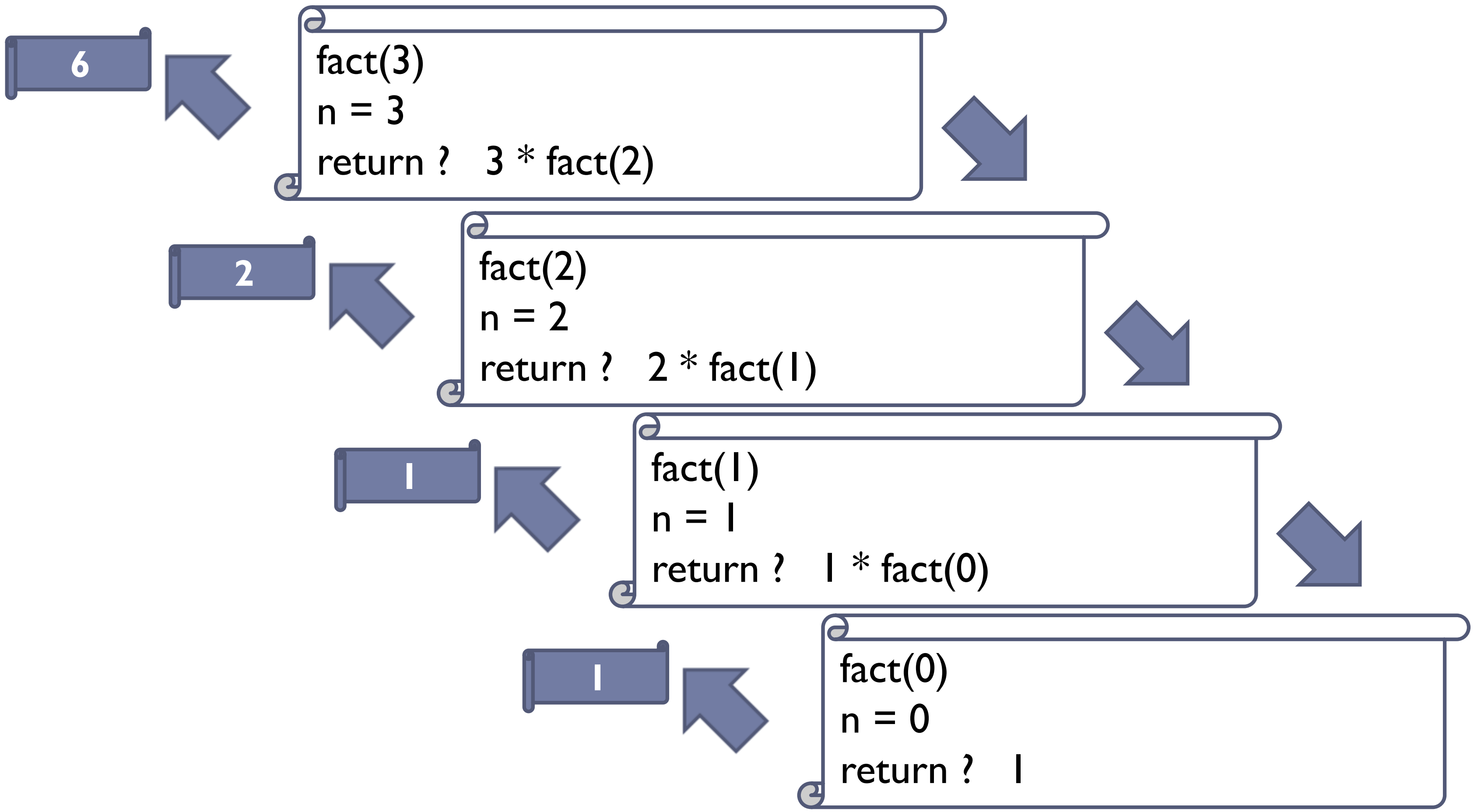
- A method's local environment includes:
  - The method's local variables
  - A copy of the actual value arguments
  - A return address in the calling routine
  - The value of the method itself

fact(3)
n = 3
A: fact(n-1) = ?
return ?

20.4 Box Trace

Box Trace

- Example



Exercise 1

- Draw a call tree of the following method call: fact(4)

Definition

- Problem:
  - Given a string of characters, write it in reverse order
- Recursive solution:
  - Each recursive step of the solution diminishes by 1 the length of the string to be written backward
  - Base case:
    - Write the empty string backward
- Examples:

```
print(writeBackward("cat"))
```

```
tac
```

```
print(writeBackward("cat"))
```

```
tac
```

Implementation

- Two approaches
  - writeBackward(s)

call method recursively for the string minus the **last** character

```
if the string s is empty:
    Do nothing – base case
else:
    write the last char of s
    writeBackward(s minus its last char)
```

  - writeBackward2(s)

call method recursively for the string minus the **first** character

```
if the string s is empty:
    Do nothing – base case
else:
    writeBackward2(s minus its first char)
    write the first char of s
```

Implementation

- Example

writeBackward("cat")
s = “cat”, write t
writeBackward("ca")

t

writeBackward("ca")
s = “ca”, write a
writeBackward("c")

ta

writeBackward("c")
s = “c”, write c
writeBackward("")

tac

writeBackward("")
s = “”
return

20.5 Writing a String Backward

Implementation

- Example

tac

writeBackward2("cat")
s = “cat”
writeBackward2(“at"), write c

ta

writeBackward2(“at")
s = “at”
writeBackward2(“t"), write a

t

writeBackward2(“t")
s = “t”
writeBackward2(""), write t

writeBackward2("")
s = “”
return

Summary

- A recursive algorithm passes the buck repeatedly to the same function
- Recursive algorithms are well-suited for solving problems in domains that exhibit recursive patterns
- Recursive strategies can be used to simplify complex solutions to difficult problems