GRAPHICS GROUP

# Computer Graphics: Clipping and Viewport Transformation

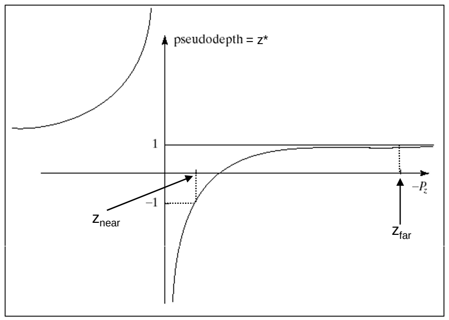
Part 2 – Lecture 3



## Today's Outline

- Pseudodepth
- Clipping
- Viewport Transformations






## PSEUDODEPTH



## Perspective Transformation

- Requirements:
  1. x and y values must be scaled by same factor as derived in perspective projection equations
  2. z values must maintain depth ordering (monotonic increasing)
  3. z values must map: $-\mathbf{z}_{\mathbf{near}}$ → -1 and $-\mathbf{z}_{\mathbf{far}}$ → +1, view volume → NDC cube
- So we need a transformation that given a point *P* results in a transformed point *P'* such that $P'_x$ and $P'_y$ meet requirement 1 and $f(p_z)$ meets requirements 2 and 3:

$$P' = \left( \frac{-near}{p_z} p_x, \quad \frac{-near}{p_z} p_y, \quad f(p_z) \right)$$

- We have already found such a transformation:
  - Multiply *P* with $\mathbf{M}_{\mathbf{proj}}$
  - Convert result to ordinary coordinates (perspective division)

# Perspective Transformation (cont'd)

- Perspective division:
  $P_{homog} = (x, y, z, w) \rightarrow P_{ord} = (x/w, y/w, z/w)$
- Thus, for these transformed points,

$$P^* = \mathbf{P}\,P = \begin{pmatrix} near\ x \\ near\ y \\ a\,z + b \\ -z \end{pmatrix} \qquad P^*_{near} = \mathbf{P}\,P_{znear} = \begin{pmatrix} near\ x \\ near\ y \\ -a\,near + b \\ near \end{pmatrix} \qquad P^*_{far} = \mathbf{P}\,P_{zfar} = \begin{pmatrix} near\ x \\ near\ y \\ -a\ far + b \\ far \end{pmatrix}$$

- Using $a = -\dfrac{far + near}{far - near}, \quad b = \dfrac{-2\ far\ near}{far - near}$

Ordinary form of the <u>x and y components</u>:
$z_{near}\,x / z = (-z_{near}/z)\,x$
$z_{near}\,y / z = (-z_{near}/z)\,y$

Ordinary form of the <u>z components</u>:
$(a\,z + b) / (-z)$
$(-a\,z_{near} + b) / z_{near} = -1.0$
$(-a\,z_{far} + b) / z_{far} = +1.0$
*Check this out!*

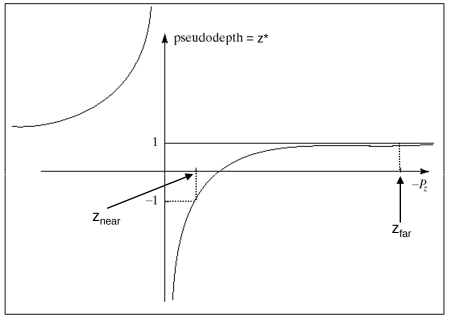


# Pseudodepth

- Transformed z* not linear function of z

$$z^* = \frac{az + b}{-z} = \frac{\left(-\dfrac{far + near}{far - near}\right) z + \dfrac{-2\,far * near}{far - near}}{-z}$$

$$z^* = \frac{(far + near)\,z + 2\,far * near}{(far - near)\,z}$$

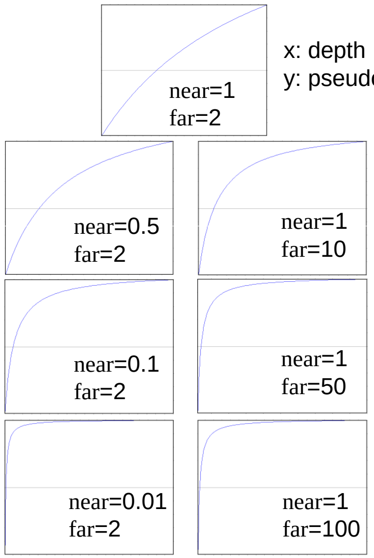



- This is OK (sort of) because z* meets our 2 requirements:
  1. monotonic increasing, and
  2. $z^* = -1$ for $z = z_{near} = -near$ and $z^* = +1$ for $z = z_{far} = -far$
- But: can cause z-buffer precision problems! (z-buffer values are usually 32 bit integers)



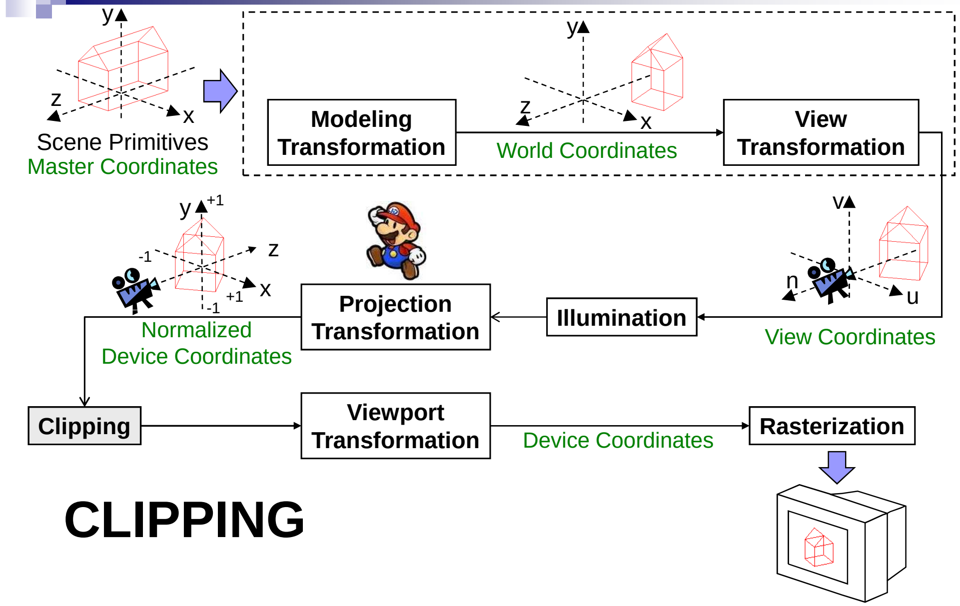
# Problems of Pseudodepth

- Points closer to near plane have highest pseudodepth resolution
- Points closer to far plane have lowest pseudodepth resolution
- Never use near = 0
  → division by zero
- Avoid very small near and very large far
  → resolution too low for points that are further away




x: depth
y: pseudodepth

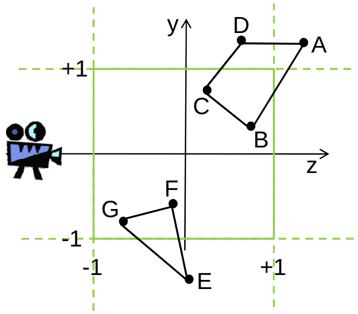





# CLIPPING

# Clipping

- Determine which lines are in the canonical view volume (using NDC)
- Outside of the view volume is given by:
  $p_x < -1$ , $p_x > +1$ , $p_y < -1$ , $p_y > +1$ ,
  $p_z < -1$ , $p_z > +1$
  (→ **clip planes**)
- Each line is either…
  1. completely inside
     → **trivial accept**
  2. completely outside
     → **trivial reject**
  3. Partially in the view volume
     → need to find out which part is inside



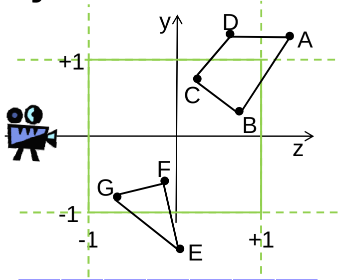

Trivial accept for:
CB and GF

Trivial reject for:
DA

Partially visible:
AB, CD, EF and EG

# Trivial Accept and Reject Tests

- For each point, check if it is outside of left (L), right (R), bottom (B), top (T), near (N) and far (F) clip plane
- Create table with **outcodes**:
  1 if point is outside, 0 if inside
- **Trivial reject** of a line PQ:
  = P and Q outside of the same clip plane
  = outcodes for same plane both 1
  = `(outcode P & outcode Q)!=0`
- **Trivial accept** of a line PQ:
  = both endpoints inside of all clip planes
  = all outcodes 0
  = `(outcode C | outcode D)==0`

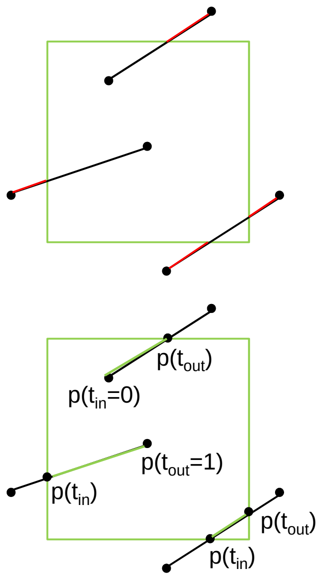



| | L | R | B | T | N | F |
|---|---|---|---|---|---|---|
| A | 0 | 0 | 0 | **1** | 0 | **1** |
| B | 0 | 0 | 0 | 0 | 0 | 0 |
| C | 0 | 0 | 0 | 0 | 0 | 0 |
| D | 0 | 0 | 0 | **1** | 0 | 0 |
| E | 0 | 0 | **1** | 0 | 0 | 0 |
| F | 0 | 0 | 0 | 0 | 0 | 0 |
| G | 0 | 0 | 0 | 0 | 0 | 0 |



# Nontrivial Clipping

- **Idea**: find intersection point of line with each clipping plane
- Each line can only enter and leave the view volume once
- For each intersection X of line PQ with a clipping plane:
  - If P outside, then clip off PX
  - If P inside, the clip off XQ
- We use parametric line equation
  $p(t) = p_0 + t(p_1 - p_0)$ with $0 <= t <= 1$
- Clipping by finding $t_{in}$ and $t_{out}$ parameter values for line segment in view volume

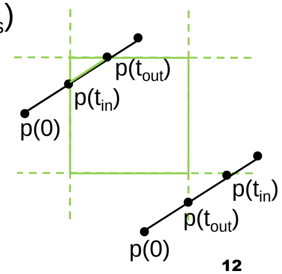





# Liang-Barsky Clipping Algorithm

Clip a line from point $p_0$ to $p_1$, represented as $p(t) = p_0 + t(p_1 - p_0)$

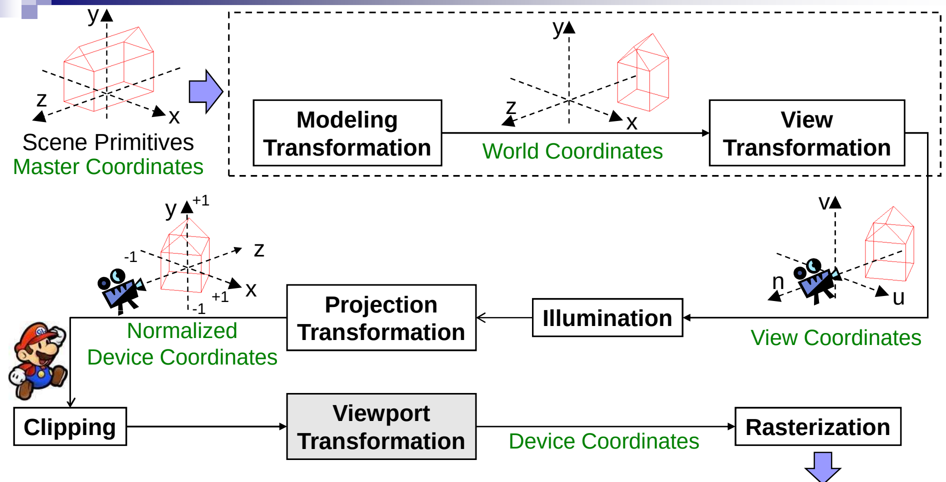
1. Perform trivial reject and accept tests, stop if trivial
2. Initialize $t_{in}$=0 and $t_{out}$=1
3. For each halfspace {x>– 1, x<+1, y>–1, y<+1, z>–1, z<+1} do
   1. Compute $t_{cross}$ where (extended) line crosses halfspace
   2. If entering half-space then $t_{in} = \max(t_{in}, t_{cross})$
      else $t_{out} = \min(t_{out}, t_{cross})$
   3. Stop if $t_{in} > t_{out}$
4. if $t_{in} > t_{out}$ then line is outside viewing volume
   else $p_0 = p(t_{in})$ and $p_1 = p(t_{out})$

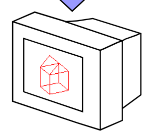


# Clipping with Homogeneous Coordinates

- OpenGL actually performs clipping before perspective division, i.e. using homogeneous coordinates
- One reason: perspective division only necessary for vertices that are in view volume
- Differences in clipping algorithm:
  - Point p is outside of view volume if
    $p_x / p_w < -1 \iff p_x < -p_w \iff p_x + p_w < 0$
    Other planes:
    $p_x - p_w > 1$, $p_y + p_w < 0$, $p_y - p_w > 0$ , $p_z + p_w < 0$ , $p_z - p_w > 0$
  - Compute $p_x(t)$, $p_y(t)$, $p_z(t)$, **<u>and</u> $p_w(t)$**



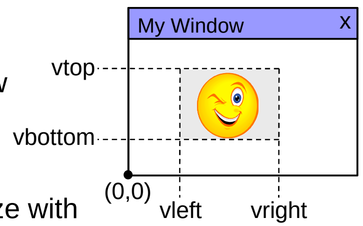



# VIEWPORT TRANSFORMATIONS



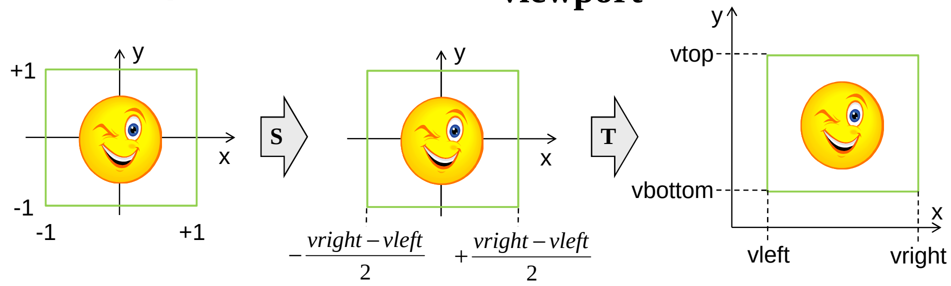
# Viewport Transformation

- Mapping from Normalized Device Coordinates (NDC) to device coordinates (DC) aka viewport coordinates
- For NDC: $x,y,z \in (-1, +1)$
- For DC: $x \in$ (vleft, vright), $y \in$ (vbottom, vtop), $z \in$ (0, maxz)
  - x and y are 2D window coordinates
  - vleft, vright, vbottom, vtop are the boundaries of the viewport in the window
  - maxz depends on type used for depth buffer values (e.g. uint32)
  - In OpenGL: set viewport position and size with `glViewport(x, y, width, height);`




- NDCs are multiplied with **viewport matrix $\mathbf{M}_{viewport}$** which maps NDC boundaries onto viewport boundaries

# Viewport Matrix $\mathbf{M}_{viewport}$




$$\mathbf{M}_{viewport} = \mathbf{T}\,\mathbf{S} = \begin{pmatrix} 1 & 0 & 0 & \frac{vright+vleft}{2} \\ 0 & 1 & 0 & \frac{vtop+vbottom}{2} \\ 0 & 0 & 1 & \frac{maxz}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{vright-vleft}{2} & 0 & 0 & 0 \\ 0 & \frac{vtop-vbottom}{2} & 0 & 0 \\ 0 & 0 & \frac{maxz}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Multiple Viewports

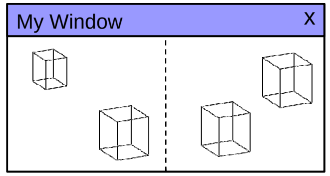
- **Problem:** How to write a GL program that displays multiple views of a scene, each one in a different viewport?
- **Solution: Multiple viewports**
  Multiple views of a scene, e.g., architectural drawing front, side, and top views
  Loop: repeat for each viewport
  1. Set this viewport:
     `glViewport( x, y, width, height );`
  2. Set view projection for this viewport (might be the same for all viewports, if so do this before loop):
     `glOrtho(left, right, bottom, top, zNear, zFar );`
     or other such as `gluPerspective( … );`
  3. Set camera view position and orientation for this viewport
     `gluLookAt(left, right, bottom, top, zNear, zFar );`
     or other such as `glTranslatef( … ); glRotatef( … );`
  4. Draw scene



# Multiple Viewports Code Example

```
// left: perspective
glViewport(0, 0, 100, 100);
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
gluPerspective(yfov, aspect,
   zNear, zFar);
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
// do view transformations…
drawScene();
```

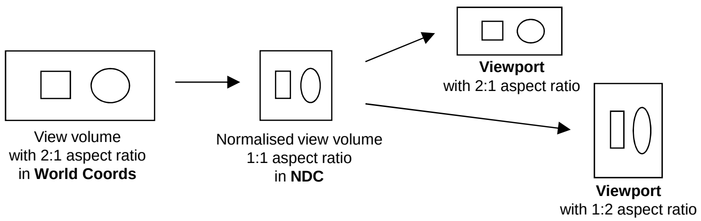
```
// right: orthographic
glViewport(100, 0, 100, 100);
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left, right, bottom,
   top, near, far);
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
// do view transformations…
drawScene();
```






# Aspect Ratio of View Volume and Viewport

- Final pipeline transformation step is viewport transformation
  ```
  glViewport(GLint x, GLint y,
             GLsizei width, GLsizei height);
  ```
  Default viewport is entire drawing window, (0, 0, winWidth, winHeight).
- **Aspect ratio** of view volume and viewport should be same




- **Problem:** How to write a GLUT program that automatically resets the view volume aspect ratio when window (viewport) is resized?



# Aspect Ratio: reshape callback function

**Solution:** in GLUT, use **reshape callback** to adjust viewport and view volume aspect ratio after a **window resize event**

- Register reshape callback function (in main at prog. init.)
  ```
  void reshape(GLsizei width, GLsizei height); // prototype
  glutReshapeFunc( reshape );      // callback registration
  ```
- Define reshape callback function (in main prog. module)
  ```
  // left, right, bottom, top = class member or global variables
  void reshape( GLsizei width, GLsizei height ) {
     glViewport(0, 0, width, height );    // set viewport size
     GLfloat aspect = (GLfloat)width /(GLfloat)height;  //NOT int!
     GLdouble center = (left + right) / 2.0;
     GLdouble newHalfWidth = aspect * (top - bottom) / 2.0;
     left = center - newHalfWidth;  right = center + newHalfWidth;
     glMatrixMode(GL_PROJECTION);  // reset proj matrix
     glLoadIdentity();
     glOrtho(left, right, bottom, top, near, far);
     drawSceneObjects();          // redraw all objects
  }
  ```

## SUMMARY



# Summary

- Pseudodepth
  - Used to normalize z with matrix
  - For small near and large far resolution problems
- Clipping removes lines outside of view volume
  - Trivial accept and reject tests using outcodes
  - Check $t_{in}$ and $t_{out}$ values of parametric line equation
- Viewport Transformation: maps NDCs to DCs using $\mathbf{M}_{viewport}$

References:


- Pseudeodepth: Hill, Chapter 7.4.3, pp. 349-351
- Clipping: Hill, Chapter 7.4.3, pp. 356-361
- Viewport Transformation: Hill, Chapter 7.4.3, p. 361




# Quiz

1. Why isn't it a good idea to use a very small number for near or a very large number for far?
2. How is an outcode table constructed? How is it used for trivial reject/accept?
3. How do we find $t_{in}$ and $t_{out}$ during clipping? How does it help us to clip lines?