

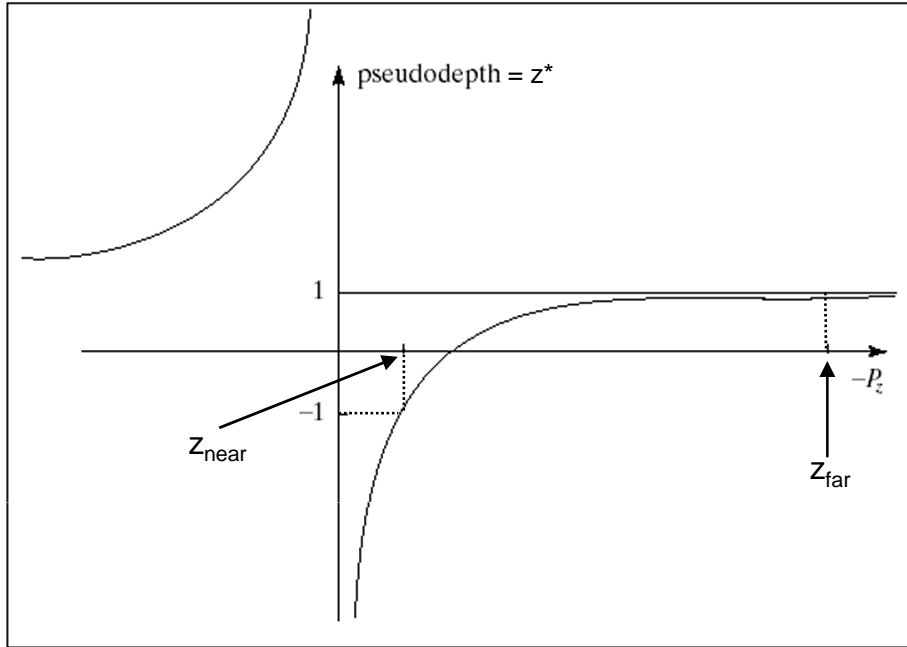
Computer Graphics: Clipping and Viewport Transformation

Part 2 – Lecture 3



Today's Outline

- Pseudodepth
- Clipping
- Viewport Transformations



PSEUDODEPTH

Perspective Transformation

- Requirements:

1. x and y values must be scaled by same factor as derived in perspective projection equations
2. z values must maintain depth ordering (monotonic increasing)
3. z values must map: $-z_{\text{near}} \rightarrow -1$ and $-z_{\text{far}} \rightarrow +1$, view volume \rightarrow NDC cube

- So we need a transformation that given a point P results in a transformed point P' such that

P'_x and P'_y meet requirement 1 and $f(p_z)$ meets requirements 2 and 3:

$$P' = \left(\frac{-near}{p_z} p_x, \frac{-near}{p_z} p_y, f(p_z) \right)$$

- We have already found such a transformation:

- Multiply P with \mathbf{M}_{proj}
- Convert result to ordinary coordinates (perspective division)

Perspective Transformation (cont'd)

- Perspective division:

$$P_{homog} = (x, y, z, w) \rightarrow P_{ord} = (x/w, y/w, z/w)$$

- Thus, for these transformed points,

$$P^* = \mathbf{P} P = \begin{pmatrix} near\ x \\ near\ y \\ a\ z + b \\ -z \end{pmatrix} \quad P^*_{near} = \mathbf{P} P_{z_{near}} = \begin{pmatrix} near\ x \\ near\ y \\ -a\ near + b \\ near \end{pmatrix} \quad P^*_{far} = \mathbf{P} P_{z_{far}} = \begin{pmatrix} near\ x \\ near\ y \\ -a\ far + b \\ far \end{pmatrix}$$

- Using $a = -\frac{far + near}{far - near}$, $b = \frac{-2\ far\ near}{far - near}$

Ordinary form of the x and y components:

$$z_{near} x / z = (-z_{near}/z) x$$

$$z_{near} y / z = (-z_{near}/z) y$$

Ordinary form of the z components:

$$(a z + b) / (-z)$$

$$(-a z_{near} + b) / z_{near} = -1.0$$

$$(-a z_{far} + b) / z_{far} = +1.0$$

} Check this out!

Pseudodepth

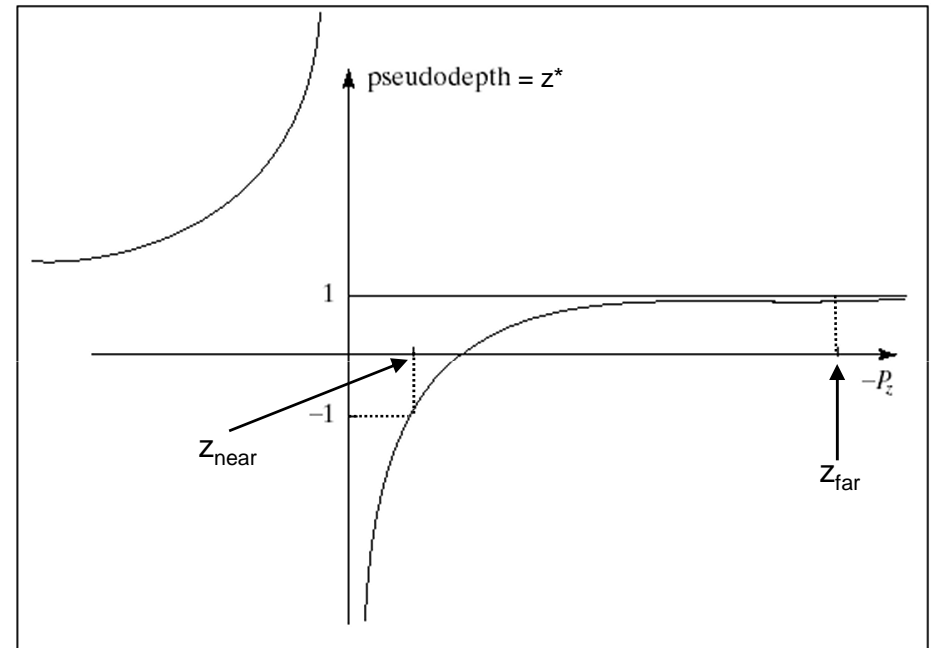
- Transformed z^* not linear function of z

$$z^* = \frac{az + b}{-z} = \frac{\left(-\frac{far + near}{far - near}\right)z + \frac{-2far * near}{far - near}}{-z}$$
$$z^* = \frac{(far + near)z + 2far * near}{(far - near)z}$$

- This is OK (sort of) because z^* meets our 2 requirements:

1. monotonic increasing, and
2. $z^* = -1$ for $z = z_{near} = -near$
and $z^* = +1$ for $z = z_{far} = -far$

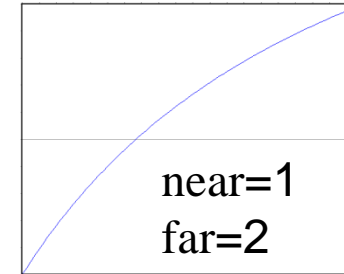
- But: can cause z-buffer precision problems!
(z-buffer values are usually 32 bit integers)



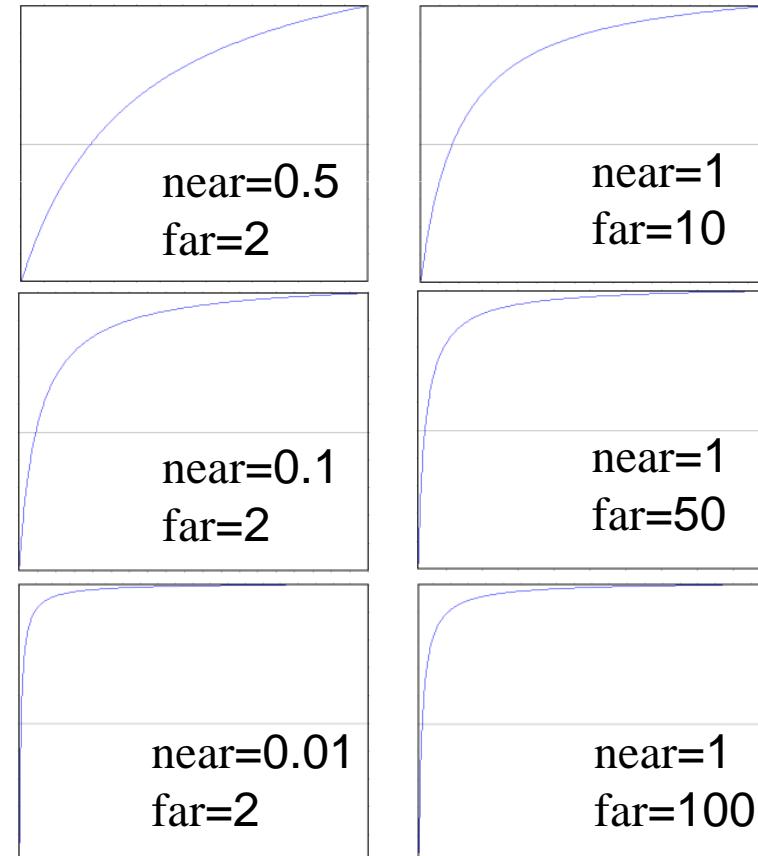
Problems of Pseudodepth

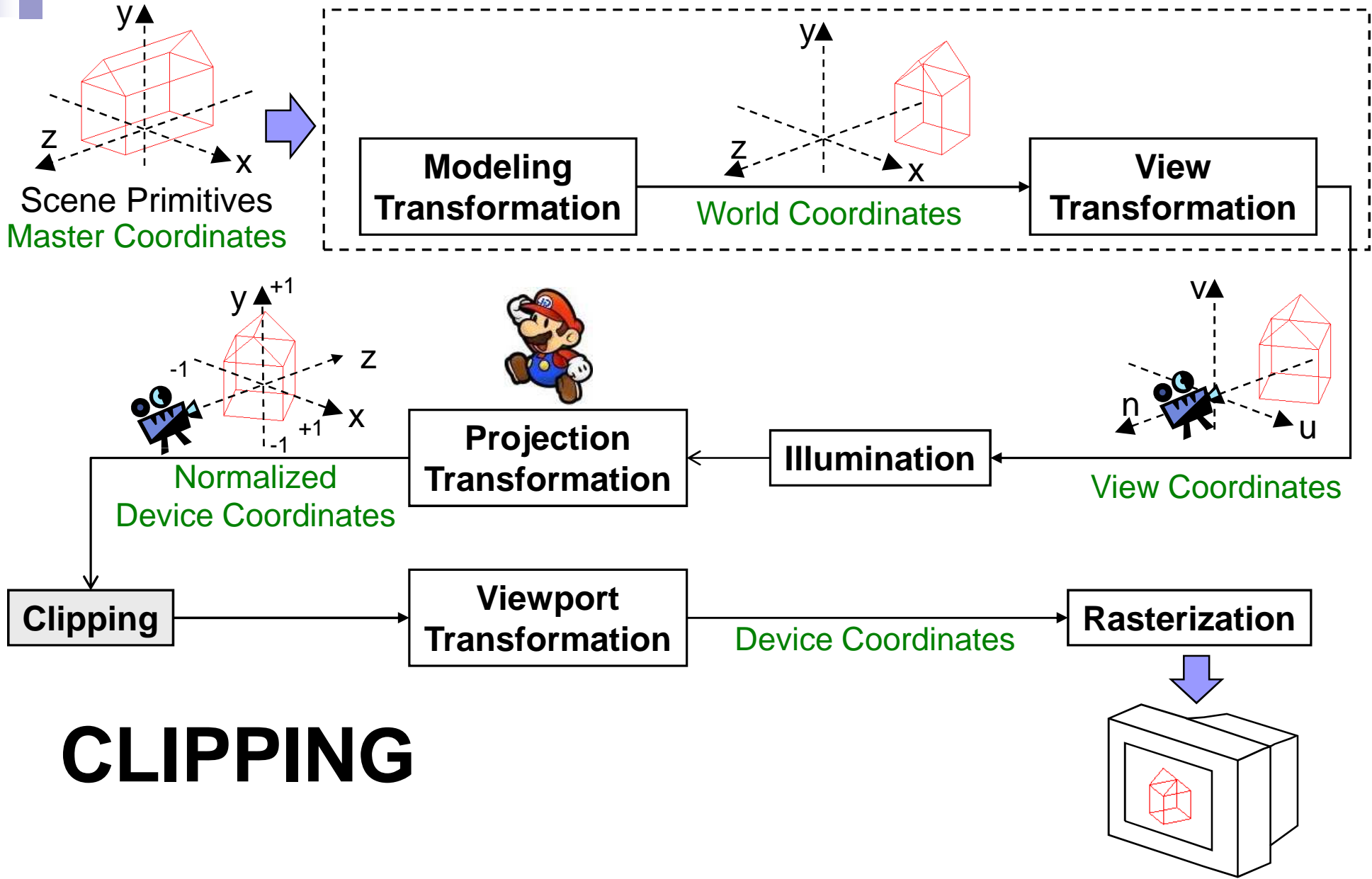
- Points closer to near plane have highest pseudodepth resolution
- Points closer to far plane have lowest pseudodepth resolution

- Never use $\text{near} = 0$
→ division by zero
- Avoid very small near and very large far
→ resolution too low for points that are further away



x: depth
y: pseudodepth

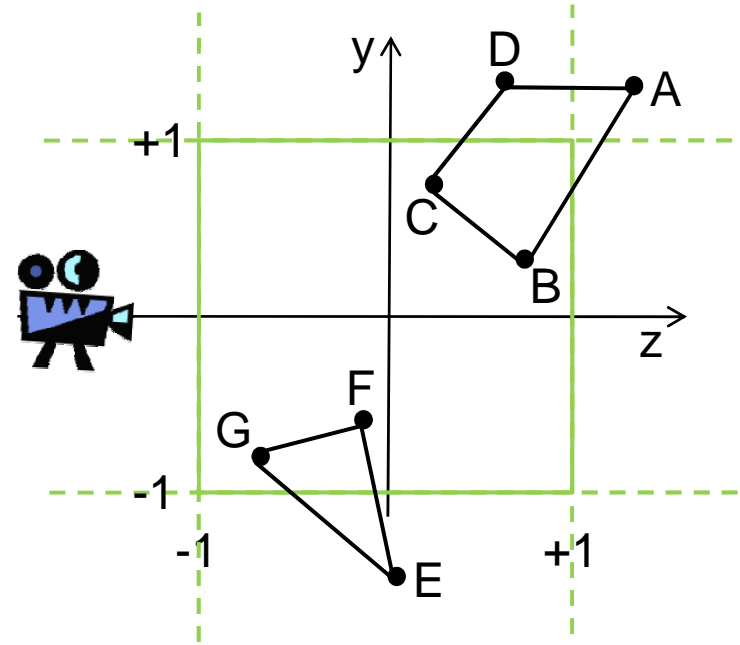




CLIPPING

Clipping

- Determine which lines are in the canonical view volume (using NDC)
- Outside of the view volume is given by:
 $p_x < -1$, $p_x > +1$, $p_y < -1$, $p_y > +1$,
 $p_z < -1$, $p_z > +1$
(→ **clip planes**)
- Each line is either...
 1. completely inside
→ **trivial accept**
 2. completely outside
→ **trivial reject**
 3. Partially in the view volume
→ need to find out which part is inside



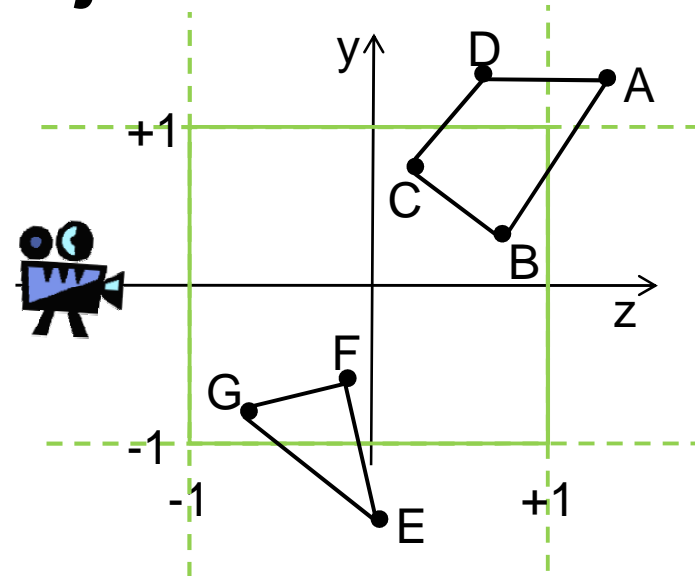
Trivial accept for:
CB and GF

Trivial reject for:
DA

Partially visible:
AB, CD, EF and EG

Trivial Accept and Reject Tests

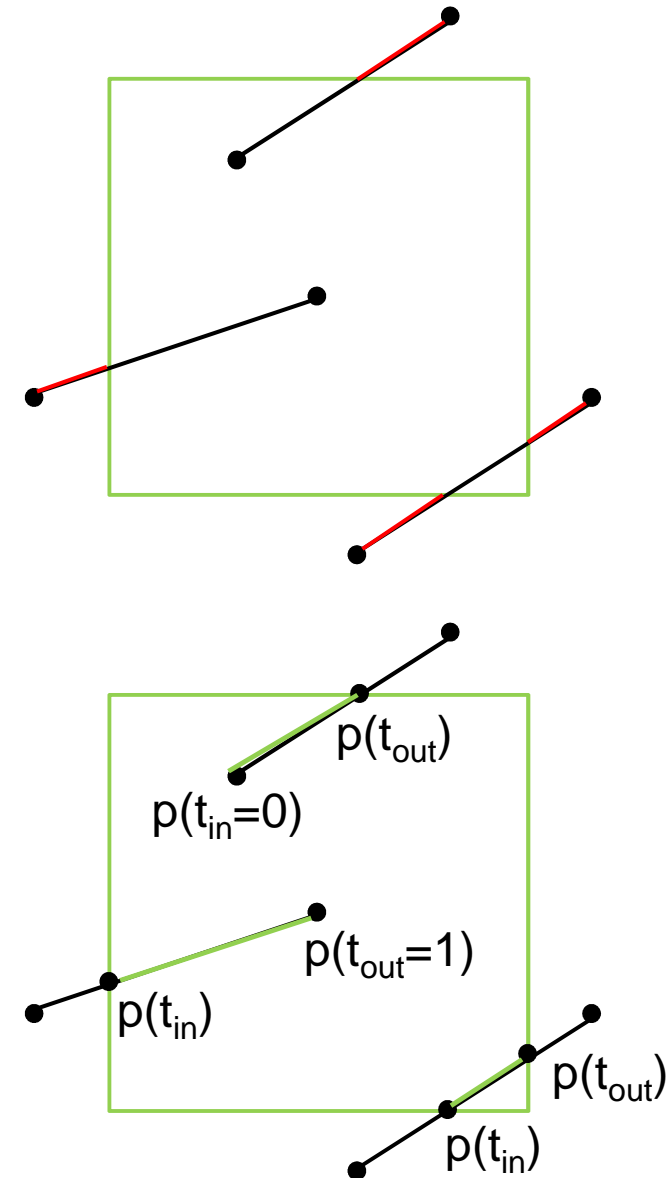
- For each point, check if it is outside of left (L), right (R), bottom (B), top (T), near (N) and far (F) clip plane
- Create table with **outcodes**:
1 if point is outside, 0 if inside
- **Trivial reject** of a line PQ:
= P and Q outside of the same clip plane
= outcodes for same plane both 1
= (outcode P & outcode Q) != 0
- **Trivial accept** of a line PQ:
= both endpoints inside of all clip planes
= all outcodes 0
= (outcode C | outcode D) == 0



	L	R	B	T	N	F
A	0	0	0	1	0	1
B	0	0	0	0	0	0
C	0	0	0	0	0	0
D	0	0	0	1	0	0
E	0	0	1	0	0	0
F	0	0	0	0	0	0
G	0	0	0	0	0	0

Nontrivial Clipping

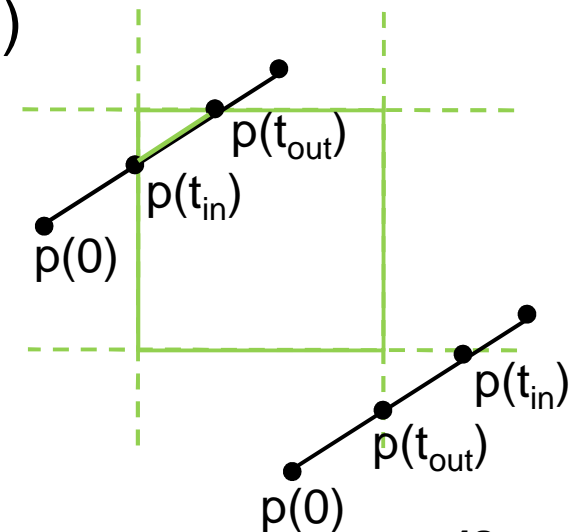
- **Idea:** find intersection point of line with each clipping plane
- Each line can only enter and leave the view volume once
- For each intersection X of line PQ with a clipping plane:
 - If P outside, then clip off PX
 - If P inside, the clip off XQ
- We use parametric line equation $p(t) = p_0 + t(p_1 - p_0)$ with $0 \leq t \leq 1$
- Clipping by finding t_{in} and t_{out} parameter values for line segment in view volume



Liang-Barsky Clipping Algorithm

Clip a line from point p_0 to p_1 , represented as $p(t) = p_0 + t(p_1 - p_0)$

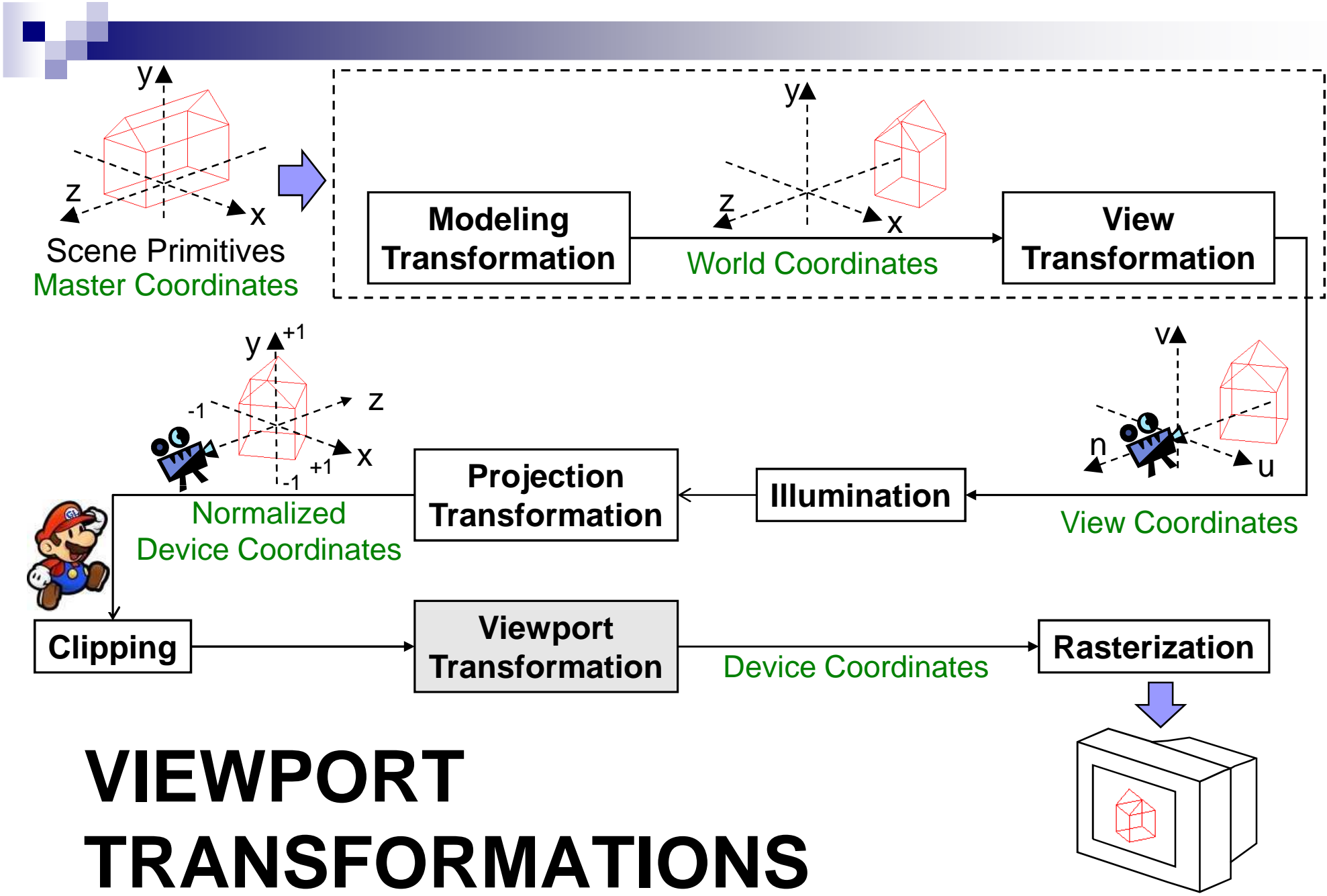
1. Perform trivial reject and accept tests, stop if trivial
2. Initialize $t_{in}=0$ and $t_{out}=1$
3. For each halfspace $\{x > -1, x < +1, y > -1, y < +1, z > -1, z < +1\}$ do
 1. Compute t_{cross} where (extended) line crosses halfspace
 2. If entering half-space then $t_{in} = \max(t_{in}, t_{cross})$
else $t_{out} = \min(t_{out}, t_{cross})$
 3. Stop if $t_{in} > t_{out}$
4. if $t_{in} > t_{out}$ then line is outside viewing volume
else $p_0 = p(t_{in})$ and $p_1 = p(t_{out})$



Clipping with Homogeneous Coordinates

- OpenGL actually performs clipping before perspective division, i.e. using homogeneous coordinates
- One reason: perspective division only necessary for vertices that are in view volume
- Differences in clipping algorithm:
 - Point p is outside of view volume if
$$p_x / p_w < -1 \Leftrightarrow p_x < -p_w \Leftrightarrow p_x + p_w < 0$$
Other planes:
$$p_x - p_w > 1, p_y + p_w < 0, p_y - p_w > 0, p_z + p_w < 0, p_z - p_w > 0$$
 - Compute $p_x(t)$, $p_y(t)$, $p_z(t)$, **and** $p_w(t)$

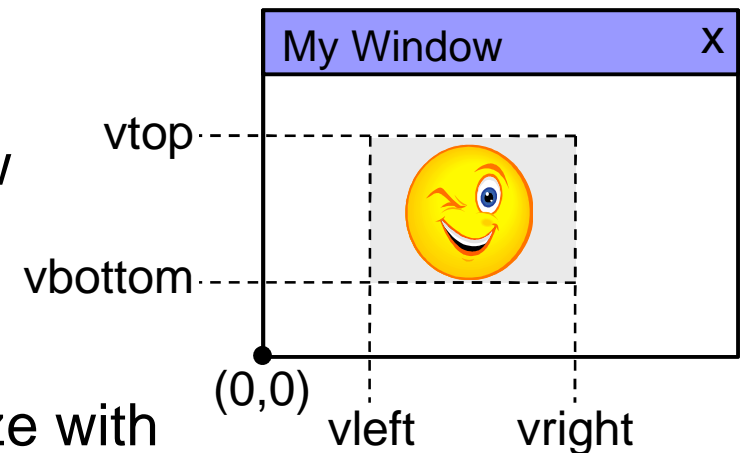




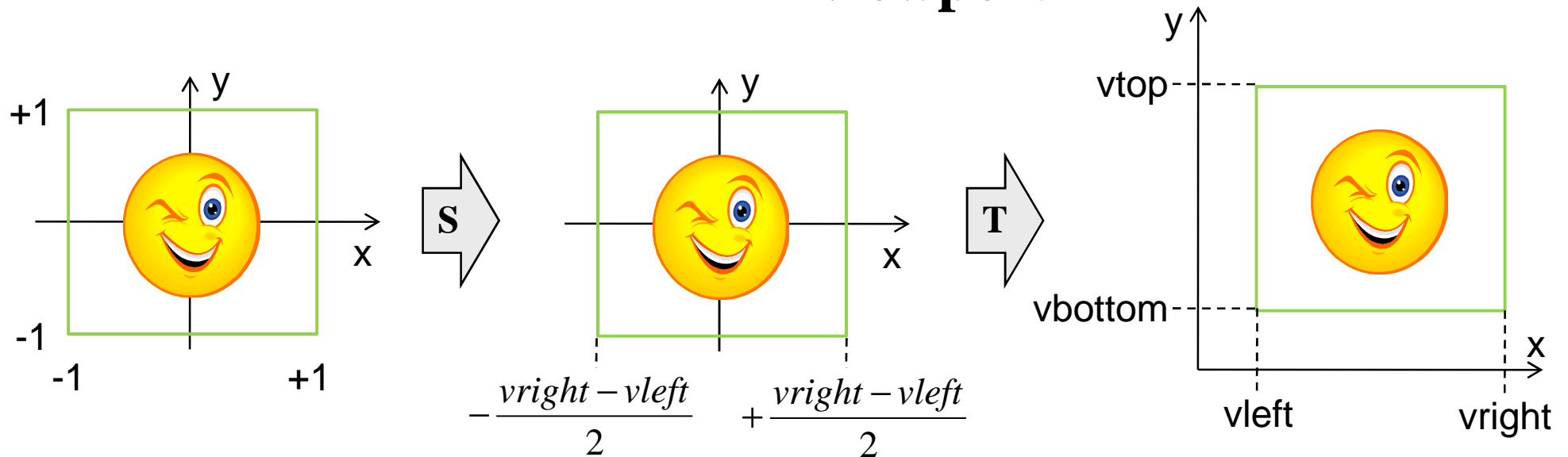
VIEWPORT TRANSFORMATIONS

Viewport Transformation

- Mapping from Normalized Device Coordinates (NDC) to device coordinates (DC) aka viewport coordinates
- For NDC: $x, y, z \in (-1, +1)$
- For DC: $x \in (vleft, vright)$, $y \in (vbottom, vtop)$, $z \in (0, maxz)$
 - x and y are 2D window coordinates
 - $vleft$, $vright$, $vbottom$, $vtop$ are the boundaries of the viewport in the window
 - $maxz$ depends on type used for depth buffer values (e.g. `uint32`)
 - In OpenGL: set viewport position and size with `glViewport(x, y, width, height);`
- NDCs are multiplied with **viewport matrix** $M_{viewport}$ which maps NDC boundaries onto viewport boundaries



Viewport Matrix M_{viewport}



$$M_{\text{viewport}} = \mathbf{T} \mathbf{S} = \begin{pmatrix} 1 & 0 & 0 & \frac{v_{\text{right}} + v_{\text{left}}}{2} \\ 0 & 1 & 0 & \frac{v_{\text{top}} + v_{\text{bottom}}}{2} \\ 0 & 0 & 1 & \frac{\text{maxz}}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{v_{\text{right}} - v_{\text{left}}}{2} & 0 & 0 & 0 \\ 0 & \frac{v_{\text{top}} - v_{\text{bottom}}}{2} & 0 & 0 \\ 0 & 0 & \frac{\text{maxz}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Multiple Viewports

- **Problem:** How to write a GL program that displays multiple views of a scene, each one in a different viewport?

- **Solution: Multiple viewports**

Multiple views of a scene, e.g., architectural drawing front, side, and top views

Loop: repeat for each viewport

1. Set this viewport:

```
glViewport( x, y, width, height );
```

2. Set view projection for this viewport (might be the same for all viewports, if so do this before loop):

```
glOrtho(left, right, bottom, top, zNear, zFar );
```

```
or other such as gluPerspective( ... );
```

3. Set camera view position and orientation for this viewport

```
gluLookAt(left, right, bottom, top, zNear, zFar );
```

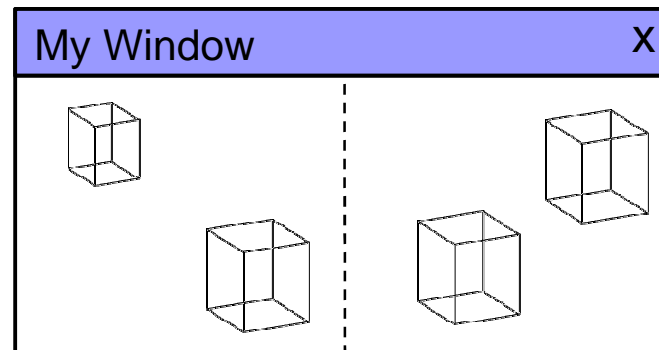
```
or other such as glTranslatef( ... ); glRotatef( ... );
```

4. Draw scene

Multiple Viewports Code Example

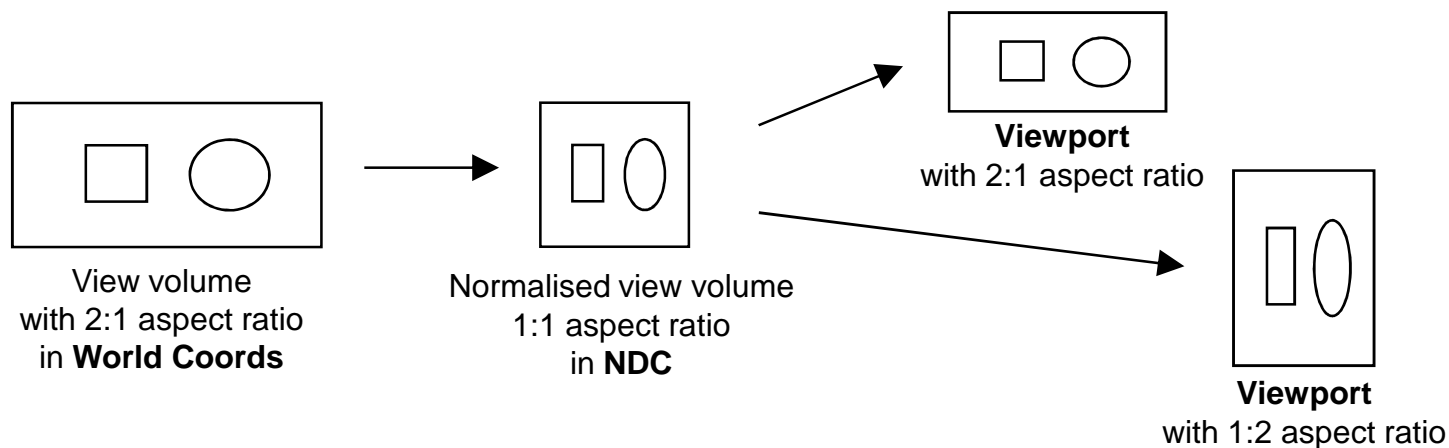
```
// left: perspective
glViewport(0, 0, 100, 100);
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
gluPerspective(yfov, aspect,
              zNear, zFar);
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
// do view transformations...
drawScene();
```

```
// right: orthographic
glViewport(100, 0, 100, 100);
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glOrtho(left, right, bottom,
        top, near, far);
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
// do view transformations...
drawScene();
```



Aspect Ratio of View Volume and Viewport

- Final pipeline transformation step is viewport transformation
`glViewport(GLint x, GLint y,
 GLsizei width, GLsizei height);`
Default viewport is entire drawing window, (0, 0, winWidth, winHeight).
- **Aspect ratio** of view volume and viewport should be same



- **Problem:** How to write a GLUT program that automatically resets the view volume aspect ratio when window (viewport) is resized?

Aspect Ratio: reshape callback function

Solution: in GLUT, use **reshape callback** to adjust viewport and view volume aspect ratio after a **window resize event**

- Register reshape callback function (in main at prog. init.)

```
void reshape(GLsizei width, GLsizei height); // prototype
glutReshapeFunc( reshape ); // callback registration
```

- Define reshape callback function (in main prog. module)

```
// left, right, bottom, top = class member or global variables
void reshape( GLsizei width, GLsizei height ) {
    glViewport(0, 0, width, height ); // set viewport size
    GLfloat aspect = (GLfloat)width / (GLfloat)height; //NOT int!
    GLdouble center = (left + right) / 2.0;
    GLdouble newHalfWidth = aspect * (top - bottom) / 2.0;
    left = center - newHalfWidth; right = center + newHalfWidth;
    glMatrixMode(GL_PROJECTION); // reset proj matrix
    glLoadIdentity();
    glOrtho(left, right, bottom, top, near, far);
    drawSceneObjects(); // redraw all objects
}
```



SUMMARY



Summary

- Pseudodepth
 - Used to normalize z with matrix
 - For small $near$ and large far resolution problems
- Clipping removes lines outside of view volume
 - Trivial accept and reject tests using outcodes
 - Check t_{in} and t_{out} values of parametric line equation
- Viewport Transformation: maps NDCs to DCs using $M_{viewport}$

References:

- Pseudodepth: Hill, Chapter 7.4.3, pp. 349-351
- Clipping: Hill, Chapter 7.4.3, pp. 356-361
- Viewport Transformation: Hill, Chapter 7.4.3, p. 361



Quiz

1. Why isn't it a good idea to use a very small number for near or a very large number for far?
2. How is an outcode table constructed? How is it used for trivial reject/accept?
3. How do we find t_{in} and t_{out} during clipping? How does it help us to clip lines?