Computer Science

COMPSCI 372 S2 C – Exercise Sheet 4

6th August 2008

Q1: Let $\mathbf{M} = \begin{pmatrix} 1 & 5 & 3 \\ 3 & 0 & 2 \end{pmatrix}$, $\mathbf{v} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

Please compute:

1. $5\mathbf{v}-3\mathbf{u}$
2. $\mathbf{v}\cdot\mathbf{u}$
3. $\mathbf{u}\mathbf{v}^T$
4. $|\mathbf{v}|$
5. $\hat{\mathbf{v}}$
6. $\mathbf{M}^T\mathbf{M}$
7. MM^T
8. The angle between **u** and **v**
9. The angle between **v** and the x-axis

Q2: Given a matrix **M** the matrix $\mathbf{M}^{-1}$ is called the inverse of **M** if and only if $\mathbf{M}^{-1}\mathbf{M} = \mathbf{M}\mathbf{M}^{-1} = \mathbf{I}$ where **I** is the identity matrix (i.e. **I** is the matrix where all diagonal elements are 1 and all off-diagonal elements are zero).

For $\mathbf{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ the inverse is computed by $\mathbf{M}^{-1} = \frac{1}{|\mathbf{M}|}\begin{pmatrix} m_{22} & -m_{12} \\ -m_{21} & m_{11} \end{pmatrix}$

(a) Show that $\mathbf{M}^{-1}\mathbf{M} = \mathbf{M}\mathbf{M}^{-1} = \mathbf{I}$.

(b) Let $\mathbf{S} = \begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$, $\mathbf{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, $\mathbf{H} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

Compute

1. $\mathbf{S}^{-1}$
2. $\mathbf{R}^{-1}$ (use the fact that $\cos\theta\cos\theta + \sin\theta\sin\theta = 1$)
3. $\mathbf{H}^{-1}$

Q3: Given are two vectors $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ with a common origin. Find all vectors orthogonal to **a**. Decompose **b** into two components $\mathbf{b}_a$ *and* $\mathbf{b}_{a^\perp}$ parallel and perpendicular to a, respectively.

Q4: Given is a triangle with the corners $(0,0)^T$, $(3,0)^T$ and $(3,2)^T$ made out of a reflective material. A light ray originates at the point $(-2,1)^T$ and travels in the direction $(1,0)^T$. Compute the direction of the light ray after hitting the triangle.