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Foundations of Artificial Intelligence
—Knowledge Representation in Logic—

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Propositional Logic

Advantages:

- It is simple to deal with.
- There is a decision procedure for it.

Propositional Semantics

P	Q	$\neg$ P	P $\wedge$ Q	P $\vee$ Q	P $\rightarrow$ Q	P $\leftrightarrow$ Q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Propositional Semantics (cont'd)

- $P \rightarrow Q$ is called a conditional and $Q \rightarrow P$ its converse.
- $\neg Q \rightarrow \neg P$ is called contrapositive.
- $P \leftrightarrow Q$ holds if and only if $P \rightarrow Q$ and its converse both hold.
- The contrapositive of $P \rightarrow Q$ holds if and only if $P \rightarrow Q$ holds.
- $P \vee \neg P$ is called a tautology.
- $P \wedge \neg P$ is a contradiction. An expression that is not a contradiction is satisfiable.

Example

Facts	Proposition
It is raining.	$RAINING$
It is sunny.	$SUNNY$
If it is raining, then it is not sunny.	$RAINING \rightarrow \neg SUNNY$

Rules of Inference

Modus Ponens:

Assume:	$P \rightarrow Q$	If it is snowing then school will be cancelled,
And:	P	and I also know it is snowing.
Then:	Q	

Disjunctive Syllogism:

Assume:	P	If I know it is snowing I can truthfully say
Then:	$P \vee Q$	that it is snowing or I have long hair.

Rules of Inference (cont'd)

Resolution:

Assume:	$P \vee Q$
And:	$\neg P \vee Z$
Then:	$Q \vee Z$

It is snowing or it is raining. It is not snowing or it is cold. So it is raining or it is cold.

A Shortcoming

Example:

Facts	Proposition
Socrates is a man.	$SOCRATESMAN$
Plato is a man.	$PLATOMAN$
All men are mortal.	$MORTALMAN$

Observation:

- This does not capture the relationship between the sentences.
- More powerful logic is needed.

(First-Order) Predicate Logic

$\forall, \exists$, variables

Positive aspects:

- Real-world facts are represented as statements written as well-formed formulas (wff's).
- These statements may contain variables and quantification.

Negative aspect:

- Predicate logic is only semidecidable (halting problem).

Example

- Marcus was a man.
  $man(Marcus)$
- Marcus was a Pompeian.
  $Pompeian(Marcus)$
- All Pompeians were Romans.
  $\forall x : Pompeian(x) \rightarrow Roman(x)$
- Caesar was a ruler.
  $ruler(Caesar)$

Example (cont'd)

- All Romans were loyal to Caesar or hated him.
  $\forall x : Roman(x) \rightarrow loyalto(x, Caesar) \vee hate(x, Caesar)$

- All Romans were either loyal to Caesar or hated him.
  $\forall x : Roman(x) \rightarrow$
  $[(loyalto(x, Caesar) \vee hate(x, Caesar)) \wedge$
  $\neg(loyalto(x, Caesar) \wedge hate(x, Caesar))]$

- Everyone is loyal to someone.
  $\forall x : \exists y : loyalto(x, y)$

Example (cont'd)

- People only try to assassinate rulers they are not loyal to.
  $\forall x : \forall y : person(x) \wedge ruler(y) \wedge tryassassinate(x, y) \rightarrow \neg loyalto(x, y)$

- Marcus tried to assassinate Caesar.
  $tryassassinate(Marcus, Caesar)$

- There exist precisely two individuals that are parents of Marcus.
  $\exists x : \exists y : \neg sameperson(x, y) \wedge parent(x, Marcus) \wedge parent(y, Marcus) \wedge \forall z : parent(z, Marcus) \rightarrow (sameperson(z, x) \vee sameperson(z, y))$

Was Marcus Loyal to Caesar?

$$\neg loyalto(Marcus, Caesar)$$
$$\uparrow$$
$$person(Marcus) \wedge ruler(Caesar) \wedge tryassassinate(Marcus, Caesar)$$
$$\uparrow$$
$$person(Marcus) \wedge tryassassinate(Marcus, Caesar)$$
$$\uparrow$$
$$person(Marcus)$$

Additional wff necessary:

$$person(Marcus)$$

Observations

- Proving means searching an AND-OR graph.
- Many English sentences are ambiguous:
  $The\ spy\ saw\ the\ cop\ with\ binoculars.$
- Often there is a choice of how to represent a sentence.
- The set of wff's is likely to be incomplete because commonsense knowledge is often lacking from them.
- It is not obvious which statements to deduce:
  $\neg loyalto(Marcus, Caesar)$, $loyalto(Marcus, Caesar)$