COMPSCI.220.S1 – Algorithms and Data Structures

Assignment 1 – Analysing Time Complexity of Algorithms

Out: Monday, 07th of March, 2016

Due: Monday, 21st of March, 2016

Assignment 1 (Lectures 1 – 6) is worth **70 marks** representing **7%** of your total course grade.

Objectives. Learning basic techniques for analysing time complexity, in particular, evaluating complexity of a given pseudocode; exploring algorithm performance with "Big-Oh", "Big-Omega", and "Big-Theta" tools, and solving basic recurrences describing the performance.

Requirements. You should give clear and detailed answers to the following questions:

1. (*10 marks*) Work out time complexity $T(n)$ of the following piece of pseudocode in terms of a total number of specified elementary operations for a given integer n; $n \geq 5$:

   for $i = 1$ **step** $i \leftarrow 2 \cdot i$ **while** $i < 2^n$ **do**
       if $i \leq 4$ **OR** $i \geq 2^{n-2}$
           for $j = n$ **step** $j \leftarrow j - 2$ **while** $j \geq 0$ **do**
               Constant number C of elementary operations
           end for
       else
           for $j = n$ **step** $j \leftarrow \lceil \frac{j}{2} \rceil$ **while** $j > 1$ **do**
               Constant number C of elementary operations
           end for
       end if
   end for

2. (*10 marks*) You have found empirically that the methods A of complexity $\Theta(n^{1.5})$ and B of complexity $\Theta(n \log n)$ process a list of 100 records for $T_A(100) = 1$ and $T_B(100) = 10$ microseconds, respectively. Find their processing times, $T_A(n)$ and $T_B(n)$, for n records and decide which of them will process faster a list of $100,000$ records.

3. (*15 marks*) Assuming $n = 3^m$ with the integer $m = \log_3 n$, derive a closed-form formula for $T(n)$ by solving the recurrence $T(n) = 3T\left(\frac{n}{3}\right) + 6$ with the base condition $T(1) = 0$.

4. (*15 marks*) **Proposition**: Given two nonnegative-valued functions, $\varphi(n)$ and $\psi(n)$, defined on nonnegative integers, n; $n = 0, 1, \ldots$, prove that $\varphi(n)$ is $\Theta(\psi(n))$ if and only if $\varphi(n)$ is $\Omega(\psi(n))$ and $\psi(n)$ is $\Omega(\varphi(n))$.

5. (*20 marks*) Processing time $T(n)$ of a certain algorithm is $\Omega(n)$ and $\mathrm{O}\left(n^3\right)$. Decide whether this implies the conclusion that $T(n)$ is $\Theta\left(n^2\right)$ and prove your decision.

Submission: Your report should answer in detail to Questions 1–3; prove Proposition 4, as well as detail and prove your decision for Question 5. The report should be submitted as a single Adobe PDF file `CS220assign1.pdf` (only this file will be marked, so check that it can be read by PCs in the departmental Computer Labs). **Scanned handwritten documents are strictly forbidden** (even as images in a pdf file) and will not be accepted for marking.

Submit your file electronically to the Assignment Drop Box (`https://adb.auckland.ac.nz`). Note that due date is Monday, 21st of March 2016, 09:00 pm (ADB time). If submitted after the due date, the penalty of 10% will be before 22nd of March 2016, 09:00 p.m.; then the penalty of 50% will be before 23rd of March 2016, 09:00 p.m., and no submission afterwards.

Marking scheme

For Questions 1 − 3	% of marks
Clear structure of your report and detailed explanations	up to 20
Correctness of the final answers	up to 30
Correctness of the intermediate steps in deriving the answers	up to 30
Detailed explanations of all steps with references, if necessary, to the textbook	up to 20
	Total: up to 100

For Proposition 4 and Question 5	
Clear structure of your report and detailed explanations	up to 20
Correct usage of Big-{Oh, Omega, Theta} tools with due references to the textbook	up to 30
Correctness of the proofs	up to 50
	Total: up to 100