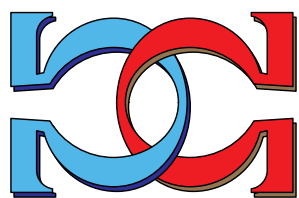
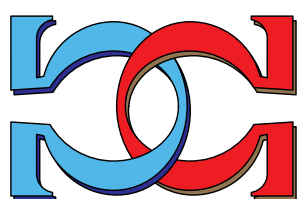


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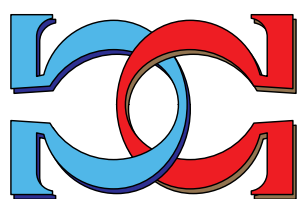


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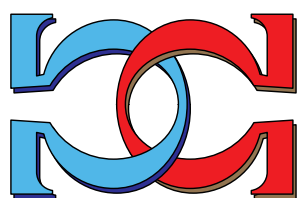


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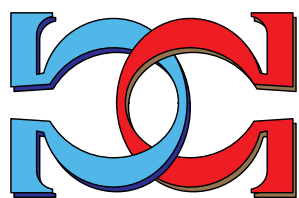


**Proposed direct test of quantum
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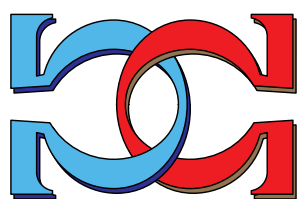
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Proposed direct test of quantum contextuality

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Abstract

Quantum contextually can be directly tested by an Einstein-Podolsky-Rosen-type experiment of two spin one and higher particles in a singlet state. The two associated contexts are “interlinked” by a common observable.

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Keywords: quantum contextuality

Since its invention, the quantum formalism confronted its creators and recipients with seemingly “mindboggling” features [1], including the discreteness of the quantum occupation per field mode [2, 3], entanglement [1, 4, 5], complementarity [6–9], randomness [10, 11], as well as local [12–20] and nonlocal [21–26] value indefiniteness. Value indefiniteness, which is often referred to as the *Kochen-Specker theorem*, is formally associated with the “scarcity” or even total absence of two-valued (also called dispersionless) states identifiable as (classical) truth assignments on certain (finitely many) quantum propositions. This mathematical result is obtained by a proof by contradiction and has given rise to various interpretations; the most prominent being *quantum contextuality* [23, 27–29] claiming that the result of an observation (among other entities such as the quantum state) depends on what observables are co-measured alongside of it. Indeed, despite of its interpretative character, contextuality is effectively used as a synonym for value indefiniteness, in the sense that “the immense majority of the experimental violations of Bell inequalities does not prove quantum nonlocality, but just quantum contextuality” [30–33].

A quantum mechanical context [34] can be formalized by a single “maximal” self-adjoint operator. Every collection of mutually compatible co-measurable operators (such as projections corresponding to yes–no propositions) are functions of such a maximal operator (e.g., Ref. [35, Sec. II.10, p. 90, English translation p. 173], Ref. [13, § 2], Ref. [36, pp. 227,228], and Ref. [37, § 84]).

A necessary condition for the *interlinking* of two or more contexts by one or more *link observable(s)* is the requirement that the dimensionality of the Hilbert space must exceed two, since for lower dimensional Hilbert spaces the maximal operators “decay” into separate, isolated “trivial” Boolean sublogics without any common observable. This is also the reason for similar requirements in the theorems by Gleason [38–41] and Kochen and Specker.

In what follows we propose an experiment capable of directly (alas via counterfactual elements of physical reality [42]) testing the contextuality hypothesis. In the proposed experiment, two different contexts or, equivalently, two non-commuting maximal observables, are simultaneously measured on a pair of spin one particles in a singlet state [23, 43] in an Einstein-Podolsky-Rosen type configuration. The contexts are fine-tuned to a common *single observable interlinking* them.

We shall first consider the contexts originally proposed by Kochen and Specker [13, pp. 71-73], referring to the change in the energy of the lowest orbital state of orthohelium resulting from the application of a small electric field with rhombic symmetry. The terms *Kochen-Specker contexts* and (maximal) *Kochen-Specker operators* will be used synonymously. More explicitly, the maxi-

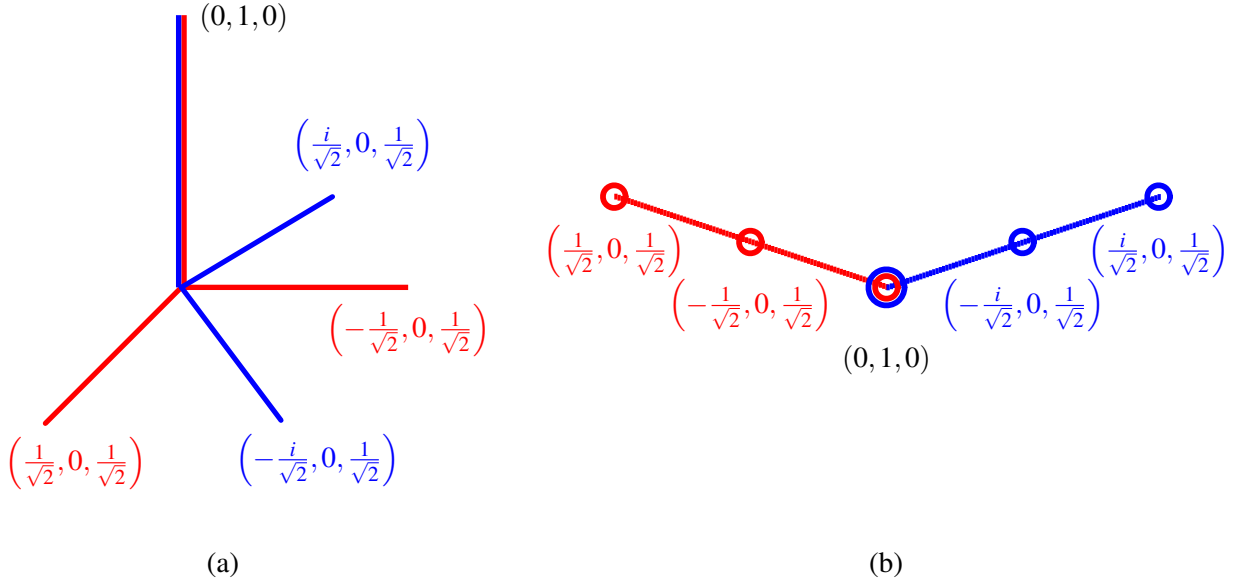


FIG. 1: Two equivalent diagrammatical representations of a configuration of two interlinked Kochen-Specker contexts: (a) Two tripods with a common leg; (b) Resulting Greechie (orthogonality) diagram: points stand for individual basis vectors, and entire contexts — in this case the one-dimensional linear subspaces spanned by the vectors of the orthogonal tripods — are drawn as smooth curves.

mal Kochen-Specker operators associated with this link configuration can be constructed from the spin one observables (e.g., Ref. [44]) in arbitrary directions measured in spherical coordinates

$$J(\theta, \phi) = \begin{pmatrix} \cos \theta & \frac{e^{-i\phi} \sin \theta}{\sqrt{2}} & 0 \\ \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & 0 & \frac{e^{-i\phi} \sin \theta}{\sqrt{2}} \\ 0 & \frac{e^{i\phi} \sin \theta}{\sqrt{2}} & -\cos \theta \end{pmatrix}, \quad (1)$$

where $0 \leq \theta \leq \pi$ stands for the polar angle in the x - z -plane taken from the z -axis, and $0 \leq \phi < 2\pi$ is the azimuthal angle in the x - y -plane taken from the x -axis. The orthonormalized eigenvectors associated with the eigenvalues $+1, 0, -1$ of $J(\theta, \phi)$ in Eq. (1) are

$$\begin{aligned} x_{+1} &= e^{i\delta_{+1}} \left(e^{-i\phi} \cos^2 \frac{\theta}{2}, \frac{1}{\sqrt{2}} \sin \theta, e^{i\phi} \sin^2 \frac{\theta}{2} \right), \\ x_0 &= e^{i\delta_0} \left(-\frac{1}{\sqrt{2}} e^{-i\phi} \sin \theta, \cos \theta, \frac{1}{\sqrt{2}} e^{i\phi} \sin \theta \right), \\ x_{-1} &= e^{i\delta_{-1}} \left(e^{-i\phi} \sin^2 \frac{\theta}{2}, -\frac{1}{\sqrt{2}} \sin \theta, e^{i\phi} \cos^2 \frac{\theta}{2} \right), \end{aligned} \quad (2)$$

where $\delta_{\pm 1}$, and δ_0 stand for arbitrary phases.

Consider a configuration with two tripods rotated by the azimuthal angle $\phi = \pi/4$ (indeed, any angle which is not zero or a multiple of $\pi/2$ would do) around a common leg located along the z -axis ($\theta = \phi = 0$). The Hilbert space configuration and the resulting logic are sketched in Fig. 1.

For $\alpha \neq \beta \neq \gamma \neq \alpha$, the maximal Kochen and Specker operators [13] are defined by

$$\begin{aligned}
C_{KS}(\alpha, \beta, \gamma) &= \frac{1}{2} [(\alpha + \beta - \gamma)J^2(\frac{\pi}{2}, 0) + (\alpha - \beta + \gamma)J^2(\frac{\pi}{2}, \frac{\pi}{2}) + (\beta + \gamma - \alpha)J^2(0, 0)] \\
&= \alpha \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \beta \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} + \gamma \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 0 \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}, \\
C'_{KS}(\alpha, \beta, \gamma) &= \frac{1}{2} [(\alpha + \beta - \gamma)J^2(\frac{\pi}{2}, \frac{\pi}{4}) + (\alpha - \beta + \gamma)J^2(\frac{\pi}{2}, \frac{3\pi}{4}) + (\beta + \gamma - \alpha)J^2(0, 0)] \\
&= \alpha \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \beta \begin{pmatrix} \frac{1}{2} & 0 & -\frac{i}{2} \\ 0 & 0 & 0 \\ \frac{i}{2} & 0 & \frac{1}{2} \end{pmatrix} + \gamma \begin{pmatrix} \frac{1}{2} & 0 & \frac{i}{2} \\ 0 & 0 & 0 \\ -\frac{i}{2} & 0 & \frac{1}{2} \end{pmatrix}.
\end{aligned} \tag{3}$$

Their common spectrum of eigenvalues is α , β and γ , corresponding to the eigenvectors $(0, 1, 0)$, $(1, 0, 1)$, $(-1, 0, 1)$ of C_{KS} , and $(0, 1, 0)$, $(-i, 0, 1)$, $(i, 0, 1)$ of C'_{KS} , respectively. If we identify with the spin states the directions in Hilbert space according to Eqs. (2); i.e., with $|+\rangle = (1, 0, 0)$, $|0\rangle = (0, 1, 0)$, and $|-\rangle = (0, 0, 1)$, then the eigenstates of C_{KS} (and similar for C'_{KS}) can be written as $|\alpha\rangle = |0\rangle$, $|\beta\rangle = (1/\sqrt{2})(|+\rangle + |-\rangle)$, and $|\gamma\rangle = (1/\sqrt{2})(|-\rangle - |+\rangle)$, which amounts to a rotation of the original basis by the angle $\pi/4$ in the x - z -plane.

In order to be able to use the type of counterfactual inference employed by an Einstein-Podolsky-Rosen setup, a multipartite quantum state has to be chosen which satisfies the *uniqueness property* [45] with respect to the two Kochen-Specker contexts such that knowledge of a measurement outcome of one particle entails the certainty that, if this observable were measured on the other particle(s) as well, the outcome of the measurement would be a unique function of the outcome of the measurement actually performed. Consider the two spin-one particle singlet state $|\Phi_s\rangle = (1/\sqrt{3})(-|00\rangle + |+-\rangle + |-+\rangle)$ which, in terms of the eigenstates of Kochen-Specker maximal operators C_{KS} (and C'_{KS}), can be rewritten form invariantly [23]; i.e., $(1/\sqrt{3})(-|\alpha\alpha\rangle + |\beta\gamma\rangle + |\gamma\beta\rangle)$. It is form invariant under rotations and satisfies the uniqueness property, just as the ordinary Bell singlet state of two spin one-half quanta (we cannot use these because they are limited to $2 \otimes 2$ dimensions, with merely two dimensions per quantum). Hence, it is possible to employ a similar counterfactual argument and establish two elements of physical reality according to the Einstein-Podolsky-Rosen criterion for the two interlinked Kochen-Specker contexts C_{KS} as well as C'_{KS} .

We are now in the position to formulate a testable criterion for contextuality: Contextuality predicts that there exist outcomes associated with α on one context which are accompanied by the

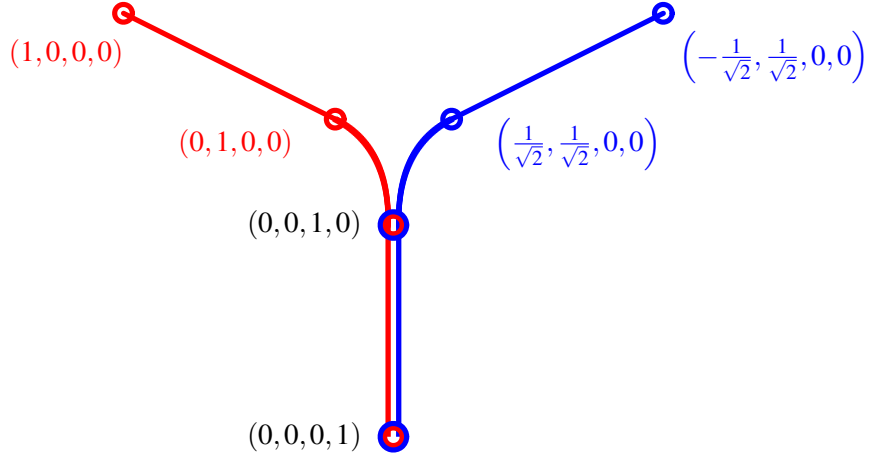


FIG. 2: Greechie diagram of two contexts in four-dimensional Hilbert space interconnected by two link observables.

outcomes β or γ for the other context. The quantum mechanical expectation values can be obtained from

$$\text{Tr} \{ |\varphi_s\rangle \langle \varphi_s| \cdot [C_{KS}(\alpha, \beta, \gamma) \otimes C'_{KS}(\delta, \epsilon, \zeta)] \} = \frac{1}{6} [2\alpha\delta + (\beta + \gamma)(\epsilon + \zeta)]. \quad (4)$$

As a consequence, the outcomes α - ϵ , α - ζ , as well as β - δ and γ - δ indicating contextuality do not occur. This is in contradiction to the contextuality hypothesis.

Another context configuration in four-dimensional Hilbert space drawn in Fig. 2 consists of two contexts which are interconnected by *two* common link observables. The two context operators

$$C(\alpha, \beta, \gamma, \delta) = \text{diag}(\alpha, \beta, \gamma, \delta), \quad C'(\alpha, \beta, \gamma, \delta) = \text{diag} \left(\frac{\alpha+\beta}{2}, \frac{\alpha-\beta}{2}, \frac{\alpha-\beta}{2}, \frac{\alpha+\beta}{2}, \gamma, \delta \right) \quad (5)$$

have identical eigenvalue spectra containing mutually different eigenvalues α , β , γ and δ .

Consider the singlet state of two spin-3/2 observables $|\psi_s\rangle = \frac{1}{2} (|\frac{3}{2}, -\frac{3}{2}\rangle - |-\frac{3}{2}, \frac{3}{2}\rangle - |\frac{1}{2}, -\frac{1}{2}\rangle + |-\frac{1}{2}, \frac{1}{2}\rangle)$ satisfying the uniqueness property. The four different spin states can be identified with the cartesian basis of fourdimensional Hilbert space $|\frac{3}{2}\rangle = (1, 0, 0, 0)$, $|\frac{1}{2}\rangle = (0, 1, 0, 0)$, $|-\frac{1}{2}\rangle = (0, 0, 1, 0)$, and $|-\frac{3}{2}\rangle = (0, 0, 0, 1)$, respectively. They are eigenstates of the context C . Likewise, a rotation in the first two components around the angle $\pi/4$ yields the eigenstates of C' . Thus, we can again counterfactually infer elements of physical reality for both of the contexts C and C' .

Compared to the previous Kochen-Specker contexts, this configuration has the additional ad-

vantage that — in the absence of any criterion for outcome preference — Jayne’s principle [46] suggests that contextuality predicts totally uncorrelated outcomes associated with a maximal unbiased of the two common link observables, resulting in the equal occurrence of the joint outcomes $\gamma\text{--}\eta$, $\gamma\text{--}\nu$, $\delta\text{--}\eta$, and $\delta\text{--}\nu$. The quantum mechanical predictions are based on the expectation values

$$\text{Tr} \{ |\psi_s\rangle \langle \psi_s| \cdot [C(\alpha, \beta, \gamma, \delta) \otimes C'(\epsilon, \zeta, \eta, \nu)] \} = \frac{1}{8} [(\gamma + \delta)(\epsilon + \zeta) + 2(\beta\eta + \alpha\nu)]. \quad (6)$$

As a consequence, there are no outcomes $\gamma\text{--}\eta$, $\gamma\text{--}\nu$, $\delta\text{--}\eta$, and $\delta\text{--}\nu$, which is in contradiction to the contextuality postulate.

Let us summarize the situation as follows. Insofar as we are able to perform counterfactual and actual measurements on pairs of singlets consisting of spin-one and spin three-half quanta, quantum mechanics seems to predict noncontextual behavior.

However, in order to cope with the kind of value indefiniteness inferred from the absence of (enough) “classical” two-valued states, it appears that, granted that quantum mechanics is valid, classical realism has to be adapted in one way or another. One of these proposed adaptations is contextuality; the idea that the *outcomes* of one and the same observable — represented identically in the quantum formalism — could and should be different, depending on its “context;” i.e., what other observables are measured alongside of it. An alternative among others [47–50] is the abandonment of classical omniscience [51] and the context translation principle [52].

One of the conceivable criticisms against the presented arguments is that the configurations considered, although containing complementary contexts, still allow even a full, separable set of two-valued states, and therefore need no contextual interpretation. However, it is exactly these Kochen-Specker type contexts which enter the Kochen-Specker argument. Hence, they should not be interpreted as separate, isolated sublogics, but as parts of a continuum of sublogics, containing the finite structure devised by Kochen and Specker and others.

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