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A Repository of Compound Graphs for use in Large Network Design

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Abstract

In a field of network design, engineers desire better ways to design efficient communication network. While designing such network, we are restricted with engineering constraints such as communication delays and hardware costs. Many construction techniques have been proposed. In this paper we focus on compounding techniques and provide (Δ, D) tables that contain the largest compound graphs for given degree Δ and diameter D. We also empirically verify a few of the recently discovered large compound graphs.

1 Introduction

In the design of interconnection networks, we are restricted by engineering limitations and hardware costs of adding communication links. That is, nodes of a network are restricted to have at most a fixed number of communication links. Due to this restriction in most cases transmitting data between two nodes require data to be traversed between several nodes before reaching its destination node. When data is traversed between any two nodes communication delays must happen, and it is a cost measure to minimize the number of nodes needed to transmit data. Hence, it is desirable to optimize both connection costs and communication delays when designing an efficient network.

Graph theory has been used to model interconnection networks, where vertices of the graph represent nodes and edges of the graph represent communication links. Furthermore, the maximum communication delay is represented by the diameter of a graph and the maximum connecting links for nodes is the maximum vertex degree. Constructing a large network under these two network constraints leads us to the following graph problem.

> **The (Δ, D) Problem:** Construct the largest possible graph with maximum degree Δ and diameter at most D.

A (Δ, D) *graph* is a graph with maximum degree Δ and diameter at most D.

There exists an easily computable bound on the largest order of the graph for a given maximum degree Δ and diameter D. Such bound is given by

$$1 + \Delta + \Delta(\Delta-1) + \ldots + \Delta(\Delta-1)^{D-1} = \frac{\Delta(\Delta-1)^D}{\Delta-2}, \Delta > 2$$

This value is called the *Moore bound*, and a graph which satisfies the bound is called a Moore graph. However there are only few graphs known to achieve the Moore bound. Hence, in most cases various graph construction techniques have been used to produce a graph whose order is closest to the Moore bound as possible.

There has been several graph construction techniques to obtain large dense graphs (see [MS]). One popular technique[1] is compounding (see [CG, GF, GFS, GM, GPB]), and it consists of replacing vertices of given graph by graph or copies of graph and rearranging edges suitably. Compounding has been proved useful for producing a large graph, and some of the largest (Δ, D) graphs known today are produced from compounding. Compounding has also been used in construction of minimal broadcast networks (see, [DVWZ]). In this paper, we refer *compound graphs* as graphs produced from using the compounding technique.

2 Graph theory preliminaries

This section contains some basic graph theory terms that are used in this paper. Most of terms follow those in [CL].

A *graph* $G = (V, E)$ is a finite non empty set V of vertices (the singular is vertex) and (possibly empty) set E of unordered pairs of distinct vertices called edges. The *order* of a graph $G = (V, E)$ is the cardinality of the vertex set V. The *degree* of a vertex is the number of edges incident to the vertex, and two vertices are *adjacent* if there is an edge connecting them. The *degree* ΔG *of a graph* is the maximum degree over all vertices. A *path* in a graph $G = (V, E)$ is a sequence of vertices $v_0 v_1 \ldots v_n$ such that every consecutive pair of a sequence is an edge in G and no vertex in the sequence is repeated. The length of a path is the number n. The *distance* between two vertices x and y of graph G is

[1] We also mention there is another popular construction technique that uses Cayley graphs (see for example [Di, DH, Ha, Lo]),

the length of a shortest path between x and y. The *diameter* D of G is the maximum distance between any two vertices of G. A graph $G = (V_0 \cup V_1, E)$ is a *bipartite graph* if its set of vertices can be partitioned into two disjoint subsets, such that no vertices of a given subset are adjacent. For any bipartite graph $G = (V_0 \cup V_1, E)$ of even diameter, distance between two vertices $x \in V_0$ and $y \in V_1$ is at most $D - 1$, and distance between two vertices $x \in V_0$ and $y \in V_0$ (or $x \in V_1$ and $y \in V_1$) is at most D. Similarly for any bipartite graph $G = (V_0 \cup V_1, E)$ of odd diameter, distance between two vertices $x \in V_0$ and $y \in V_1$ is at most D, and distance between two vertices $x \in V_0$ and $y \in V_0$ (or $x \in V_1$ and $y \in V_1$) is at most $D - 1$ (see [GPB]).

3 Generalized polygons

A *generalized n-gon* is a connected bipartite graph whose vertices are the points and lines of a non-degenerate quadric surface in n dimensional space $PG(n, q)$ and have been frequently used in the construction of compound graphs. For more information on generalized polygons, we refer the reader to [Va, DV, Be].

Generalized n-gon with $n = 3, 4, 6$ are called *generalized triangle* (denoted by T_q), *generalized quadrangle* (denoted by Q_q) and *generalized hexagon* (denoted by H_q) respectively. Generalized n-gons (P_q, Q_q and H_q) only exist if and only if q is a prime power. Degree, diameter and order of these generalized n-gons are shown in Table 1.

Table 1: Degree, diameter and order for generalized polygons.

	Degree Δ	Diameter D	Order N
P_q	$\Delta = q + 1$	$D = 3$	$N = 2(q^2 + q^1 + 1)$
Q_q	$\Delta = q + 1$	$D = 4$	$N = 2(q^3 + q^2 + q^1 + 1)$
H_q	$\Delta = q + 1$	$D = 6$	$N = 2(q^5 + q^4 + q^3 + q^2 + q^1 + 1)$

4 Compound graphs

Using the compounding technique, several internal configurations were constructed which can be used to generate compound graphs. In this paper we focus on configurations $G \wedge B$, $B_0 \Theta B_1$, $G \kappa_5 B$, $B_0 \Sigma_6 B_1$, $B_0 \Theta_4 B_1$ and $B_0 \Sigma_7 B_1$ (see [CG, GFS, GF, GM, GPB, Ki]). These configurations are used to produce large compound graphs, and some of the compound graphs produced from these configurations still remain as largest known graph for given degree and diameter.

Table 2: Standard notation for compound graph configurations.

	Description of graphs used in the compounding configuration.
$G \wedge B$	G is any graph $G=(V,E)$ with diameter D_G, degree Δ_G and order N_G. B is any bipartite graph $B=(V_0 \cup V_1, E)$ with even diameter D_B, degree Δ_B, order N_B and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_B}{2}$.
$G\kappa_5 B$	G is any graph $G=(V,E)$ with diameter D_G, degree Δ_G and order N_G. B is any bipartite graph $B=(V_0 \cup V_1, E)$ with even diameter D_B, degree Δ_B, order N_B and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_B}{2}$.
$B_0\Theta_1 B_1$	B_0 is any bipartite graph $B_0=(V_0 \cup V_1, E)$ with even diameter D_0, degree Δ_0, order N_0 and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_0}{2}$. B_1 is any bipartite graph $B_1=(V_0 \cup V_1, E)$ with degree even diameter D_1, Δ_1, order N_1 and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_1}{2}$.
$B_0\Sigma_6 B_1$	B_0 is any bipartite graph $B_0=(V_0 \cup V_1, E)$ with even diameter D_0, degree Δ_0, order N_0 and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_0}{2}$. B_1 is any bipartite graph $B_1=(V_0 \cup V_1, E)$ with even diameter D_1, degree Δ_1, order N_1 and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_1}{2}$.
$B_0\Theta_4 B_1$	B_0 is any bipartite graph $B_0=(V_0 \cup V_1, E)$ with even diameter D_0, degree Δ_0, order N_0 and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_0}{2}$. B_1 is any bipartite graph $B_1=(V_0 \cup V_1, E)$ with even diameter D_1, degree Δ_1, order N_1 and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_1}{2}$.
$B_0\Sigma_7 B_1$	B_0 is any bipartite graph $B_0=(V_0 \cup V_1, E)$ with odd diameter D_0, degree Δ_0, order N_0 and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_0}{2}$. B_1 is any bipartite graph $B_1=(V_0 \cup V_1, E)$ with even diameter D_1, degree Δ_1, order N_1 and two disjoint subsets V_0 and V_1 such that $\|V_0\|=\|V_1\|=\frac{N_1}{2}$.

Each of these configurations requires two graphs and produces a compound graph by making copies of the two graphs with additional edges between vertices. The type of graphs used in each configuration are described in Table 2.

Using the graphs, as described in Table 2, the degree, diameter and order of the compound graphs, which are constructed from each configuration type, are shown in Table 3.

Table 3: Degree, diameter and order for various types of compound graphs.

	Degree Δ	Diameter D	Order N
$G \wedge B$	$\Delta = max\{\Delta_G + 2, \Delta_B + 1\}$	$D \leq D_G + D_B + 1$	$N = \frac{3}{2} N_G N_B$
$G \kappa_5 B$	$\Delta = max\{\Delta_G + 6, \Delta_B + 2\}$	$D \leq D_G + D_B + 1$	$N = \frac{15}{2} N_G N_B$
$B_0 \Theta_1 B_1$	$\Delta = max\{\Delta_0 + 2, \Delta_1 + 2\}$	$D \leq D_0 + D_1$	$N = N_0 N_1$
$B_0 \Sigma_6 B_1$	$\Delta = max\{\Delta_0 + 3, \Delta_1 + 2\}$	$D \leq D_0 + D_1$	$N = 3 N_0 N_1$
$B_0 \Theta_4 B_1$	$\Delta = max\{\Delta_0 + 3, \Delta_1 + 3\}$	$D \leq D_0 + D_1$	$N = 4 N_0 N_1$
$B_0 \Sigma_7 B_1$	$\Delta = max\{\Delta_0 + 3, \Delta_1 + 2\}$	$D \leq D_0 + D_1$	$N = \frac{5}{2} N_0 N_1$

All of the configurations described in this paper have similar patterns, however for a given degree and diameter some configurations generates larger graphs than others. This is due to the type of graphs used in the graph construction and configuration design. As shown in Table 3, graphs produced from configurations $B_0 \Theta B_1$, $B_0 \Sigma_6 B_1$, $B_0 \Theta_4 B_1$ and $B_0 \Sigma_7 B_1$ do not require any additional path length in their diameter, hence we can produce large graphs with diameter being the sum of diameters from the two graphs used in construction. However, a limitation for configurations $B_0 \Theta B_1$, $B_0 \Sigma_6 B_1$ and $B_0 \Theta_4 B_1$ is that we can only generating graphs with even diameter, while with configuration $B_0 \Sigma_7 B_1$ we can only generate graphs with odd diameter. Configurations $G \wedge B$ and $G \kappa_5 B$ do require one additional path length for the diameter, but these configurations can be used to generate a graph of any diameter greater than four.

We now provide a concrete example of a compound graph constructed by using one of configurations mentioned in this paper. The two graphs used in construction of the compound graph is shown in Figure 1 and these graphs are used to construct the compound graph $K_3 \wedge K_{2,2}$. Such construction uses configuration $G \wedge B$ and it is generated by taking two copies of graph K_3 and three copies of bipartite graph $K_{2,2}$ with extra adjacencies between copies of the graphs. The graph $K_3 \wedge K_{2,2}$ has degree $\Delta = max\{2+2, 2+1\} = 4$, diameter $D = 4$ and order $N = 18$. Construction details are shown in Figure 2.

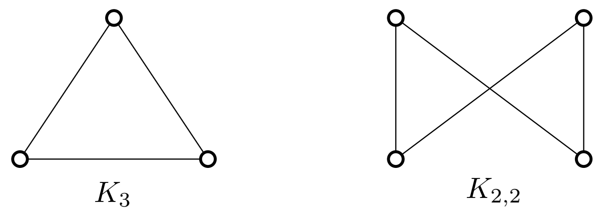

Figure 1: Graph K_3 and bipartite graph $K_{2,2}$

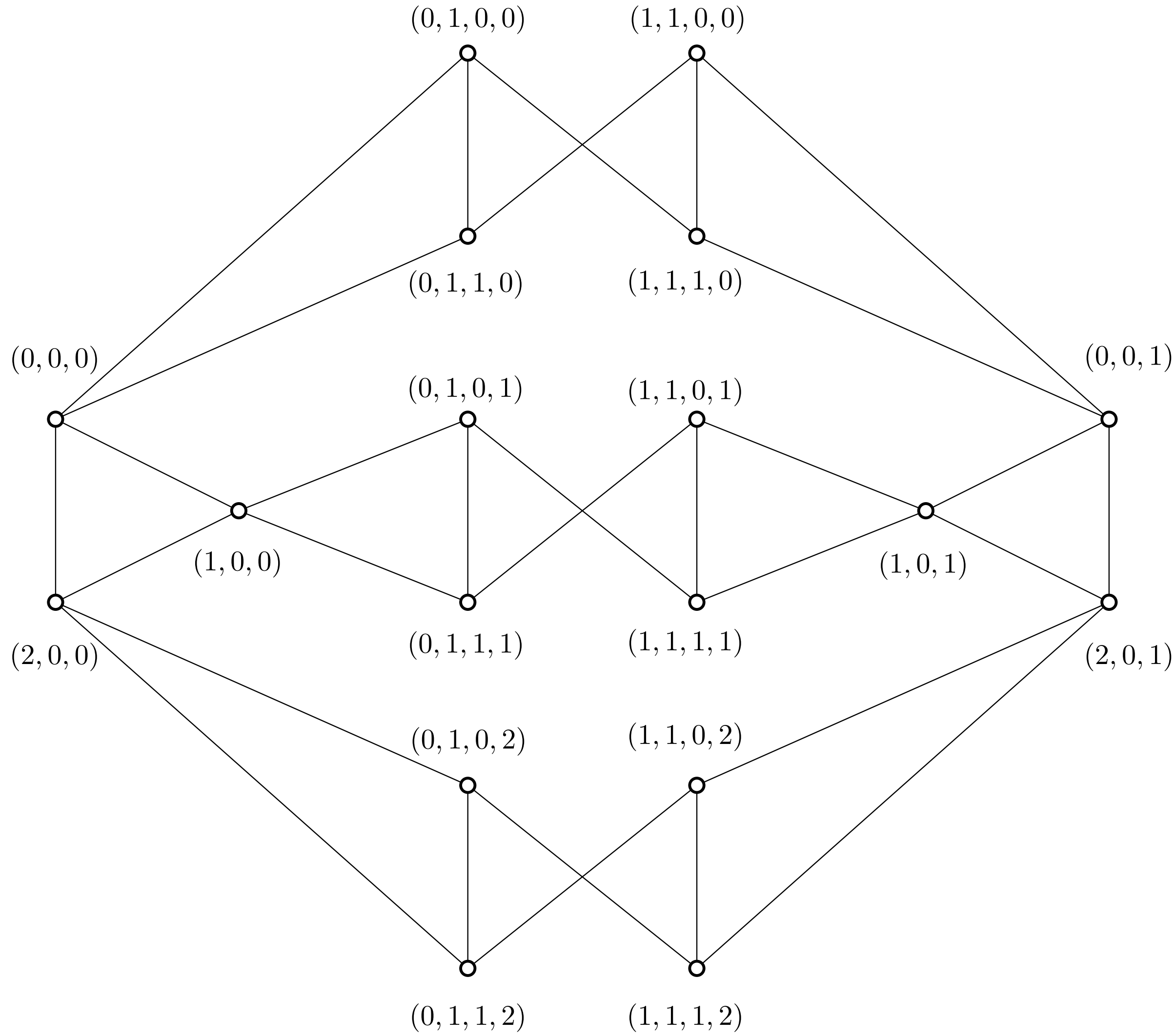

Figure 2: Construction of the compound graph $K_3 \wedge K_{2,2}$

5 (Δ, D) tables of largest compound graphs

For each configuration $G\kappa_5 B$, $B_0\Sigma_6 B_1$, $B_0\Theta_4 B_1$ and $B_0\Sigma_7 B_1$, we provide a (Δ, D) table containing the best (largest) graphs produced from its specified configuration. These graphs and orders for configurations $G\kappa_5 B$, $B_0\Sigma_6 B_1$, $B_0\Theta_4 B_1$ and $B_0\Sigma_7 B_1$ are shown in Tables 4, 5, 6, 7 and 8, respectively.

Some entries given in the of tables contain notations of configurations that have not been mentioned in the previous section. They are configurations that have been constructed by applying modifications to the existing configurations. The modifications involve adding additional copies of graph and adjacencies to original configuration, and in some cases these modified configurations can generate larger graph than the original configuration on a given degree and diameter. For configurations $G\kappa_5 B$, $B_0\Sigma_6 B_1$ and $B_0\Sigma_7 B_1$, one or more modified configurations exist and they are denoted by $G\kappa_5 B(n)$, $B_0\Sigma_6' B_1$, $B_0\Sigma_6 B_1(n)$ and $B_0\Sigma_7^1 B_1$, $B_0\Sigma_7^2 B_1$ and $B_0\Sigma_7 B_1(n)$. We refer readers to [GFS], [GM] and [Ki] for construction details of these modified configurations. In these tables, $T(m, n)$ refers to the largest known graphs of degree m and diameter n as given in [CD] (as of November 2007).

6 Verification of compound graphs

We empirically verified using a computer a few compound graphs given by Gómez and Miller, [GM] and Gómez, Fiol, and Serra, [GFS]. The $(14, 7)$ compound graph $K_1\Sigma_8 H_{11}$ of order 6200460 and some representatives, highlighted in Table 9, for various compound configerations where checked. All of the parameter values from the theoretical formula and algorithmic results on the adjacency lists[2] are the same, which empirically indicates that the construction technicques of all the compound graph described in this repository are correct.

[2]These graph adjacency lists are availble by request from the authors.

Table 4: A (Δ, D) table for compound graphs $B_0\Sigma_6 B_1$, $B_0\Sigma_6' B_1$ and $B_0\Sigma_6 B_1(n)$.

$\Delta\backslash D$	4	6	8	10
6	$K_{3,3}\Sigma_6 K_{4,4}$ 144	$K_{3,3}\Sigma_6 Q_3$ 1440	$K_{3,3}\Sigma_6 H_3$ 13104	$Q_2\Sigma_6 H_3$ 65520
7	$K_{4,4}\Sigma_6 K_{5,5}$ 240	$K_{4,4}\Sigma_6 Q_4$ 4080	$K_{4,4}\Sigma_6 H_4$ 65520	$Q_3\Sigma_6 H_4$ 655200
8	$K_{5,5}\Sigma_6 K_{6,6}$ 360	$K_{5,5}\Sigma_6 Q_5$ 9360	$K_{5,5}\Sigma_6 H_5$ 234360	$Q_4\Sigma_6 H_5$ 3984120
9	$K_{6,6}\Sigma_6 K_{7,7}$ 504	$K_{6,6}\Sigma_6' Q_5$ 13104	$Q_5\Sigma_6' Q_5$ 340704	$Q_5\Sigma_6' H_5$ 8530704
10	$K_{7,7}\Sigma_6' K_{7,7}$ 686	$K_{5,5}\Sigma_6 Q_7(2)$ 36000	$K_{5,5}\Sigma_6 H_7(2)$ 1764720	$Q_4\Sigma_6 H_7(2)$ 30000240
11	$K_{8,8}\Sigma_6' K_{8,8}$ 896	$K_{6,6}\Sigma_6 Q_8(2)$ 63180	$K_{6,6}\Sigma_6 H_8(2)$ 4044492	$Q_7\Sigma_6 H_8$ 179755200
12	$K_{9,9}\Sigma_6' K_{9,9}$ 1134	$K_{7,7}\Sigma_6 Q_9(2)$ 103320	$K_{7,7}\Sigma_6 H_9(2)$ 8370180	$Q_8\Sigma_6 H_9$ 466338600
13	$K_{10,10}\Sigma_6' K_{10,10}$ 1400	$K_{10,10}\Sigma_6' Q_9$ 114800	$Q_9\Sigma_6' Q_9$ 9413600	$Q_9\Sigma_6' H_9$ 762616400
14	$K_{11,11}\Sigma_6' K_{11,11}$ 1694	$K_{7,7}\Sigma_6 Q_{11}(3)$ 245952	$K_{7,7}\Sigma_6 H_{11}(3)$ 29762208	$Q_8\Sigma_6 H_{11}(2)$ 1865452680
15	$K_{12,12}\Sigma_6' K_{12,12}$ 2016	$K_{12,12}\Sigma_6' Q_{11}$ 245952	$Q_{11}\Sigma_6' Q_{11}$ 30006144	$Q_{11}\Sigma_6' H_{11}$ 3630989376
16	$K_{13,13}\Sigma_6' K_{13,13}$ 2366	$K_{9,9}\Sigma_6 Q_{13}(3)$ 514080	$K_{9,9}\Sigma_6 H_{13}(3)$ 86882544	$Q_{11}\Sigma_6 H_{13}$ 7066446912

Table 5: A (Δ, D) table for compound graphs $B_0\Theta_4B_1$.

$\Delta\backslash D$	4	6	8	10
6	$K_{3,3}\Theta_4K_{3,3}$ 144	$K_{3,3}\Theta_4Q_2$ 720	$Q_2\Theta_4Q_2$ 3600	$Q_2\Theta_4H_2$ 15120
7	$K_{4,4}\Theta_4K_{4,4}$ 256	$K_{4,4}\Theta_4Q_3$ 2560	$Q_3\Theta_4Q_3$ 25600	$Q_3\Theta_4H_3$ 232960
8	$K_{5,5}\Theta_4K_{5,5}$ 400	$K_{5,5}\Theta_4Q_4$ 6800	$Q_4\Theta_4Q_4$ 115600	$Q_4\Theta_4H_4$ 1856400
9	$K_{6,6}\Theta_4K_{6,6}$ 576	$K_{6,6}\Theta_4Q_5$ 14976	$Q_5\Theta_4Q_5$ 389376	$Q_5\Theta_4H_5$ 9749376
10	$K_{7,7}\Theta_4K_{7,7}$ 784	$K_{7,7}\Theta_4Q_5$ 17472	$K_{7,7}\Theta_4H_5$ 437472	$Q_5\Theta_4H_5$ 9749376
11	$K_{8,8}\Theta_4K_{8,8}$ 1024	$K_{8,8}\Theta_4Q_7$ 51200	$Q_7\Theta_4Q_7$ 2560000	$Q_7\Theta_4H_7$ 125491200
12	$K_{9,9}\Theta_4K_{9,9}$ 1296	$K_{9,9}\Theta_4Q_8$ 84240	$Q_8\Theta_4Q_8$ 5475600	$Q_8\Theta_4H_8$ 350522640
13	$K_{10,10}\Theta_4K_{10,10}$ 1600	$K_{10,10}\Theta_4Q_9$ 131200	$Q_9\Theta_4Q_9$ 10758400	$Q_9\Theta_4H_9$ 871561600
14	$K_{11,11}\Theta_4K_{11,11}$ 1936	$K_{11,11}\Theta_4Q_9$ 144320	$K_{11,11}\Theta_4H_9$ 11691680	$Q_9\Theta_4H_9$ 871561600
15	$K_{12,12}\Theta_4K_{12,12}$ 2304	$K_{12,12}\Theta_4Q_{11}$ 281088	$Q_{11}\Theta_4Q_{11}$ 34292736	$Q_{11}\Theta_4H_{11}$ 4149702144
16	$K_{13,13}\Theta_4K_{13,13}$ 2704	$K_{13,13}\Theta_4Q_{11}$ 304512	$K_{13,13}\Theta_4H_{11}$ 36848448	$Q_{11}\Theta_4H_{11}$ 4149702144

Table 6: A (Δ, D) table for compound graphs $B_0\Sigma_7 B_1$, $B_0\Sigma_7^1 B_1$, $B_0\Sigma_7^2 B_1$ and $B_0\Sigma_7 B_1(n)$.

$\Delta\backslash D$	5	7	9
6	$K_{1,1}\Sigma_7^2 Q_3$ 560	$K_{1,1}\Sigma_7^2 H_3$ 5096	$P_2\Sigma_7 H_3$ 25480
7	$K_{1,1}\Sigma_7 Q_4(2)$ 1360	$K_{1,1}\Sigma_7 H_4(2)$ 21840	$P_3\Sigma_7 H_4$ 177450
8	$K_{1,1}\Sigma_7 Q_5(2)$ 2496	$K_{1,1}\Sigma_7 H_5(2)$ 62496	$P_4\Sigma_7 H_5$ 820260
9	$K_{1,1}\Sigma_7^1 Q_5(2)$ 3120	$K_{1,1}\Sigma_7^1 H_5(2)$ 78120	$P_5\Sigma_7 H_5$ 1212860
10	$K_{1,1}\Sigma_7 Q_7(3)$ 8800	$K_{1,1}\Sigma_7 H_7(3)$ 431376	$P_5\Sigma_7^1 H_7$ 7294176
11	$K_{1,1}\Sigma_7^2 Q_8(2)$ 14040	$K_{1,1}\Sigma_7^2 H_8(2)$ 898776	$P_7\Sigma_7 H_8$ 21345930
12	$K_{1,1}\Sigma_7^2 Q_9(2)$ 19680	$K_{1,1}\Sigma_7^2 H_9(2)$ 1594320	$P_8\Sigma_7 H_9$ 48493900
13	$K_{1,1}\Sigma_7 Q_9(4)$ 22960	$K_{1,1}\Sigma_7 H_9(4)$ 1860040	$P_9\Sigma_7 H_9$ 60451300
14	$K_{1,1}\Sigma_7 Q_{11}(4)$ 40992	$K_{1,1}\Sigma_7 H_{11}(4)$ 4960368	$P_9\Sigma_7^1 H_{11}$ 193454352
15	$K_{1,1}\Sigma_7 Q_{11}(4)$ 40992	$K_{1,1}\Sigma_7 Q_{11}(4)$ 4960368	$P_{11}\Sigma_7 H_{11}$ 235617480
16	$K_{1,1}\Sigma_7 Q_{13}(5)$ 80920	$K_{1,1}\Sigma_7 H_{13}(5)$ 13675956	$P_{11}\Sigma_7^1 H_{13}$ 641965464

Table 7: A (Δ, D) table for compound graphs $G\kappa_5 B$ and $G\kappa_5 B(n)$.

$\Delta\backslash D$	4	5	6	7
6	$K_1\kappa_5 K_{4,4}$ 60	$K_1\kappa_5 Q_3$ 600	$K_1\kappa_5 Q_3$ 600	$K_1\kappa_5 H_3$ 5460
7	$K_1\kappa_5 K_{5,5}$ 75	$K_1\kappa_5 Q_4$ 1275	$K_2\kappa_5 Q_4$ 2550	$K_1\kappa_5 H_4$ 20475
8	$K_1\kappa_5 K_{6,6}$ 90	$K_1\kappa_5 Q_5$ 2340	$K_3\kappa_5 Q_5$ 7020	$K_1\kappa_5 H_5$ 58590
9	$K_1\kappa_5 K_{7,7}$ 105	$K_1\kappa_5 Q_5$ 2340	$K_4\kappa_5 Q_5$ 9360	$K_1\kappa_5 H_5$ 58590
10	$K_1\kappa_5 K_{8,8}(2)$ 200	$K_1\kappa_5 Q_7(2)$ 10000	$K_5\kappa_5 Q_7$ 30000	$K_1\kappa_5 H_7(2)$ 490200
11	$K_1\kappa_5 K_{9,9}(2)$ 225	$K_1\kappa_5 Q_8(2)$ 14625	$K_6\kappa_5 Q_8$ 52650	$K_1\kappa_5 H_8(2)$ 936225
12	$K_1\kappa_5 K_{10,10}(2)$ 250	$K_1\kappa_5 Q_9(2)$ 20500	$K_7\kappa_5 Q_9$ 86100	$K_1\kappa_5 H_9(2)$ 1660750
13	$K_1\kappa_5 K_{11,11}(2)$ 275	$K_1\kappa_5 Q_9(2)$ 20500	$K_8\kappa_5 Q_9$ 98400	$K_1\kappa_5 H_9(2)$ 1660750
14	$K_1\kappa_5 K_{12,12}(3)$ 420	$K_1\kappa_5 Q_{11}(3)$ 51240	$K_9\kappa_5 Q_{11}$ 197640	$K_1\kappa_5 H_{11}(3)$ 6200460
15	$K_1\kappa_5 K_{13,13}(3)$ 455	$K_1\kappa_5 Q_{11}(3)$ 51240	$K_{10}\kappa_5 Q_{11}$ 219600	$K_1\kappa_5 H_{11}(3)$ 6200460
16	$K_1\kappa_5 K_{14,14}(3)$ 490	$K_1\kappa_5 Q_{13}(3)$ 83300	$K_7\kappa_5 Q_{13}(2)$ 416500	$K_1\kappa_5 H_{13}(3)$ 14078190

Table 8: A (Δ, D) table for compound graphs $G\kappa_5 B$ and $G\kappa_5 B(n)$.

$\Delta \backslash D$	8	9	10
6	$K_1\kappa_5 H_3$ 5460	$K_1\kappa_5 H_3$ 5460	$K_1\kappa_5 H_3$ 5460
7	$K_2\kappa_5 H_4$ 40950	$K_2\kappa_5 H_4$ 40950	$K_2\kappa_5 H_4$ 40950
8	$K_3\kappa_5 H_5$ 175770	$C_5\kappa_5 H_5$ 292950	$C_7\kappa_5 H_5$ 410130
9	$K_4\kappa_5 H_5$ 234360	$T(3,2)\kappa_5 H_5$ 585900	$T(3,3)\kappa_5 H_5$ 1171800
10	$K_5\kappa_5 H_7$ 1470600	$T(4,2)\kappa_5 H_7$ 4411800	$T(4,3)\kappa_5 H_7$ 12058920
11	$K_6\kappa_5 H_8$ 3370410	$T(5,2)\kappa_5 H_8$ 13481640	$T(5,3)\kappa_5 H_8$ 40444920
12	$K_7\kappa_5 H_9$ 6975150	$T(6,2)\kappa_5 H_9$ 31886400	$T(6,3)\kappa_5 H_9$ 109609500
13	$K_8\kappa_5 H_9$ 7971600	$T(7,2)\kappa_5 H_9$ 49822500	$T(7,3)\kappa_5 H_9$ 167403600
14	$K_9\kappa_5 H_{11}$ 23916060	$T(8,2)\kappa_5 H_{11}$ 151468380	$T(8,3)\kappa_5 H_{11}$ 672307020
15	$K_6\kappa_5 H_{11}(2)$ 26573400	$T(9,2)\kappa_5 H_{11}$ 196643160	$T(9,3)\kappa_5 H_{11}$ 1554543900
16	$K_7\kappa_5 H_{13}(2)$ 70390950	$T(10,2)\kappa_5 H_{13}$ 549049410	$T(10,3)\kappa_5 H_{13}$ 3921781500

Table 9: Some computer verified (Δ, D) graphs.

Compound graph	Theoretical formula	Verified theoretical parameters
$K_{2,2}\Sigma_6 K_{3,3}$	$\Delta = max\{\Delta_0 + 3, \Delta_1 + 2\}$ $D \leq D_0 + D_1$ $N = 3N_0N_1$	$\Delta = max\{2+3, 3+2\} = 5$ $D \leq 2+2 = 4$ $N = 3 \times 4 \times 6 = 72$
$K_{2,2}\Sigma_6' K_{2,2}$	$\Delta = max\{\Delta_0 + 3, \Delta_1 + 3\}$ $D \leq D_0 + D_1$ $N = \frac{7}{2}N_0N_1$	$\Delta = max\{2+3, 2+3\} = 5$ $D \leq 2+2 = 4$ $N = \frac{7}{2} \times 4 \times 4 = 56$
$K_{2,2}\Theta_4 K_{2,2}$	$\Delta = max\{\Delta_0 + 3, \Delta_1 + 3\}$ $D \leq D_0 + D_1$ $N = 4N_0N_1$	$\Delta = max\{2+3, 2+3\} = 5$ $D \leq 2+2 = 4$ $N = 4 \times 4 \times 4 = 64$
$K_{1,1}\Sigma_7 K_{2,2}$	$\Delta = max\{\Delta_0 + 3, \Delta_1 + 2\}$ $D \leq D_0 + D_1$ $N = \frac{5}{2}N_0N_1$	$\Delta = max\{1+3, 2+2\} = 4$ $D \leq 1+2 = 3$ $N = \frac{5}{2} \times 2 \times 4 = 20$
$K_{1,1}\Sigma_7^1 K_{3,3}$	$\Delta = max\{\Delta_0 + 4, \Delta_1 + 2\}$ $D \leq D_0 + D_1$ $N = 3N_0N_1$	$\Delta = max\{1+4, 3+2\} = 5$ $D \leq 1+2 = 3$ $N = 3 \times 2 \times 6 = 36$
$K_{1,1}\Sigma_7^2 K_{4,4}$	$\Delta = max\{\Delta_0 + 5, \Delta_1 + 2\}$ $D \leq D_0 + D_1$ $N = \frac{7}{2}N_0N_1$	$\Delta = max\{1+5, 4+2\} = 6$ $D \leq 1+2 = 3$ $N = \frac{7}{2} \times 2 \times 8 = 56$
$K_1\kappa_5 K_{4,4}$	$\Delta = max\{\Delta_G + 6, \Delta_B + 2\}$ $D \leq D_G + D_B$ $N = \frac{5}{2}N_GN_B + 5N_GN_B$	$\Delta = max\{0+6, 4+2\} = 6$ $D \leq 0+2+1 = 3$ $N = \frac{5}{2} \times 1 \times 8 + 5 \times 1 \times 8 = 60$
$K_1\kappa_5 K_{8,8}(2)$	$\Delta = max\{\Delta_G + 10, \Delta_B + 2\}$ $D \leq D_G + D_B + 1$ $N = \frac{5}{2}N_GN_B + 10N_GN_B$	$\Delta = max\{0+10, 8+2\} = 10$ $D \leq 0+2+1 = 3$ $N = \frac{5}{2} \times 1 \times 16 + 10 \times 1 \times 16 = 200$
$K_{2,2}\Sigma_6 K_{5,5}(2)$	$\Delta = max\{\Delta_0 + 5, \Delta_1 + 2\}$ $D \leq D_0 + D_1$ $N = \frac{9}{2}N_0N_1$	$\Delta = max\{2+5, 5+2\} = 7$ $D \leq 2+2 = 4$ $N = \frac{9}{2} \times 4 \times 10 = 180$
$K_{1,1}\Sigma_7 K_{5,5}(2)$	$\Delta = max\{\Delta_0 + 6, \Delta_1 + 2\}$ $D \leq D_0 + D_1$ $N = 4N_0N_1$	$\Delta = max\{1+6, 5+2\} = 7$ $D \leq 1+2 = 3$ $N = 4 \times 2 \times 10 = 80$

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