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Science at the Crossroad Between Randomness and Determinism

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Abstract

Time and again, man's understanding of Nature is at the crossroad between total world-comprehension and total randomness. It is suggested that not only are the preferences influenced by the theories and models of today, but also by the very personal subjective inclinations of the people involved. The second part deals with the principle of self-consistency and its consequences for totally deterministic systems.

1 Who is more afraid of what?

Let me start with a question to you, the reader of this article.

> *"What appears to be more frightening: a clocklike universe which is totally governed by deterministic laws, or a lawless universe which is totally unpredictable and random?"*

1.1 Clocklike universe

In a totally deterministic "clocklike" universe, every single phenomenon is predetermined by its previous state. Once the initial stage is "set up," its creator gets detached from it and watches—without in any way influencing it—as time and events go by.

In particular, no room is left for free will at all. To any kind of personality and conscious agent imprisoned in such a universe, free will must be a subjective impression which is an illusion—*Maya*. If these agents could only look behind the scene, then they would know. But as the clocklike universe is hermetic, to them any such beyond does not make any operational sense.

Clocklike universes are nowadays best described by the term "algorithm" [1, 2]. Via the Church-Thesis, they can be even formalized by recursive function theory [3, 4]. From this point of view, the universe appears as a gigantic (from our perspective), presumably universal, computer. Conscious agents are just temporary imprints or patterns on whatever "hyper-substance" it may be made of.

If this indeed would be the case with the universe we are living in, then what appears to be amazing is the mere possibility of our self to imagine these scenarios; to phantasize about free will being an illusion and about a hierarchical organization of reality; to express *Maya*. This is not totally new: already von Neumann considered the possibilities of implanting agents in a universal cellular automata substratum capable of self-reproduction and introspection [5]. Fredkin has developed "digital mechanics" [6] and "digital soal."

Within totally deterministic systems, subjective indeterminism may result from intrinsic undecidability. There exist various forms of intrinsic indeterminism (see [2] for a review); among them undecidability analogous to the recursive unsolvability of the halting problem, and computational complementarity [7].

Let me gear up this scenario by purporting that not only might the universe be clocklike, but *reversibile.* That means that every process therein, every single evolution step, is one-to-one; in more formal terms, the evolution map between initial and final state is bijective.

In such a reversible hermetic prison, the time evolution is a constant permutation of one and the same "message" which always remains the same but expresses itself through different forms. Information is neither created nor discarded but remains constant at all times. The implicit time symmetry spoils the very notion of "progress" or "achievement," since what is a valuable output is purely determined by the subjective meaning the observer associates with it and is devoid of any syntactic relevance. In such a scenario, any gain in knowledge remains a merely subjective impression of ignorant observers.

Let us now turn to the other extreme.

1.2 Lawless universe

Both chaos theory and quantum mechanics assert that there is an irreducible randomness in nature.

One concrete example of this allegedly irreducible randomness is the "quantum coin toss" [8] realized recently be the group of Anton Zeilinger [9]. It is a which-way detection of a single photon passing through a semitransparent mirror or a calcit crystal.

A lawless universe is characterized by the—admittedly highly nonconstructive—property that it is not governed by any law at all. There could be no principle which could in any way "explain" or "predict" the performance of such a universe. More importantly: there could be no control over events. Formally, a lawless universe can be represented by a Martin-Löf/Solovay/Chaitin random [10, 11, 12] bit string [13, 14].

This does not mean that on a *local* scale, say, for any finite number of phenomenologic occurrences or evolution steps, the lawless universe cannot appear to be governed by laws. Indeed, some observers embedded in a totally lawless universe [15, 16, 17, 18, 19, 2, 20] might figure out some local structure and believe that this could persist for any finite time for any finite extension. They, like us, might call this the cosmological principle.

Because of the lack of meaning, observers could experience total freedom. This resembles the absurd freedom of existentialism. Because if there is no law, there cannot be any convincing moral codex, at least globally. Any kind of behaviour or decision would at most make local sense, but would be devoid of any deeper, permanent relevance. From a global ethical point of view, any decision would be reduced to the throwing of a fair coin.

It is not totally unreasonable to speculate that the cosy little lawful local worlds some observers appear to be living in could be a mere subjective fantasy, a subjective impression which is an illusion—*Maya* again. And physics and all natural sciences may just amount to pretentious talk about finite lawful bubbles within an endless ocean of chaos.

This may be not the full story. Consider a related question, namely

> *"Can there be order out of chaos?"*

As of today, the answer to this question is unknown. A quite straightforward positive answer can be given by applying the law of large numbers: if, for instance, one is measuring the output of a random source emitting the binary symbols "0" and "1", and if one just waits long enough, then each one of these binary symbols occurs with probability 1/2.

Formally, Martin-Löf/Solovay/Chaitin random sequences are Borel normal; i.e., contain the code of any finite universe an infinite number of times. By the very way it was defined, any Martin-Löf/Solovay/Chaitin random sequence obeys all statistical laws associated with randomness.

If we are justified to derive more lawful structures out of such random sources is debatable but challenging. The most radical answer I can think of is that there is a *unique* and *robust* class of laws emerging, and that these laws correspond to the physical universe we are living in. Robust in this context means that the laws are not changed "very much" if we focus on different finite parts of the source code.

1.3 Miracles

Besides the clocklike and the lawless universe there appears to be at least another variant: A clocklike universe inspired by miracles. In what follows, we shall denote by "miracle" all *ad hoc* occurrances which can in no way be explained in an otherwise clocklike universe. Miracles have been studied by the Vienna Circle, in particular by Philip Franck [21].

Imagine the following example. Suppose you are an actor in a virtual computer game (such as Quake) in which a number of persons interact collectively. Their virtual reality environment is totally lawful: it is created by a single computer or a network of computers. Yet, what is going on in this virtual environment is not totally determined by the computer system alone, but decisively by the constant input of the players. The players act and input via interfaces. Since the interface is not total, "part of" the player will always be beyond the scope of the game. Thus many of the interventions of the players are beyond the scope of the limited domain of the virtual reality interface through which they interact.

Let us consider a trivial example: one player feels hungry and decides to take a break and order some Pizza in the "real world." This act may come as a total surprise and cannot be precisely predicted or predetermined within the "virtual world" of the game.

Almost needless to say, this picture is an old idea in a relatively new context—dualism.

1.4 Personal preferences

As the topic is far from being settled, it is not unreasonable to assume that each individual researcher has his or her personal preferences. We take the position here that these preferences are mostly determined by the person's fears and desires.

Clocklike universes may appear monotonic and dull, without any possibility to act freely. Lawless universes may appear totally incomprehensive, arbitrary and weird.

On the other hand, at least to a certain extend, clocklike universes appear (subjectively) controllable and predictable. This possibility may bring about a certain kind of dignity felt by the Enlightenment: man is not confronted with a totally random environment but can influence the world according to his own desires.

Lawless universes seem to guarantee spontaneity and freedom. They don't appear to be hermetic prisons and have an open future which is constantly created.

2 Limits to forcast and event control

Are there limits to event forecast and event control for observers embedded in totally deterministic systems?

Here we shall argue for *complementarity* in such systems. It is a robust notion insofar this feature does not depend on the particular type of deterministic system.

Intuitively, complementarity states that it is impossible to (irreversibly) observe certain observables simultaneously with arbitrary accuracy. The more precisely one of these observables is measured, the less precisely can be the measurement of other—complementary—observables. Typical examples of complementary observables are position/momentum (velocity), angular momentum in the x/y/z direction, and particle number/phase [22, 23].

Let us develop *computational complementarity,* as it is often called [24, 25], as a game between you as the reader and me as the author. The rules of the game are as follows. I first give you all you need to know about the intrinsic workings of the automaton. For example, I tell you, "if the automaton is in state 1 and you input the symbol 2, then the automaton will make a transition into state 2 and output the symbol 0;" and so on. Then I present you a black box which contains a realization of the automaton. The black box has a keyboard, with which you input the input symbols. It has an output display, on which the output symbols appear. No other interfaces are allowed. Suppose that I can choose in which initial state the automaton is at the beginning of the game. I do not tell you this state. Your goal is to find out by experiment which state I have chosen. You can simply guess or relying on your luck by throwing a dice. But you can also perform clever input-output experiments and analyze your data in order to find out. You win if you give the correct answer. I win if you guess incorrectly. (So, I have to be mean and select worst-case examples).

Suppose that you try very hard. Is cleverness sufficient? Will you always be able to uniquely determine the initial automaton state?

The answer to that question is "no." The reason for this is that there may be situations when the input causes an irreversible transition into a state which does not allow any further queries about the initial state. This is the meaning of the term "self-interference" mentioned above. Any such irreversible loss of information about the initial value of the automaton can be traced back to many-to-one operations [26]: different states are mapped onto a single state with the same output. Many-to-one operations such as "deletion of information" are the only source of entropy increase in mechanistic systems [26, 27].

The reader is refered to much more detailed accounts in refs. [2, 28, 7].

3 Principle of self-consistency

Let us assume, for the rest of the article, that the universe is clocklike.

In this part we shall review consequences of the basic and most evident consistency requirement—that measured events cannot happen and not happen at the same time. As a consequence, particular, very general bounds on the forecast and control of events within the known laws of physics are derived. These bounds are of a global, statistical nature and need not affect singular events or groups of events.

An irreducible, atomic physical phenomenon manifests itself as a click of some detector. There can either be a click or there can be no click. This yes-no scheme is experimental physics in-a-nutshell (at least according to a theoretician). From this type of elementary observation, all of our physical evidence is accumulated. Irreversibly observed events of physical reality (in the context in which they can be defined [29, 30, 31]) are subject to the primary condition of *consistency* or *self-consistency.*

> *Any particular irreversibly observed event can either happen or cannot happen, but it must not both happen and not happen.*

Indeed, so trivial seems the requirement of consistency for the set of physically recorded events that David Hilbert polemicised against "another author" with the following words [32], "...for me, the opinion that the [[physical]] facts and events themselves can be contradictory is a good example of thoughtlessness."

Just as in mathematics, inconsistency, i.e., the coexistence of truth and falseness of propositions, is a fatal property of any physical theory. Nevertheless, in a certain very precise sense, quantum mechanics incorporates inconsistencies in a very subtle way which assures overall consistency. For instance, a particle wave function or quantum state is said to "pass" a double slit through both slits, which is classically impossible. (Such considerations may, however, be considered as mere trickery quanum talk, devoid of any operational meaning.) Yet, neither a particle wave function nor quantum states are directly associable with any sort of irreversible observed event of physical reality.

And just as in mathematics it can be argued that too strong capacities of event forecast and event control renders the system overall inconsistent.

3.1 Strong forecasting

Let us consider forecasting the future first. Even if physical phenomena occur deterministically and can be accounted for ("computed") on a higher level of abstraction, from within the system such a complete description may not be of much practical, operational use.

Indeed, suppose there exists free will. Suppose further that an agent could predict *all* future events, without exceptions. We shall call this the *strong form of forecasting.* In this case, the agent could freely decide to counteract in such a way as to invalidate that prediction. Hence, in order to avoid inconsistencies and paradoxes, either free will has to be abandoned or it has to be accepted that complete prediction is impossible.

Another possibility would be to consider strong forms of forecasting which are, however, not utilized to alter the system. Effectively, this results in the abandonment of free will, amounting to an extrinsic, detached viewpoint. After all, what is knowledge and what is it good for if it cannot be applied and made to use?

It should be mentioned that the above argument is of an ancient type. It has been formalized recently in set theory, formal logic and recursive function theory, where it is called "diagonalization method."

3.2 Strong event control

A very similar argument holds for event control and the production of "miracles" [21]. Suppose there exists free will. Suppose further that an agent could entirely control the future. We shall call this the *strong form of event control.* Then this observer could freely decide to invalidate the laws of physics. In order to avoid a paradox, either free will or some physical laws would have to be abandoned, or it has to be accepted that complete event control is impossible.

> *Stated differently, forecast and event control should be possible only if this capacity cannot be associated with any paradox or contradiction.*

Thus the requirement of consistency of the phenomena seems to impose rather stringent conditions on forecasting and event control. Similar ideas have already been discussed in the context of time paradoxes in relativity theory (cf. [33] and [34, p. 272], *"The only solutions to the laws of physics that can occur locally ... are those which are globally self-consistent"*).

3.3 Weak forcast and event control

There is, however, a possibility that the forecast and control of future events *is* conceivable for *singular* events within the statistical bounds. Such occurrences may be "singular miracles" which are well accountable within classical physics. They will be called *weak forms of forecasting and event control.*

It may be argued that, in order to obey overall consistency, such a framework should not be extendable to any forms of strong forecast or event control, because, as has been argued before, this could either violate global consistency criteria or would make necessary a revision of the known laws of physics.

It may be argued that weak forms of forecasting and event control amount to nothing else than the impossibility of *any forms of forecasting and event control* at all.

This, however, needs not to be the case. The laws of statistics impose rather lax constraints and do not exclude local, singular, improbable events. For example, a binary sequence such as

$$1111111111111111111111111111111$$

is just as probable as the sequences

$$1110010111010100011100000110101 01$$

$$0101010101010101010101010101010 1$$

and its occurrence in a test is equally likely, although its statistical property and the "meaning" an observer could ascribe to it is rather outstanding.

Just as it is perfectly all right to consider the statement "This statement is true" to be true, it may thus be perfectly reasonable to speculate that certain events are forecasted and controlled within the domain of statistical laws. But in order to be within the statistical laws, any such method *needs not to be guaranteed* to work all the time.

To put it pointedly: it may be perfectly reasonable to become rich, say, by singular forecasts of the stock and future values or in horse races, but such an ability must necessarily be irreproducible and secretive. At least to such an extend that no guarantee of an overall strategy can be derived from it.

The associated weak forms of forecasting and event control are thus beyond any global statistical significance. Their importance and meaning seem to lie mainly on a very subjective level of singular events. This comes close to one aspect of what Jung imagined as the principle of "Synchronicity" [35].

3.4 Against the odds

This final paragraphs review a couple of experiments which suggest themselves in the context of weak forecast and event control. All are based on the observation that an agent forcasts or controls correctly future events such as, say, the tossing of a fair coin.

In the first run of the experiment, no consequence is derived from the agent's capacity despite the mere recording of the data.

The second run of the experiment is like the first run, but the *meaning* of the forecasts or controlled events are different. They are taken as outcomes of, say gambling, against other individuals (i) with or (ii) without similar capacities, or against (iii) an anonymous "mechanic" agent such as a casino or a stock exchange.

As a variant of this experiment, the partners or adversaries of the agent are informed about the agent's intentions.

In the third run of experiments, the experimenter attempts to counteract the agent's capacity. Let us assume the experimenter has total control over the event. If the agent predicts or attempts to bring about to happen a certain future event, the experimenter causes the event not to happen and so on.

It might be interesting to record just how much the agent's capacity is changed by the setup. Such a correlation might be defined from a dichotomic observable

$$e(A,i)=\left\{\begin{array}{ll} +1 & \text{correct guess} \\ -1 & \text{incorrect guess} \end{array}\right.$$

where i stands for the i'th experiment and A stands for the agent A. A correlation function can then be defined as usual by the average over N experiments; i.e.,

$$C(A)=\frac{1}{N}\sum_{i=1}^{N} e(A,i).$$

From the first to the second type of experiment it should become more and more unlikely that the agent operates correctly, since his performance is leveled against other agents with more or less the same capacities. The third type of experiment should produce a total uncorrelation.

Postscript

Instead of a summary, let me cite from a 1983 poem by Erich Christian Schreibmller.

> *Er nennt sich heimlich den ausgelassensten Dentisten der Galaxie, doch wei er natrlich nichts von den wahren Verhltnissen.*
>
> English translation: *Secretly he calls himself the most flamboyant dentist of the galaxy, but of course he does not realize the true circumstances.*

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