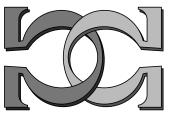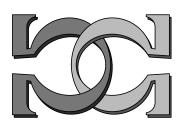# CDMTCS
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Quantum Correlations Conundrum: An Automaton-Theoretic Approach

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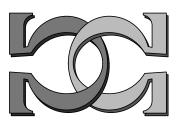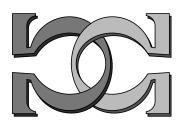# Quantum Correlations Conundrum: An Automaton-Theoretic Approach

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Abstract. We develop an automatic-theoretic analysis of *Einstein-Podolsky-Rosen* conundrum on the basis of two simple devices introduced by Mermin [10, 11].

1 Introduction

Quantum entanglement [16] and nonlocal correlations [1, 6] are "mindboggling" features of quantized systems. They cannot be expected nor explained in the context of classical 19th century physics. Currently, a "classical understanding" is often interpreted mechanistically (algorithmically) in terms of computation theory. If the domain is restricted even further, one is lead to a notion of classicality based on finite deterministic automata. One of the most radical views would be based on the assumption that indeed, theoretical physics can be "reduced" to finite automata.

As highly speculative as these attempts towards a finite physical understanding may appear, they have been very stimulating for automata theory. Moore [13] developed his finite automaton model with an analogy to quantum theory in mind; recent developments include the study of computational complementarity [9, 17, 3, 4] and empirical propositional calculus of finite and reversible automata [17, 18].

In what follows we shall concentrate on one of the biggest enigma of quantum mechanics, the *Einstein-Podolsky-Rosen (EPR)*. Our automatic-theoretic analysis will be based on Mermin's simple devices [10, 11] designed to explain quantum correlations.

2 EPR Conundrum and Bell's Theorem

According to the philosophical view called realism, *reality* exists and has definite properties irrespective whether they are observed by some agent. Motivated by this view point, Einstein, Podolsky and Rosen [8] suggested a classical argument showing that quantum mechanics is incomplete. EPR

assumed a) the non-existence of action-at-a-distance, b) that some of the statistical predictions of quantum mechanics are correct, and c) a reasonable criterion defining the existence of "an element of physical reality".[1] They considered a system of two spatially separated but quantum mechanically correlated particles. A "mysterious" feature appears: By counterfactual reasoning, quantum mechanical experiments yield outcomes which cannot be predicted by quantum theory; hence the quantum mechanical description of the system is incomplete!

One possibility to complete the quantum mechanical description is to postulate additional "hidden-variables" in the hope that completeness, determinism and causality will be thus restored.

But then, another conundrum occurs: Using basically the same postulates as those of EPR, Bell [1, 2] showed that no deterministic local hidden-variables theory can reproduce all statistical predictions of quantum mechanics. Bell's argument applied to an EPR-type *Gedanken experiment* of Bohm; later, Bell's analysis was extended to actual systems and experimental tests were suggested and performed (see, for example, [5]). Essentially, the particles on either side appear to be "more correlated" than can be expected by a classical analysis assuming locality (i.e., the impossibility of any kind of information or correlation transfer faster than light). In more concrete terms, if each single one of the particles can be in either one of the two states "+" or "-", then, for almost all setups, more "++" and "−−" (and less "+-" and "−−") coincidences are recorded than can be explained by any local classical analysis.

3   Mermin's Devices

Mermin [10, 11] imagined two simple devices to explain EPR conundrum without using the classical quantum mechanical notions of wave functions, superposition, wave-particle duality, uncertainty principle, etc.

3.1   Mermin's EPR Device

Mermin's EPR device [10] has three "completely unconnected"[2] parts, two detectors (D1) and (D2) and a source (S) emitting particles. The source is placed between the detectors: whenever a button is pushed on (S), shortly thereafter two particles emerge, moving off toward detectors (D1) and (D2). Each detector has a switch that can be set in one of three possible positions–labeled 1,2,3–and a bulb that can flash a red (R) or a green (G) light. The purpose of lights is to "communicate" information to the observer. Each detector flashes either red or green whenever a particle reaches it. Because of the lack of any relevant connections between any parts of the device, the link between the emission of particles by (S), i.e., as a result of pressing a button, and the subsequent flashing of detectors can only be provided by the passage of particles from (S) to (D1) and (D2). Additional tools can be used to check and confirm the lack of any communication, cf. [10], p. 941.

The device is repeatedly operated as follows:

1. the switch of either detector (D1) and (D2) is set randomly to 1 or 2 or 3, i.e., the settings or states 11, 12, 13, 21, 22, 23, 31, 32, 33 are equally likely,
2. pushing a button on (S) determines the emission toward both (D1) and (D2),
3. sometime later, (D1) and (D2) flash one of their lights, G or R,
4. every run is recorded in the form $ijXY$, meaning that D1 was set to state i and flashed X and (D2) was set to j and flashed Y.

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[1] *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physica quantity. See [8], p. 777.*

[2] There are no relevant connections, neither mechanical nor electromagnetic.

For example, the record $31GR$ means "(D1) was set to 3 and flashed G and (D2) was set to 1 and flashed R".

Long recorded runs show the following pattern:

a) For records starting with ii, i.e., 11, 22, 33, both (D1) and (D2) flash the same colours, RR, GG, with equal frequency; RG and GR are never flashed.
b) For records starting with $ij, i \neq j$, i.e., 12, 13, 21, 23, 31, 32, both (D1) and (D2) flash the same colour only 1/4 of the time (RR and GG come with equal frequencies); the other 3/4 of the time, they flash different colours (RG, GR), occurring again with equal frequencies.

Of course, the above patterns are statistical, that is they are subject to usual fluctuations expected in every statistical prediction: patterns are more and more "visible" as the number of runs becomes larger and larger.

The conundrum posed by the existence of Mermin's device reveals as soon as we notice that the seemingly simplest physical explanation of the pattern a) is incompatible with pattern b). Indeed, as (D1) and (D2) are unconnected there is no way for one detector to "know", at any time, the state of the other detector or which colour the other is flashing. Consequently, it seems plausible to assume that the colour flashed by detectors is determined only by some property, or group of properties, of particles, say speed, size, shape, etc. What properties determine the colour does not really matter; only the fact that each particle carries a "program" which determines which colour a detector will flash in some state is important. So, we are led to the following two hypotheses:

H1 *Particles are classified into eight categories:*

$$GGG, GGR, GRG, GRR, RGG, RGR, RRG, RRR.^3$$

H2 *Two particles produced in a given run carry identical programs.*

According to H1–H2, if particles produced in a run are of type RGR, then both detectors will flash R in states 1 and 3; they will flash G if both are in state 2. Detectors flash the same colours when being in the same states because *particles carry the same programs.*

It is clear that from H1–H2 it follows that *programs carried by particles do not depend in any way on the specific states of detectors*: they are properties of particles not of detectors. Consequently, both particles carry the same program whether or not detectors (D1) and (D2) are in the same states.[4]

We are ready to argue that

[L]   For each type of particle, *in runs of type* b) *both detectors flash the same colour at least one third of the time.*

If both particles are of types GGG or RRR, then detectors will flash all the time the same colour. For particles carrying programs containing one colour appearing once and the other colour appearing twice, only in two cases out of six possible combinations both detectors will flash the same light. For example, for particles of type RGR, both detectors will flash R if (D1) is in state

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[3] A particle of type XYZ will cause a detector in state 1 to flash X; a detector in state 2 will flash Y and a detector in state 3 will flash Z.

[4] The emitting source (S) has no knowledge about the states of (D1) and (D2) and there is no communication among any parts of the device.

1 and (D2) is in state 3 and vice versa. In all remaining cases detectors will flash different lights. The argument remains the same for all combinations as the conclusion was solely based on the fact that one colour appears once and the other twice. So, the lights are the same one third of the time.

The conundrum reveals as a significant difference appears between the data dictated by particle programs (colours agree at least one third of the time) and the quantum mechanical prediction (colours agree only one fourth of the time):

under H1–H2, the observed pattern b) is incompatible with [L].

3.2 Mermin's GHZ Device

Based on Greenberg, Horne, and Zeilinger [6] version of EPR experiment, Mermin [11] imagined a new device, let's call it GHZ, to show quantum nonlocality. The device has a source and three widely separated detectors (A), (B), (C), each of which has only two switch settings, 1 and 2. Any detector, when triggered, flashes red (R) or green (G). Again, detectors are supposed to be far away from the source and there are no connections between the source and detectors (except those induced by a group of particles flying from the source to each detector).

The experiment runs as following. Each detector is in a randomly chosen state (1 or 2) and then by pressing a button at the source a trio of particles are released towards detectors; each particle will reach a detector and, consequently, each detector will flash a light, green or red. There are eight possible states, but for the argument we need to take into consideration only those for which the number of 1's is odd, i.e., 111, 122, 212, 221.

According to [6], a) if one detector is set to 1 (and the others to 2), then an *odd* number of red lights always flash, i.e., RRR, RGG, GRG, GGR, and they are equally likely, b) if all detectors are set to 1, then an *odd* number of red lights is *never* flashed: GRR, RGR, RRG, GGG.

It is immediate that in case a) knowing the colour flashed by two detectors, say (A) and (B), *determines uniquely* the colour flashed by the third detector, (C). The explanation can come only because particles are emitted by the same source (there are no connections between detectors). A similar conclusion as in the case of EPR device reveals: *particles carry programs instructing their detectors what colour to flash.* Any particle carries a program of the form XY telling its detector to flash colour X if in state 1 and colour Y if in state 2. There are four types of programs: GG, GR, RG, RR. A run in which programs carried by the trio of particles are of types (RG, GR, GG) will result in RRG if the states were 122, in GGG if the states were 212, and in GRG if the states were 221. This is an *illegal* set of programs as the number of R's is not odd (in RRG, for example). A *legal* set of programs is (RG, GR, GR) as it produces RRR, GGR, GRG on 122, 212, 221. There are eight legal programs, (RR, RR, RR), (RR, GG, GG), (GG, RR, GG), (GG, RR, GG), (GG, GG, RR), (RG, GR, GR), (RG, RG, RG), (GR, GR, RG), and (GR, RG, GR) out of 64 possible programs.

The conundrum reveals again as none of the above programs respects b), i.e., it is compatible with the case 111. *A single* 111 *run suffices to prove inconsistency! Particle programs require an odd number of R's to be flashed on* 111, *but quantum mechanics prohibits this in every* 111 *run.*

4 Correlations via Moore Automata

Consider now a probabilistic automaton[5] simulating Mermin's EPR device. The states of the automaton are all combinations of states of detectors (D1) and (D2), $Q = \{11, 12, 13, 21, 22, 23, 31, 32, 33\}$, the input alphabet models the lights, red and green, $\Sigma = \{G, R\}$, the output alphabet captures all combinations of lights flashed by (D1) and (D2),

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[5] See Paz [14] for a general theory of probabilistic automata.

$O = \{GG, GR, RG, RR\}$, and the output function $f : Q \to O$, modeling all combinations of green/red lights flashed by (D1) and (D2) in all their possible states, is probabilistically defined by:

$$f(ii) = XX, \text{ with probability } 1/2, \text{ for } i = 1, 2, 3, X \in \{G, R\},$$
$$f(ii) = XY, \text{ with probability } 0, \text{ for } i = 1, 2, 3, X, Y \in \{G, R\}, X \neq Y,$$
$$f(ij) = XX, \text{ with probability } 1/8, \text{ for } i, j = 1, 2, 3, i \neq j, X \in \{G, R\},$$
$$f(ij) = XY, \text{ with probability } 3/8, \text{ for } i, j = 1, 2, 3, i \neq j, X, Y \in \{G, R\},$$
$$X \neq Y.$$

For example, $f(11) = RR$ with probability $1/2$, $f(11) = GR$ with probability 0, $f(11) = RG$ with probability 0, $f(11) = RR$ with probability $1/2$, $f(12) = GG$ with probability $1/8$, $f(12) = GR$ with probability $3/8$, $f(12) = RG$ with probability $3/8$, $f(12) = RR$ with probability $1/8$, etc.

The automaton transition $\delta : Q \times \Sigma \to Q$ is *not specified*. In fact, varying all transition functions δ we get a class of Mermin EPR automata:

$$\mathcal{M}(EPR) = (Q, \Sigma, O, \delta, (p_{ij}^{XY}, i, j = 1, 2, 3, X, Y \in \{G, R\})),$$

where $p_{ii}^{XX} = 1/2$, $p_{ii}^{XY} = 0$, $X \neq Y$, $p_{ij}^{XX} = 1/8$, $p_{ij}^{XY} = 3/8$, $X \neq Y$.

Are there two identical, spatially separated, probabilistic automata with identical initial states, whose direct product "simulates" a Mermin's EPR automaton? More formally, are there two probabilistic automata

$$\mathcal{M}_i = (\{1, 2, 3\}, \{G, R\}, \{G, R\}, \delta_i, (\alpha_{i,j}^X, j = 1, 2, 3, X \in \{G, R\}))$$

such that their direct product $\mathcal{M}_1 \otimes \mathcal{M}_2$ is isomorphic to a Mermin's automaton $\mathcal{M}(EPR)$, i.e., $\delta(ij, X) = \delta_1(i, X)\delta_2(j, X)$, and $p_{ij}^{XY} = \alpha_{1,i}^X \alpha_{2,j}^Y$, for all $j = 1, 2, 3, X, Y \in \{G, R\}$?

The answer is *negative*. In fact, a stronger result is true:

no single state of any Mermin's EPR probabilistic automaton $\mathcal{M}(EPR)$ can be simulated by the product of the corresponding states of any probabilistic automata $\mathcal{M}_i$.

Indeed, $\alpha_{i,j}^G = 1 - \alpha_{i,j}^R$. For a state ii we get the following contradictory relations:

$$\alpha_{1,i}^G \, \alpha_{2,i}^G = (1 - \alpha_{1,i}^G)(1 - \alpha_{2,i}^G) = 1/2,$$

$$\alpha_{1,i}^G \, (1 - \alpha_{2,i}^G) = (1 - \alpha_{1,i}^G) \, \alpha_{2,i}^G = 0.$$

For a state kl with $k \neq l$ we, again, get two contradictory relations:

$$\alpha_{1,k}^G \, \alpha_{2,l}^G = (1 - \alpha_{1,k}^G)(1 - \alpha_{2,l}^G) = 1/8,$$

$$\alpha_{1,k}^G \, (1 - \alpha_{2,l}^G) = (1 - \alpha_{1,k}^G) \, \alpha_{2,l}^G = 3/8.$$

Every Mermin's EPR probabilistic automaton $\mathcal{M}(EPR)$ has strong correlations preventing it from being decomposed as a direct product of two independent probabilistic automata, no matter what transitions and output functions.

Let's turn our attention to Mermin's GHZ device and to this aim consider a probabilistic automaton simulating Mermin's GHZ device. The states of the Mermin's GHZ automaton are all combinations of states of detectors (A), (B) and (C), $Q = \{111, 112, 121, 122, 211, 212, 221, 222\}$, the input alphabet models the lights, red and green, $\Sigma = \{G, R\}$, the output alphabet captures all combinations of lights flashed by (A), (B) and (C), $O = \{GGG, GGR, GRG, GRR, RGG, RGR, RRG, RRR\}$, and the output function $f : Q \to O$, modeling all combinations of green/red lights flashed by (A), (B) and (C), is determined by the following conditions:[6]

$$f(ijk) = XYZ, \text{ with probability } 1/4, \text{ for } ijk \in \{122, 212, 221\},$$
$$XYZ \in \{RRR, RGG, GRG, GGR\},$$
$$f(ijk) = XYZ, \text{ with probability } 0, \text{ for } ijk \in \{122, 212, 221\},$$
$$XYZ \in \{GRR, RGR, RRG, GGG\},$$
$$f(111) = XYZ, \text{ with probability } 0, \text{ for } XYZ \in \{RRR, RGG, GRG, GGR\},$$
$$f(111) = XYZ, \text{ with probability } 1/4, \text{ for } XYZ \in \{GRR, RGR, RRG, GGG\}.$$

Again, the transition function $\delta : Q \times \Sigma \to Q$ is not specified. We get a class of Mermin GHZ automata

$$\mathcal{M}(GHZ) = (Q, \Sigma, O, \delta, (p_{ijk}^{XYZ}, i, j, k = 1, 2, X, Y, Z \in \{G, R\})),$$

where $p_{ijk}^{XYZ} = 1/4$, for $ijk \in \{122, 212, 221\}$, $XYZ \in \{RRR, RGG, GRG, GGR\}$ or $i = j = k = 1$, $XYZ \in \{GRR, RGR, RRG, GGG\}$, and $p_{ijk}^{XYZ} = 0$, for $ijk \in \{122, 212, 221\}$, $XYZ \in \{GRR, RGR, RRG, GGG\}$ or $i = j = k = 1$, $XYZ \in \{RRR, RGG, GRG, GGR\}$.

Is there any Mermin's GHZ automaton which can be decomposed into three identical, spatially separated, probabilistic automata with identical initial values? Rephrased, are there three probabilistic automata

$$\mathcal{M}_i = (\{1, 2\}, \{G, R\}, \{G, R\}, \delta_i, (\alpha_{i,j}^X, j = 1, 2, X \in \{G, R\}))$$

such that their direct product $\mathcal{M}_1 \otimes \mathcal{M}_2 \otimes \mathcal{M}_3$ is isomorphic to a Mermin's automaton $\mathcal{M}(GHZ)$:

$\delta(ijk, XYZ) = \delta_1(i, X)\delta_2(j, Y)\delta_3(k, Z)$ and $p_{ijk}^{XYZ} = \alpha_{1,i}^X \alpha_{2,j}^Y \alpha_{3,k}^Z$, for all $j = 1, 2, X, Y \in \{G, R\}$?

The answer is again *negative:*

> *no single state of any Mermin's GHZ probabilistic automaton $\mathcal{M}(GHZ)$ can be simulated by the product of the corresponding states of any probabilistic automata $\mathcal{M}_i$.*

We have $\alpha_{i,j}^G = 1 - \alpha_{i,j}^R$. Take the output $XYZ = GGR$. As $p_{111}^{GGR} = 0$ we deduce that

$$\alpha_{1,1}^G \alpha_{i2,1}^G (1 - \alpha_{3,1}^G) = 0,$$

which contradicts the system of equalities

$$p_{122}^{GGR} = p_{212}^{GGR} = p_{221}^{GGR} = 1/4,$$

and the same conclusion can be derived for any output.

Again, due to strong correlations, every Mermin's GHZ probabilistic automaton $\mathcal{M}(EPR)$ cannot be decomposed as a direct product of three independent probabilistic automata, no matter what transitions and output functions.

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[6] Note that the following conditions do not determine uniquely the output function.

5 Correlation for Local Parallel Composition of Deterministic Mealy Automata

First we deal with Mermin EPR device. To this aim we discuss a configuration in which two identical deterministic Mealy automata[7] $\mathcal{M}_1$ and $\mathcal{M}_2$ with unknown but identical initial states are detected in (D1) and (D2), respectively.

More precisely, let us assume that each automaton $\mathcal{M}_j$, $j = 1, 2$, has three states $Q = \{1, 2, 3\}$, the input alphabet $\Sigma = \{1, 2, 3\}$, the output alphabet $O = \{G, R\}$, as well as a(n) (irreversible, i.e., many-to-one) transition function $\delta_j(q, i) = i$ and output function $\lambda_j(q, i) = G$, if $q = i$ and $\lambda_j(q, i) = R$, otherwise; $q \in Q$ and $i \in \Sigma$. Let us further assume that there is an equidistribution of initial states, i.e., each one occurs with equal probability $1/3$.

We can construct a joint output function by the cartesian product $\lambda : Q \times \Sigma \to O \times O$, $\lambda(q, i) = (\lambda_1(q, i), \lambda_2(q, i))$.

Since both $\mathcal{M}_1$ and $\mathcal{M}_2$ are in an identical initial value, there are just three allowed categories GRR, RGR, RRG out of the conceivable ones $GGG, GGR, GRG, GRR, RGG, RGR, RRG, RRR$.

A straightforward combinatorial argument shows that with these assumptions one obtains the following probabilities:

$$\lambda(i, i) = GG, \text{ with probability } 1/3, \text{ for } i = 1, 2, 3,$$
$$\lambda(i, i) = RR, \text{ with probability } 2/3, \text{ for } i = 1, 2, 3,$$
$$\lambda(i, i) = XY, \text{ with probability } 0, \text{ for } i = 1, 2, 3, \, X, Y \in \{G, R\}, X \neq Y,$$
$$\lambda(i, j) = GG, \text{ with probability } 0, \text{ for } i, j = 1, 2, 3, i \neq j,$$
$$\lambda(i, j) = GR, \text{ with probability } 1/3, \text{ for } i, j = 1, 2, 3, \, i \neq j,$$
$$\lambda(i, j) = RG, \text{ with probability } 1/3, \text{ for } i, j = 1, 2, 3, \, i \neq j,$$
$$\lambda(i, j) = RR, \text{ with probability } 1/3, \text{ for } i, j = 1, 2, 3, \, i \neq j.$$

The automata flash the same colour (red) $1/3$ of the time and different colours $2/3$ of the time. This is not exactly the classical case as discussed by Mermin, but it comes close to it in terms of classicality and locality of the automata arrangement. To understand why, let us define the notion of *correlation function* in the automaton context. Assume again two output symbols, say R and G, and three input symbols, say $1, 2$ and 3.

Associate the numbers $n_t(i, \mathcal{M}_j) = +1$ and $n_t(i, \mathcal{M}_j) = -1$ with the outcomes R and G of the experiment with input i at discrete time t, respectively. In analogy to physical correlation functions [15] we can define a correlation function C as the weighted average over the product of the numbers associated with the outcomes of the first and second automata $\mathcal{M}_1, \mathcal{M}_2$, i.e.,

$$C(i, j) = \frac{1}{N} \sum_{t=1}^{N} n_t(i, \mathcal{M}_1) \cdot n_t(j, \mathcal{M}_2).$$

We always get $-1 \leq C(i, j) \leq +1$. In the above case, for identical inputs, $C(i, i) = 1$, $i = 1, 2, 3$. For nonidentical input $i \neq j$, $C(ij) = -1/3$. The "Bell inequality" [5] is considered a measure for classicality and locality; in particular

$$|C(1, 2) - C(1, 3)| \leq 1 + C(2, 3). \tag{1}$$

is always satisfied for classical systems. The automaton correlation functions always satisfy this inequality and the others obtained by permuting the inputs. This is an indication (although no sufficient condition) that the corresponding classical system behaves locally in the sense used in physics. That is, no causal influence such as a light signal originating from a measurement on one particle can influence the measurement on the other particle and vice versa. This comes as no

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[7] In a Mealy automaton the output function depends both on the current state and input letter.

surprise, because the way the two-automaton setup was conceived, both automata are causally separated in a classical sense.

These results are independent of the particular transition function δ involved, provided it is not a permutation (one-to-one).

A local automaton realization which comes closer to the configuration discussed by Mermin is the following one. Assume again two Mealy automata $\mathcal{M}_j$, $j = 1, 2$, with four states $Q = \{1, 2, 3, 4\}$, the input alphabet $\Sigma = \{1, 2, 3, 4\}$, the output alphabet $O = \{G, R\}$, as well as a(n) (irreversible, i.e., many-to-one) transition function $\delta_j(q, i) = i$ and output function given by the following table $(q \in Q,\ i \in \Sigma)$:

q/i	1	2	3	4
1	G	G	G	R
2	G	R	R	G
3	R	G	R	G
3	R	R	G	R

Let us further assume that there is an equidistribution of initial values; i.e., each one occurs with equal probability $1/4$.

We can construct a joint output function by the cartesian product $\lambda : Q \times \Sigma \to O \times O$. Both automata $\mathcal{M}_1$ and $\mathcal{M}_2$ are in an identical initial value, but now all conceivable categories $GGG, GGR, GRG, GRR, RGG, RGR, RRG, RRR$ are allowed.

A straightforward combinatorial argument shows that with these assumptions one obtains the following probabilities:

$$\lambda(i, i) = GG,\ \text{with probability } 1/2,\ \text{for } i = 1, 2, 3,$$
$$\lambda(i, i) = RR,\ \text{with probability } 1/2,\ \text{for } i = 1, 2, 3,$$
$$\lambda(i, i) = XY,\ \text{with probability } 0,\ \text{for } i = 1, 2, 3, X, Y \in \{G, R\}, X \neq Y,$$
$$\lambda(i, j) = GG,\ \text{with probability } 1/4,\ \text{for } i, j = 1, 2, 3, i \neq j,$$
$$\lambda(i, j) = GR,\ \text{with probability } 1/4,\ \text{for } i, j = 1, 2, 3, i \neq j,$$
$$\lambda(i, j) = RG,\ \text{with probability } 1/4,\ \text{for } i, j = 1, 2, 3, i \neq j,$$
$$\lambda(i, j) = RR,\ \text{with probability } 1/4,\ \text{for } i, j = 1, 2, 3, i \neq j.$$

The automata flash the same colour (red) $1/2$ of the time. This corresponds exactly to the classical case as discussed by Mermin. In this case we get the correlation $C(i, j) = 0$ for $i \neq j$, then the "Bell inequality"

$$|C(1, 2) - C(1, 3)| \leq 1 + C(2, 3),$$

(and the ones obtained by permutating the inputs) is always satisfied.

An automaton realization which comes close to Mermin's treatment of the GHZ experiment can be given by three identical automata $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$ with identical initial value, given by the following table $(q \in Q,\ i \in \Sigma,\ o \in O)$:

$q/i, o$	1	2	1	2
1	1	1	R	R
2	1	1	R	G

Here, in configurations like 122, there always occurs an odd number of R's, whereas for 111, only a single result RRR emerges, which has an odd number of R's and is distinct from the quantum mechanical result containing an even number of R's.

Again, the argument is independent of the transition function as long as it is not a permutation.

6 Final Remarks

Thus, in summary, insofar models are used which implement a parallel composition of "spatially" separated, noncommunicating single automata, the associated correlations obey Bell's inequality.

Yet insofar as nonseparated, global automaton models are introduced which are holistic in the sense that they cannot be parallel decomposed into proper parts, then the associated Bell-type automaton inequalities are violated; very much in the same way as in the case of its quantum mechanical double.

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