CDMTCS
Research
Report
Series

Abstracts of the 2nd Japan–New Zealand Workshop on Logic in Computer Science

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The 2nd Japan - New Zealand Workshop on Logic in Computer Science

Date: October 22, 1998
When: 1:30 – 5:00pm
Venue: Room 133 (C.S. tutorial room)

University of Auckland, New Zealand

1:30 – 2:00

Peter Schuster.
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Constructive non-nonstandard analysis without choice.

We investigate how non-nonstandard reals can be established constructively without countable choice as arbitrary infinite sequences of rationals, following the classical approach due to C. Schmieden and D. Laugwitz. A total standard part map into F. Richman's generalised Dedekind reals is constructed, and discontinuities of bounded functions are smoothed.

2:05 – 2:35

Hajime Ishihara.
<ishihara@jaist.ac.jp>

Hyperextensional functions between metric spaces.

We define a hyperextensional fuction between metric space and show a function between metric spaces is sequentially continuous if and only if it is nondiscontinuous and hyperextensional.

2:40 – 2:55

Asat Arslanov.
<aars001@cs.auckland.ac.nz>

On the question of G. Chaitin.

We discuss the following question of G. Chaitin: Which Turing degrees contain random sequences and which Turing degrees don't contain any random sequences? We give a partial answer to this question by constructing a random sequence in every jump class.

Theorem. For every Turing degree $\mathbf{a} \geq \mathbf{O}'$ there exists a random sequence X such that $deg'_T(X) = \mathbf{a}$.

3:00 – 3:15

Douglas Bridges* and Ayan Mahalanobis.
<douglas@waikato.ac.nz>

Monotone, Bounded Variation, and Regulated Functions – Constructively.

I shall talk about monotone functions and functions of bounded variation (b.v.) in constructive analysis. Among the results discussed are the following, in which the function f is always strongly extensional: that is, if $f(x) \neq f(y)$, then $x \neq y$.

Theorem.
If the (total) variation of f exists on a compact interval $I = [a, b]$, then it exists on any compact subinterval of I. If also f is sequentially continuous at $\xi \in I$, then the variation function is continuous at ξ.

Theorem.
Let f be an increasing function on an interval I in $\mathbf{R}$, let J be a proper compact interval contained in the interior of I, and let $\varepsilon > 0$. There exists a finitely enumerable set $\{x_1, \ldots, x_M\}$ of points of J such that if $x \in J^\circ$ and $x \neq x_n$ for each n, then $f(x^+) - f(x^-) < \varepsilon$.

Theorem.
If f is of bounded variation on the interval I, then it is regulated.

The last two of these results were obtained jointly with Ayan Mahalanobis.

3:20 – 3:45

Break.

3:45 – 4:00

Bakh Khoussainov.
<bmk@cs.auckland.ac.nz>

On Algorithmic Dimensions of Structures.

One of the basic issues in the theory computable data types concerns the relationship between computable presentations of structures. A fundamental notion in the study of this issue is the notion of computable isomorphism first introduced by Malcev in 1961. Two computable presentations are computably isomorphic if there exists a computable isomorphism that transforms one presentation into the other. The maximal number of pairwise noncomputably isomorphic computable presentations of a structure A is the algorithmic dimension of A, denoted by dim(A). The algorithmic dimensions of basic structures that arise in computer science equal to 1. However, for any nonzero cardinal $n \leq \omega$ there are structures common to mathematics such as groups, rings, lattices, graphs whose algorithmic dimensions are equal to n. In this talk we discuss issues related to algorithmic dimensions of structures.

4:05 – 4:20

Michael J. Dinneen.
<mjd@cs.auckland.ac.nz>

Designing and Using Graph Testset Congruences.

We present a practical theory for using t-boundaried graph testsets (for graphs of bounded combinatorial width). In addition to giving us linear-time (finite-state) membership algorithms, we can use, where applicatable, the generated automata to quickly find minor-order obstructions. We will state several open problems.

We give four simple testset examples: graph connectedness, k-maximum path, hamiltonian graphs, and k-feedback vertex set.

4:25 – 4:40

Douglas Bridges and Luminiţa Dediu*.
<simona@hoiho.math.waikato.ac.nz>

A Hahn-Banach Type Theorem for Ultraweakly Continuous Linear Functionals.

Let $\mathcal{R}$ be a linear subset of the space $\mathcal{B}(H)$ of bounded operators on a Hilbert space with an orthonormal basis. Within Bishop's constructive mathematics we prove that if the unit ball of $\mathcal{R}$ is weak-operator totally bounded, then an ultraweakly continuous linear functional on $\mathcal{R}$ extends to one on $\mathcal{B}(H)$, and that the extended functional has the form

$$T \mapsto \sum_{n=1}^{\infty} \langle Tx_n, y_n \rangle,$$

where $\sum_{n=1}^{\infty} \|x_n\|^2$ and $\sum_{n=1}^{\infty} \|y_n\|^2$ are convergent series in $\mathbf{R}$.

4:45 – 5:00

Cristian Calude, Richard Coles*, Peter Hertling, Bakh Khoussainov.
<coles@cs.auckland.ac.nz>

Representations of Computably Enumerable Reals.

Computability theory studies the relative computability of sets of natural numbers. By coding mathematical structures into the natural numbers, computability theory can be applied to many areas of mathematics. One such application almost as old as computability theory itself is to real analysis.

In this talk we will consider effective versions of Cauchy sequences to define the computable and computably enumerable (c.e.) reals. A natural way to represent a c.e. real is by an increasing convergent computable sequence of rationals. There are many such representations for a c.e. real, and we will take a look at the relative computability of them.
