Review of[1]
The Mathematics of Voting and Elections: A Hands-On Approach
by Jonathan K. Hodge and Richard E. Klima
American Mathematical Society (Mathematical World series, volume 22)
226 + xiv pages, softcover

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1 Introduction

The mathematics of collective decision-making ("social choice theory") has been studied for centuries, and until recently almost exclusively by political scientists and economists. The main areas of study concern protocols for aggregating the preferences of individual agents (of which voting is the most common type) and for fair division of resources.

In the last decade these topics, and areas of economics and political science such as mechanism design and game theory more generally, have become interesting to computer scientists. Algorithmic game theory in particular has attracted much research energy from theoretical computer scientists.

The field of Computational Social Choice has developed considerably over the last decade. For example,

- considering search engines as voters and web pages as candidates has proved fruitful;
- conferences on multiagent systems in artificial intelligence, and on electronic commerce, typically have many papers on voting rules;
- the computational complexity of various operations such as determining the winner or manipulating the result of the election has been the subject of much research;
- quantitative versions of famous social choice results such as Arrow's Theorem and the Gibbard-Satterthwaite Theorem have been proved;
- new international workshop series have started, such as COMSOC and ADT.

I see a clear need for teaching materials aimed at introducing this area of research to beginners and outsiders. No single book exists for this purpose. It is probably best to begin with a book on social choice and supplement with research articles. Most books on (classical) social choice theory are research monographs, some rather old, and not well suited to classroom use. The book under review begins to fill this gap.

2 Summary

The book is organized into 10 chapters. The core of the book consists of Chapters 1–5 and the remaining chapters depend very little on these or on each other. Chapter 1 discusses elections

with 2 candidates, desirable features of voting rule, and some possible rules. Then majority rule is treated in detail, including its characterization by May's theorem. Chapter 2 introduces elections with 3 or more candidates, and concentrates on the plurality and Borda rules. Chapter 3 deals with the Condorcet criterion and paradox, focusing on sequential pairwise rule and the Instant Runoff (also called Single Transferable Vote) rule. Chapter 4 considers the Independence of Irrelevant Alternatives criterion, and states and discusses in detail various versions of Arrow's famous impossibility theorem. Chapter 5 gives proofs of Arrow's theorem and looks at Approval Voting. Chapter 6–8 deal with weighted voting rules. Chapter 6 covers the basic definitions and properties, while Chapter 7 is concerned with voting power, and the Banzhaf and Shapley-Shubik indices. Chapter 8 covers the United States Electoral College in detail. Chapter 9 focuses on paradoxes around referenda. Chapter 10 is concerned with apportionment ("proportional" allocation of seats in a legislative body), its history and paradoxes.

The authors have tried to keep the spirit of a class taught via the "Moore" (Socratic) method. They have taught such a class for undergraduates with a wide range of backgrounds. A notable feature is the use of Questions rather than worked examples or exercises. Some of these Questions come with Answers, and many are open-ended and require substantial work by the reader.

3 Opinion

The book succeeds admirably in presenting material to its intended audience, which is, roughly speaking, North American undergraduates in the final two years of a degree who have a general interest in the topic. Explanations are careful and detailed, and the questions are very well chosen, often containing a wealth of interesting detail on voting systems used in practice. The material is developed logically and with regard to what the reader can absorb. The mathematical requirements are minimal and much of the book would be appropriate for interested high school students. It would also be very appropriate for more advanced students using it for self-study. The text is very clear and free from errors (there is an errata page at http://www.ams.org/bookpages/mawrld-22/errata1.pdf). The table of contents, index and typesetting are all good. Although some of the cultural references aimed at the target audience may be confusing to a wider audience, even these are interesting. This book should be considered by anyone teaching social choice, as a main textbook or supplemental reading.

Of course, no book can serve all audiences. For computer science students and (potential) researchers interested in proceeding further, clearly some extra reading will be required. We can only hope that a book as well written as this one can fill the gap soon. One possible minor difficulty would be that the book under review occasionally uses idiosyncratic (or at least nonstandard) terminology. Such common terms as social welfare function and profile do not appear (instead, we have societal preference order and preference schedule).

The book's topics are chosen with good taste, but of course some important ones are omitted. Standard topics not mentioned, or only mentioned in passing, and which could have been included without excessive technical requirements on the reader, include single-peaked preferences, the median voter theorem, and Condorcet's jury theorem. The most obvious omission (to this reviewer) is any discussion of manipulation, bribery, control, and strategic behaviour in general (apart from a brief mention of agenda control in sequential pairwise voting). This is a substantial part of research in the computational social choice community.

List of recommended further reading after finishing the book under review:

- A Primer in Social Choice Theory by Wulf Gaertner (revised edition), Springer, 2009.
- Handbook of Social Choice and Welfare, volume 1, Elsevier, 2002.
- Mathematics and Voting by Donald G. Saari, American Mathematical Society Notices, April 2008.

  Available from http://www.ams.org/notices/200804/tx080400448p.pdf.
- A Short Introduction to Computational Social Choice by Yann Chevaleyre, Ulle Endriss, Jérôme Lang and Nicolas Maudet, Proceedings of SOFSEM 2007.

  Available from http://staff.science.uva.nl/ ulle/pubs/files/ChevaleyreEtAlSOFSEM2007.pdf.