



Surface Curvature Maps and Michelangelo's David

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1) Curvature

Planar curves

Surfaces

2) The “David” dataset

Significance

Data structure

Challenges

3) Experimental results

Curvature calculations

Depth maps

Curvature maps

Curvature: Planar curves.

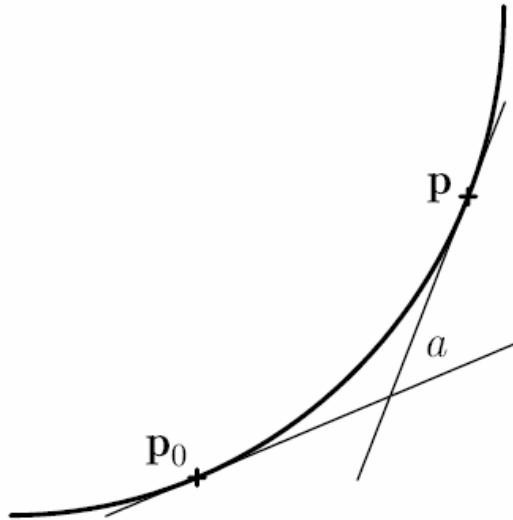


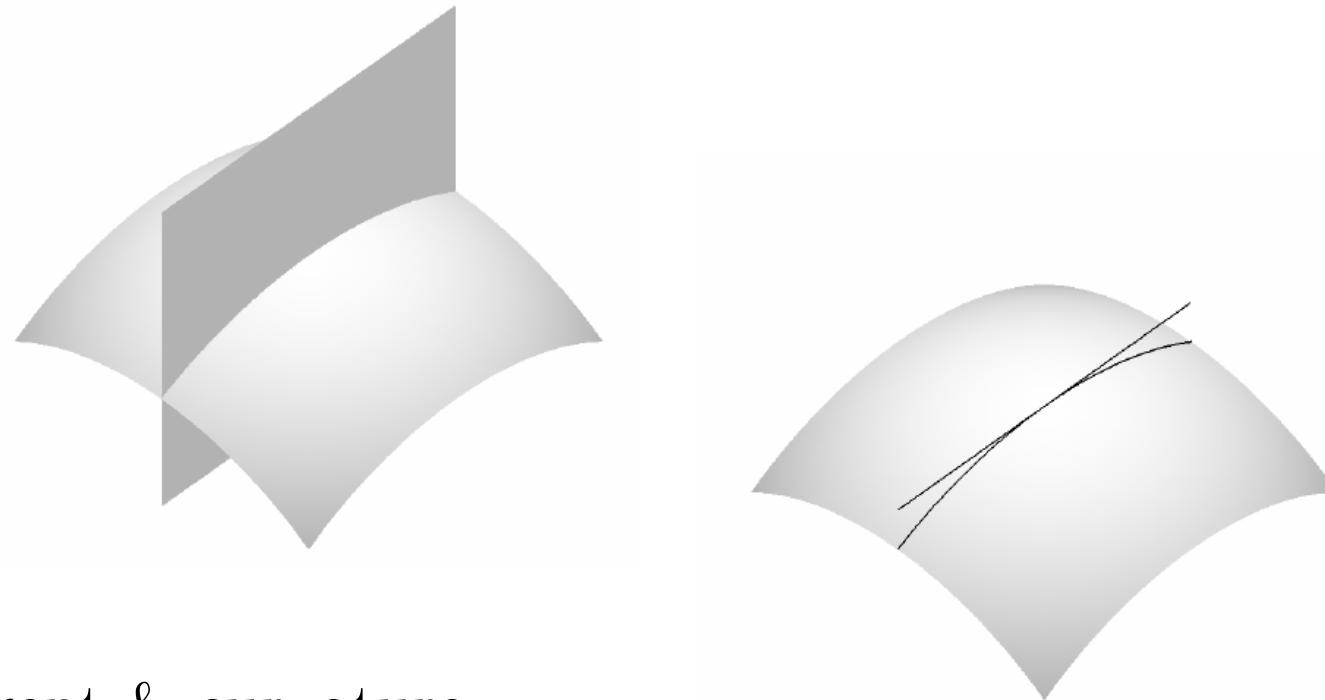
FIGURE 1. Curvature.

Consider a planar curve with arc-length l from the point p_0 to the point p , and the counter-clockwise angular advance a between the tangents at p_0 and p as illustrated, for example, in figure (1). Then the *curvature* of the curve at the point p_0 is defined to be

$$k = \lim_{p \rightarrow p_0} \frac{a}{l} = \frac{da}{dl}$$

Surfaces.

The intersection of surface and plane is a plane curve in the surface.

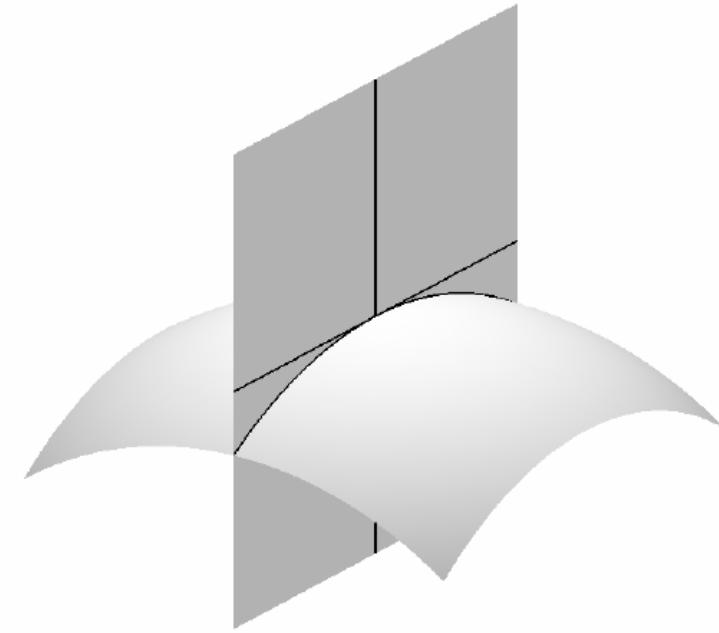
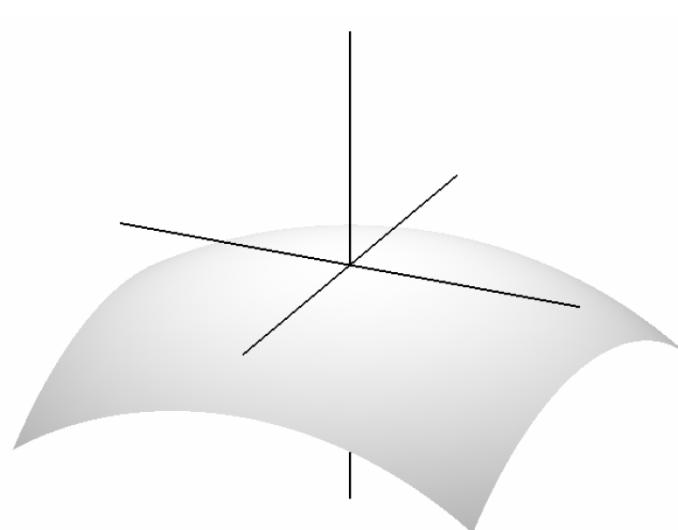


Tangent & curvature.

Surfaces.

Normal curvature.

Unique for each tangent direction!



Surfaces.

The two *principle curvatures* k_1 and k_2 are defined as the minimum and maximum normal curvatures.

The *mean curvature* is defined as: $\frac{k_1 + k_2}{2}$

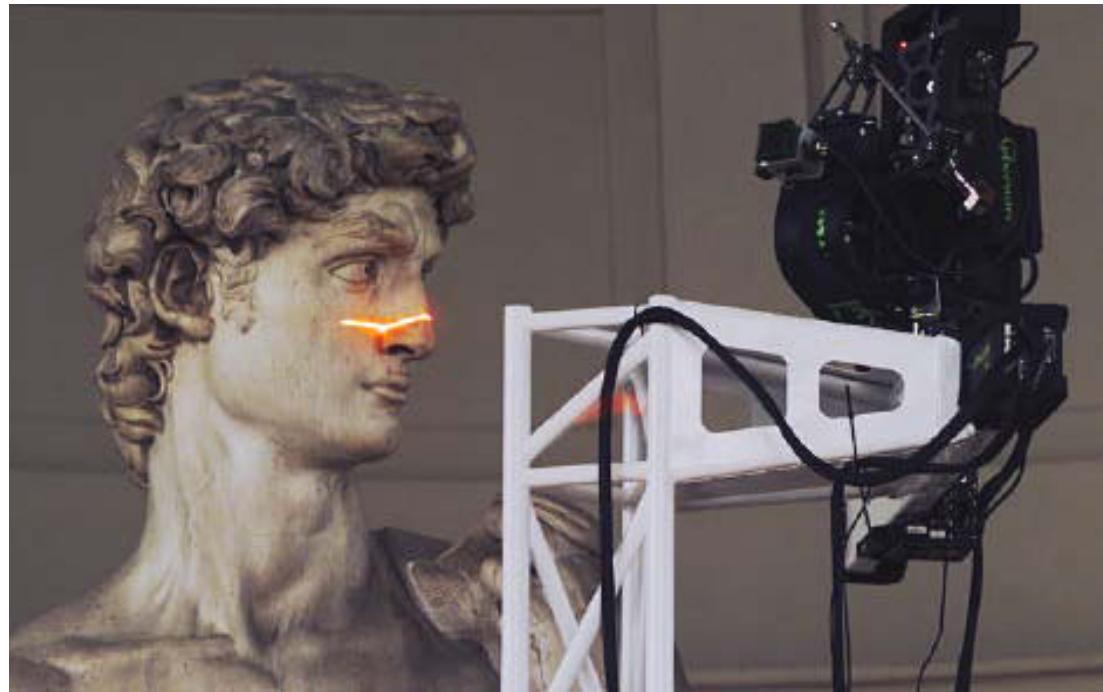
Surfaces.

For mean curvature, the normal curvatures associated with any two orthogonal *cuts* will work!

$$H = \frac{k_1 + k_2}{2}$$

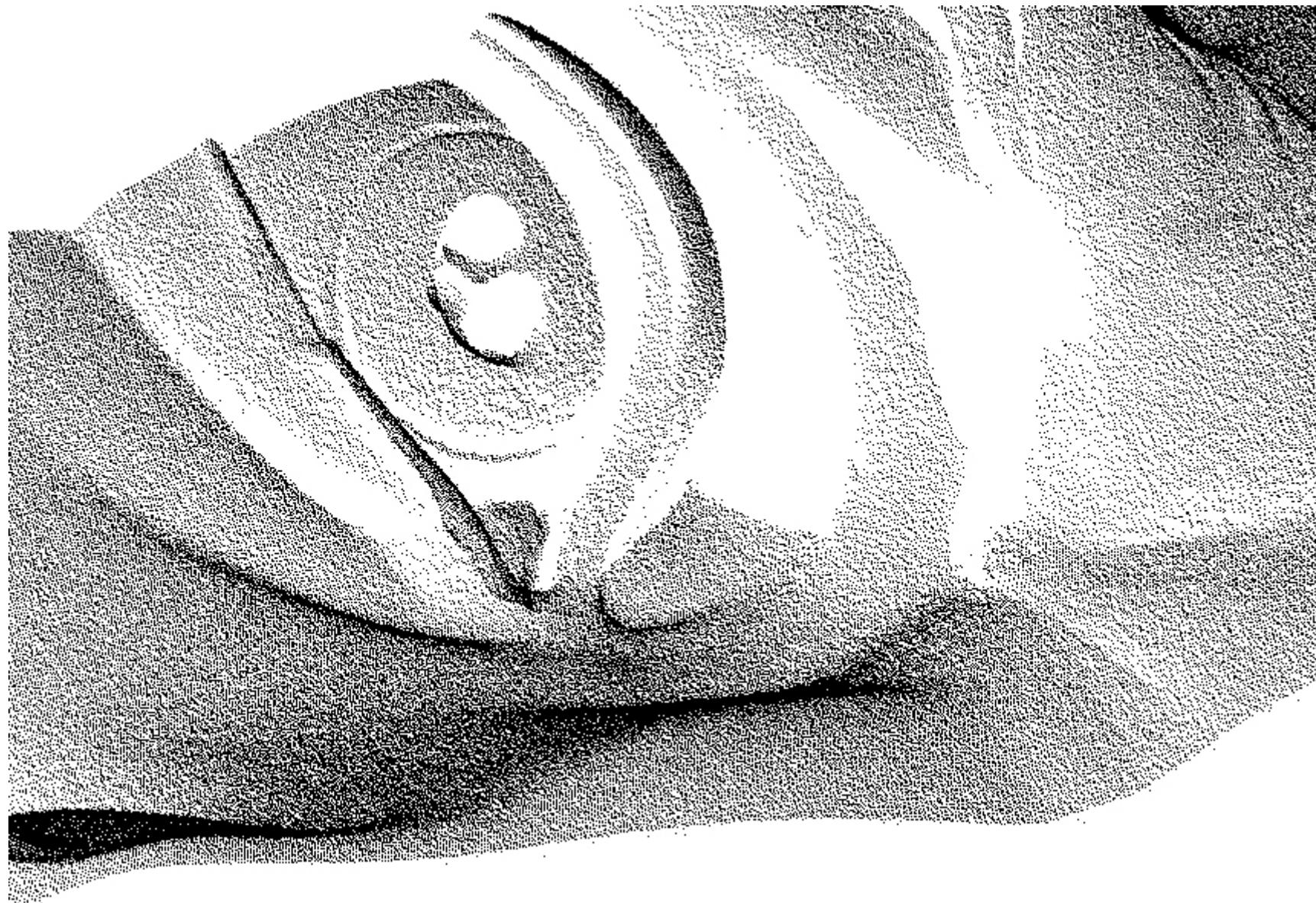


Michelangelo's David was digitized in 1998-99 by a team of 30 faculty, staff, and students from the Graphics Laboratory at University of Stanford using a custom designed laser triangulation scanner.



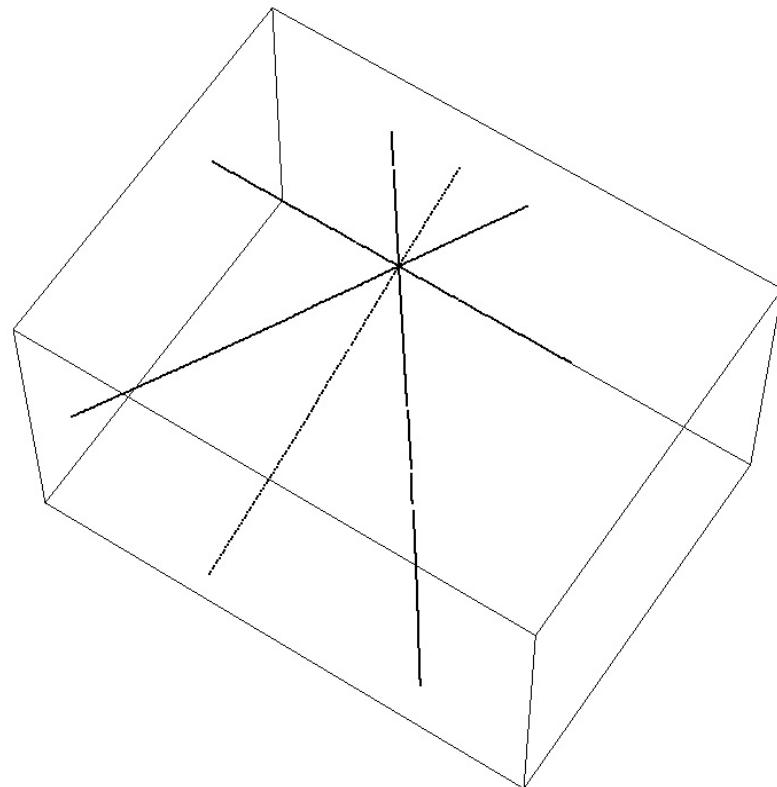
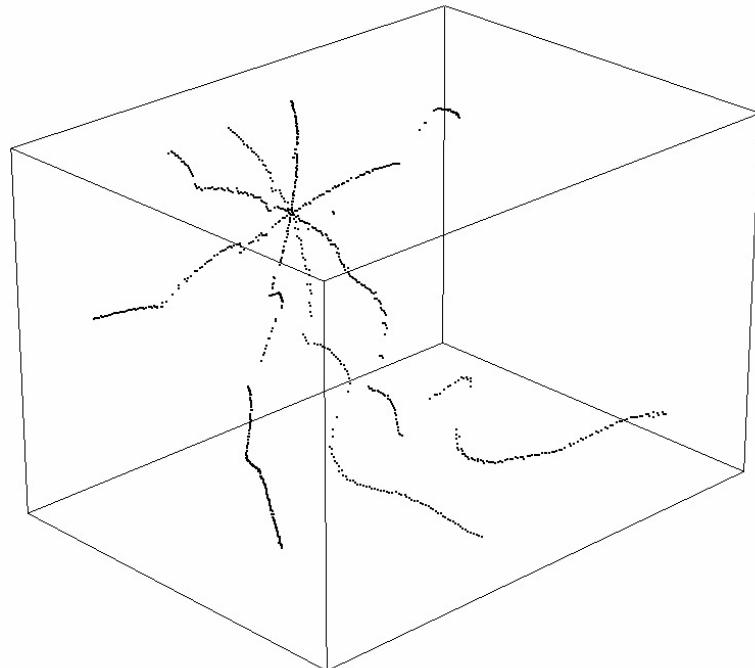
1.93 giga-bytes data, 6540 scan files, ~1.1 billion 3D points!

Scan point visualization using C++ and OpenGL.

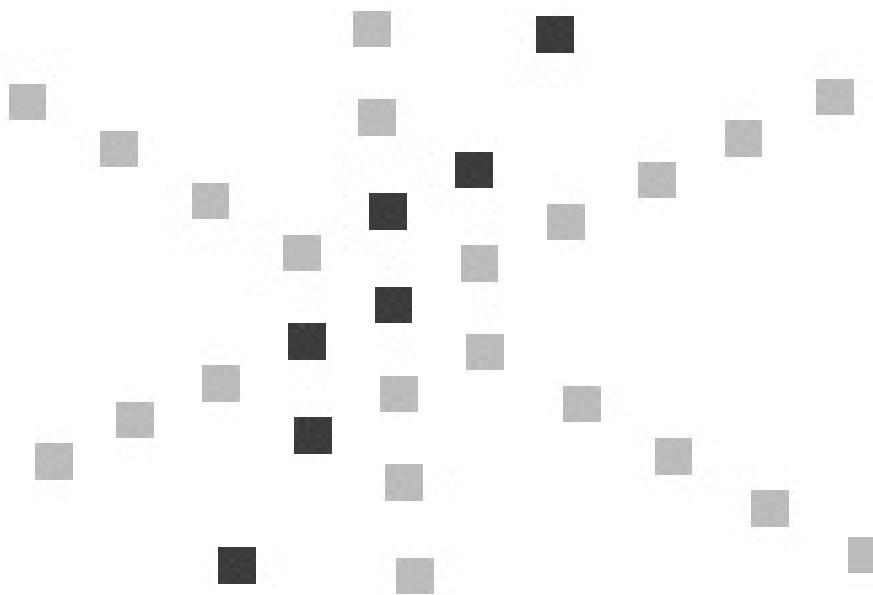


The David statue:

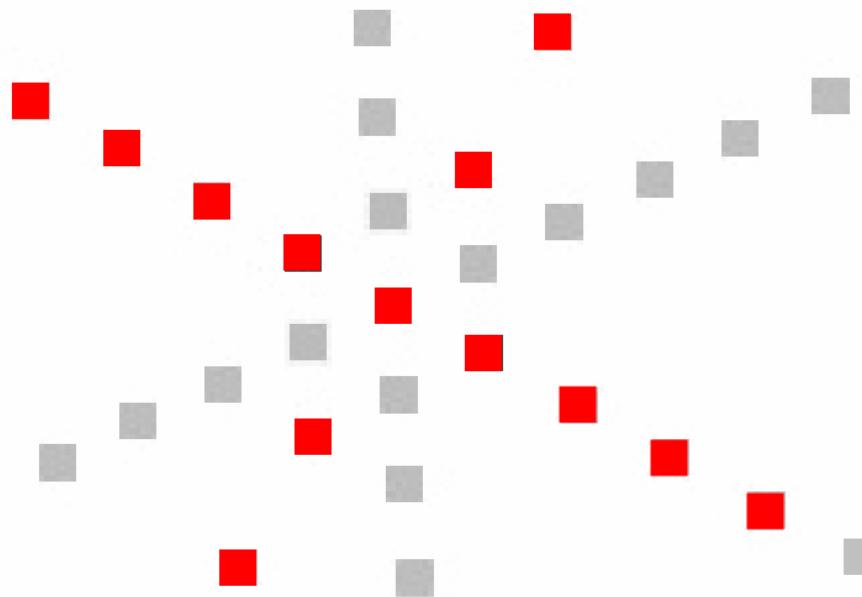
Two different views of four cuts through the same point in scan file 14.



Hexagonal adjacency pattern and interleaved acquisition.



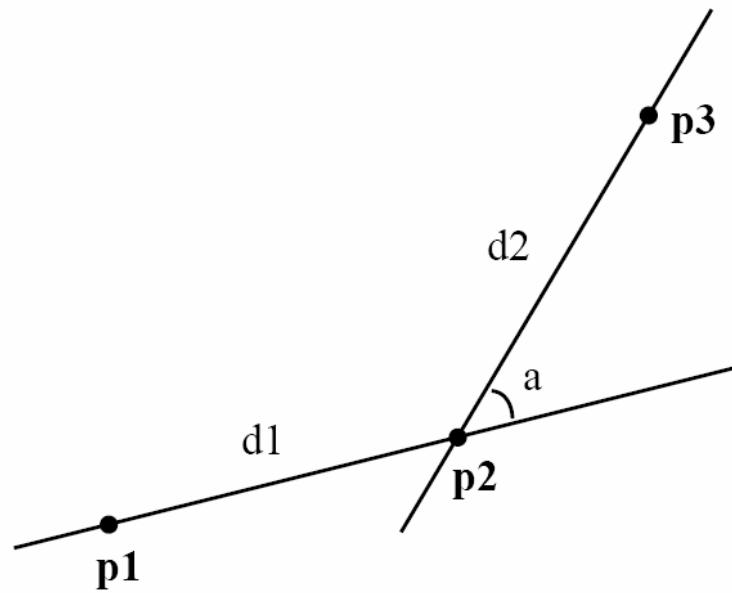
Cut selection for curvature calculations:



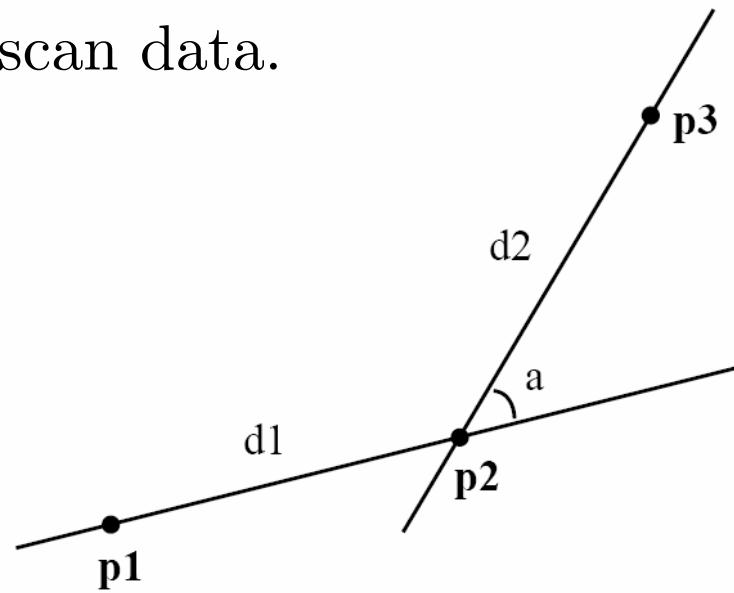
Curvature *estimation* from scan data.

Planar curvature:

$$k = \frac{a}{(d_1 + d_2)/2}$$



Curvature calculation from scan data.



$$k = \left(\frac{2}{||\mathbf{v1}|| + ||\mathbf{v2}||} \right) \cos^{-1} \left(\frac{\mathbf{v1} \cdot \mathbf{v2}}{||\mathbf{v1}|| ||\mathbf{v2}||} \right)$$

where $\mathbf{v1} = \mathbf{p2} - \mathbf{p1}$ and $\mathbf{v2} = \mathbf{p3} - \mathbf{p2}$

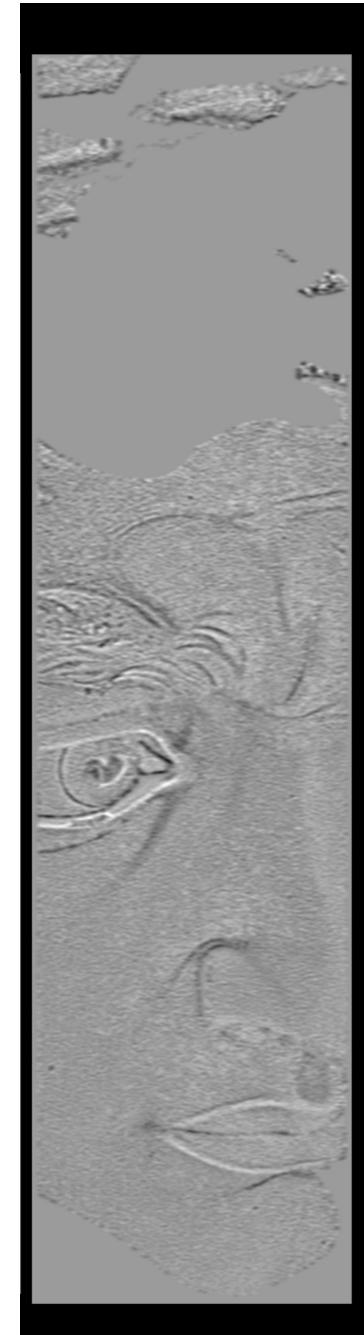
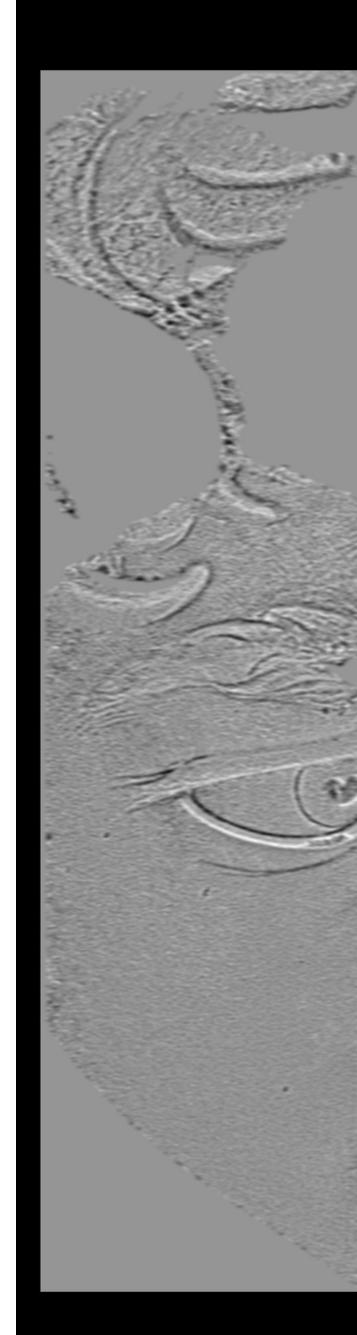
Results:

Curvature maps of scans 13 and 14.

Maximum positive curvature is shading coded as white.

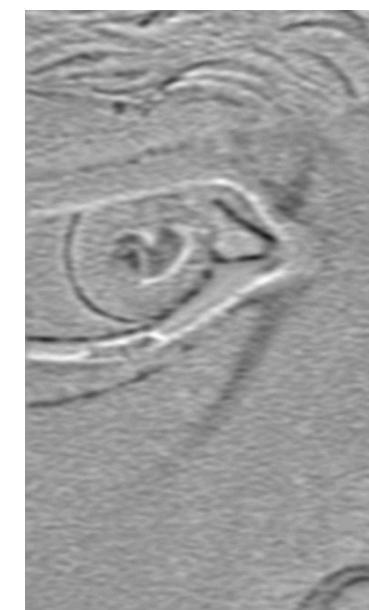
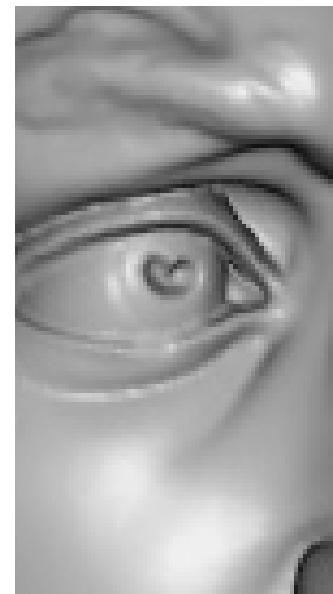
Zero curvature is shading coded as medium grey and maximum negative curvature is encoded as black.

Out of range points are encoded as medium grey.



Conclusion:

The curvature maps extract surface detail (such as the chip in the lower eyelid of David's right eye!) that is not as readily apparent in the photographic image or in the reference rendering.



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References:

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