Diophantine Representation for $\boldsymbol{\Omega}$

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The Ω number, see [2] is one of the most important concepts in Algorithmic Information Theory [1]. Chaitin has presented a first exponential Diophantine representation for a natural Ω number in [2]. His equation was automatically generated from a complex register machine program and is very large, e.g. it has approximately 17,000 unknowns. While it is has been shown that this can be reduced to just three [3], doing so would be a very challenging task.

In [4], Ord and Kieu have shown how to determine the kth bit of Ω by solving k instances of the halting problem. From this they reduce the problem of determining the kth bit of Ω to determining whether a certain Diophantine equation with two parameters, k and N, has solutions for an odd or an even number of values of N. They further construct an exponential Diophantine equation with a parameter k which has an odd number of solutions iff the kth bit of Ω is 1, and a polynomial of positive integer variables and a parameter k that takes on an odd number of positive values iff the kth bit of Ω is 1.

The projects seeks to improve the Ord and Kieu constructions and to write a program to automatically generate "smaller" Diophantine representations for Ω.

References

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[3] Y. V. Matiyasevich. *Hilbert's Tenth Problem.* MIT Press, Cambridge, Massachusetts, 1993.

[4] T. Ord and T. D. Kieu. On the existence of a new family of diophantine equations for Ω. *Fundamenta Informaticae*, 56, 3(273-284), 2003.